

Lattice Gauge Theory  
for Physics  
beyond the Standard Model

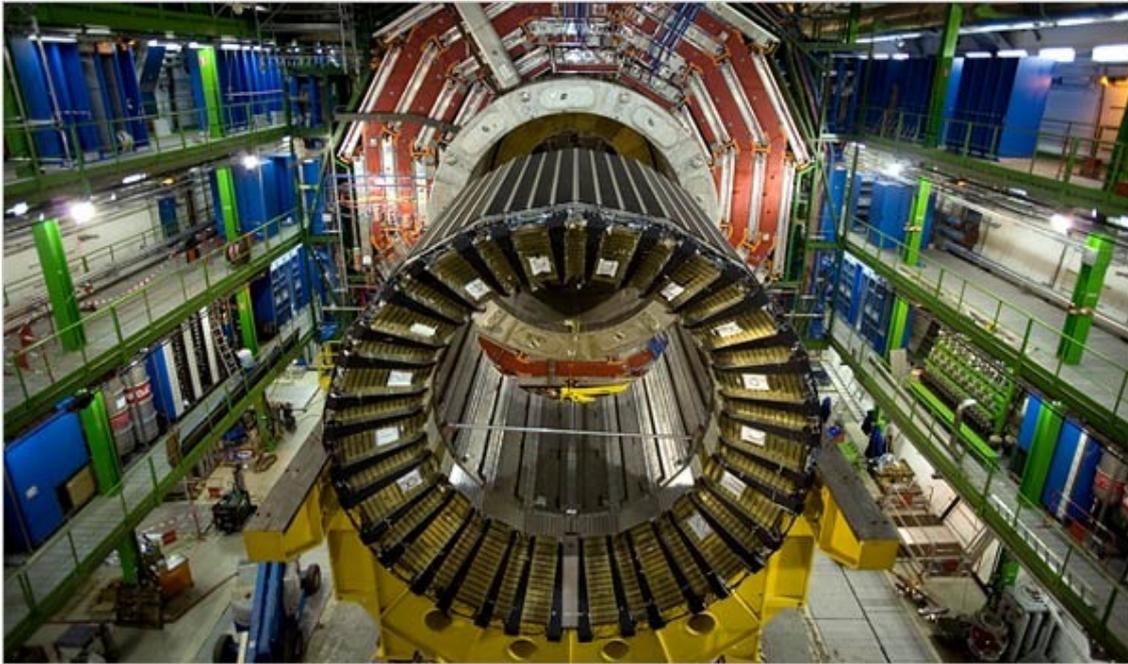
LQCD II @ DOE January 30, 2008

Richard C. Brower

# The New York Times

May 15, 2007

A Giant Takes On Physics' Biggest Questions



The LHC is coming soon!

Goal is to investigate physics of electroweak symmetry breaking.



Theorists have propose a **myriad** of models for TeV physics, often dependent on **heuristics** for non-perturbative effects in gauge theories

**Triage is needed:** Lattice field theory can help narrow the options and predict the consequences of specific models.

# Physics beyond the Standard Model

- Rough classification of LHC experimental scenarios & theoretical proposals:
  - ◆ SM Higgs (or multiple Higgs)
  - ◆ Super Symmetry (e.g. MSSM: Minimal SUSY Standard Model)
  - ◆ New TeV strong dynamics (QCD-like, technicolor, little Higgs)
- What can/should Lattice Gauge Theory contribute to unraveling the new physics? *Explore non-perturbative mechanisms assumed by model builders and predict spectra.*
- Some lattice work has begun in all three areas above, but compared with lattice QCD this project enters *terra incognita*

# Planning 1<sup>st</sup> workshop on Lattice Gauge Theory for Physics beyond the Standard Model

Place: Lawrence Livermore National Laboratory

Date: May 2, 2008.

Organizing Committee:

- ◆ Thomas Appelquist    Yale University
- ◆ Richard Brower        Boston University
- ◆ George Fleming        Yale University
- ◆ Claudio Rebbi         Boston University
- ◆ Ron Soltz              LLNL
- ◆ Pavlos Vranas         LLNL

# Outline of BSM Lattice Projects<sup>†</sup>

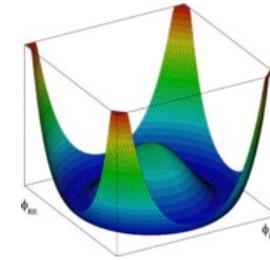
I. Higgs dynamics in the Standard Model

II. Supersymmetric field theories

III. New strong dynamics

# Does NATURE abhor a fundamental SCALAR (HIGGS)?

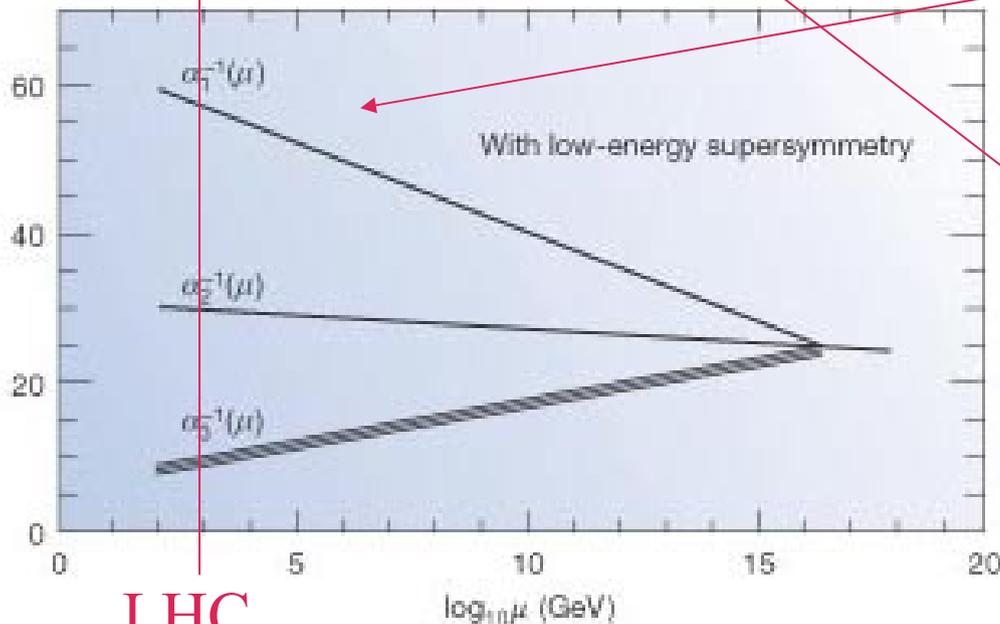
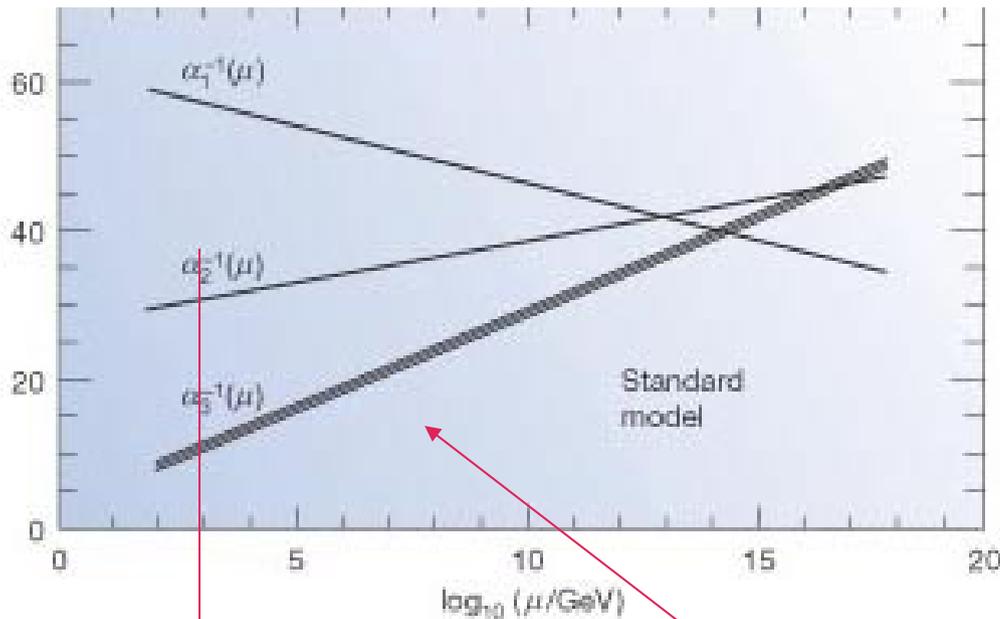
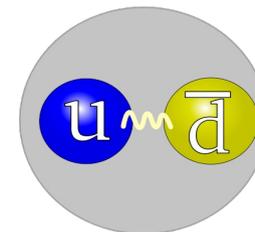
I. **NO:** Only a scalar Higgs



II. **Sort of:** Give the Higgs a “super partner”

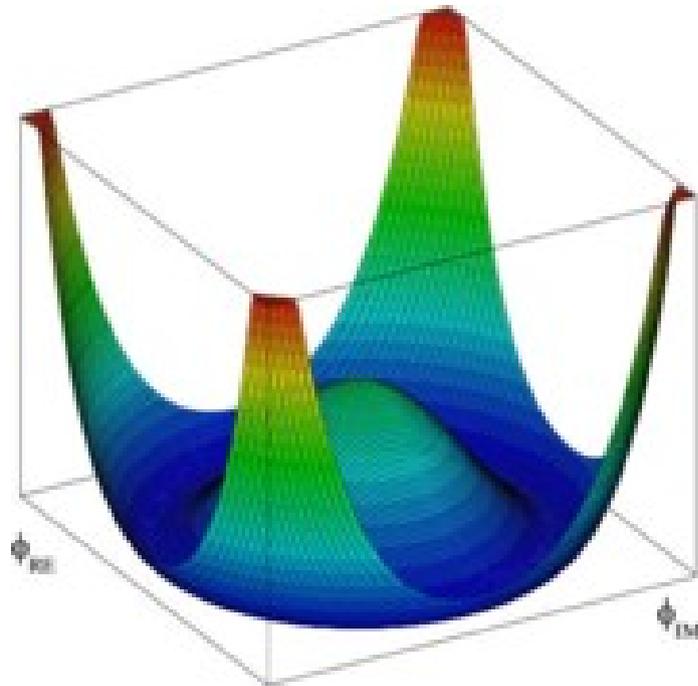
**Spin = 0 & 1/2**

III. **YES:** Build at Higgs from Heavy techni-Quarks!



LHC

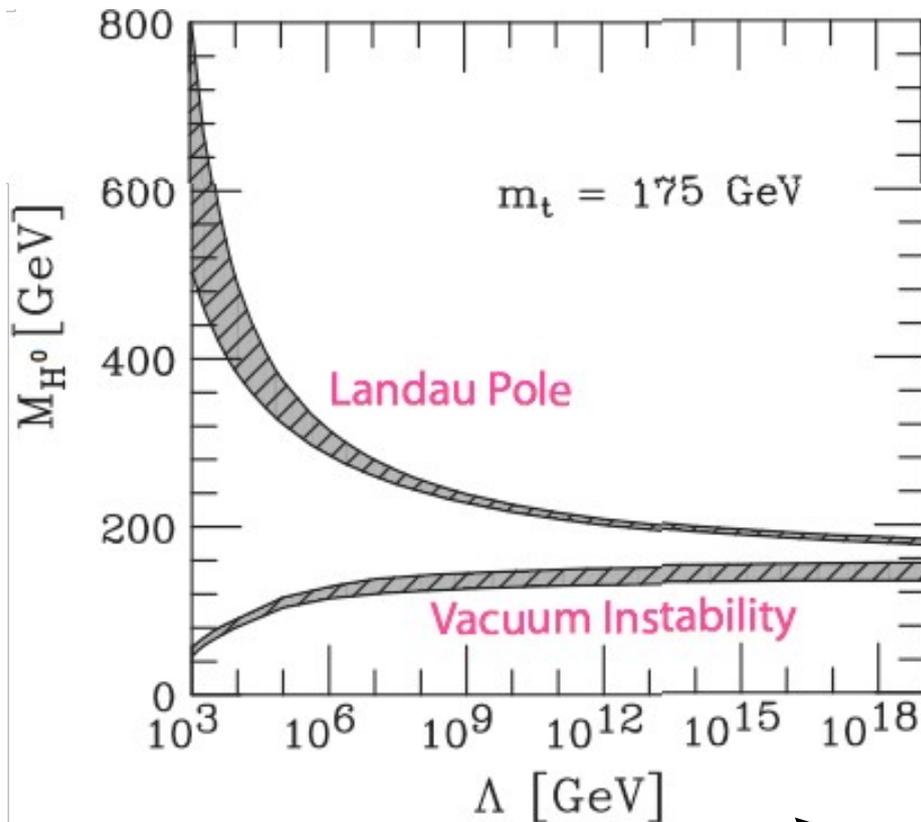
# I. Higgs dynamics in the Standard Model



# What if the LHC “only” finds the SM Higgs?

○ Standard Model is an effective theory:

○ Nature might be “Unnatural” with  $\Lambda$  at the Planck scale ?



Large  $M_H$  bound: “Landau pole”  
signal unitarity problems

Small  $M_H$  bound: Top quark  
triggers vacuum instability

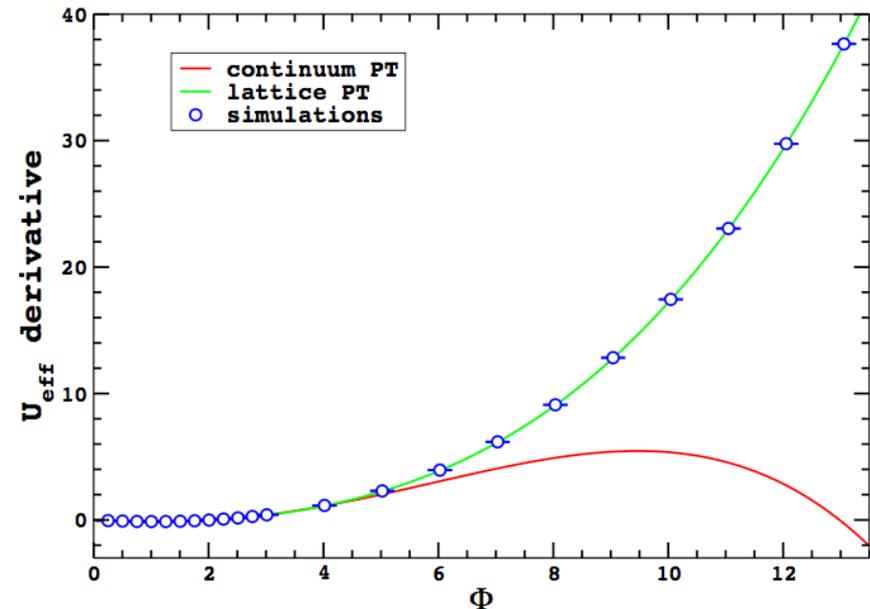
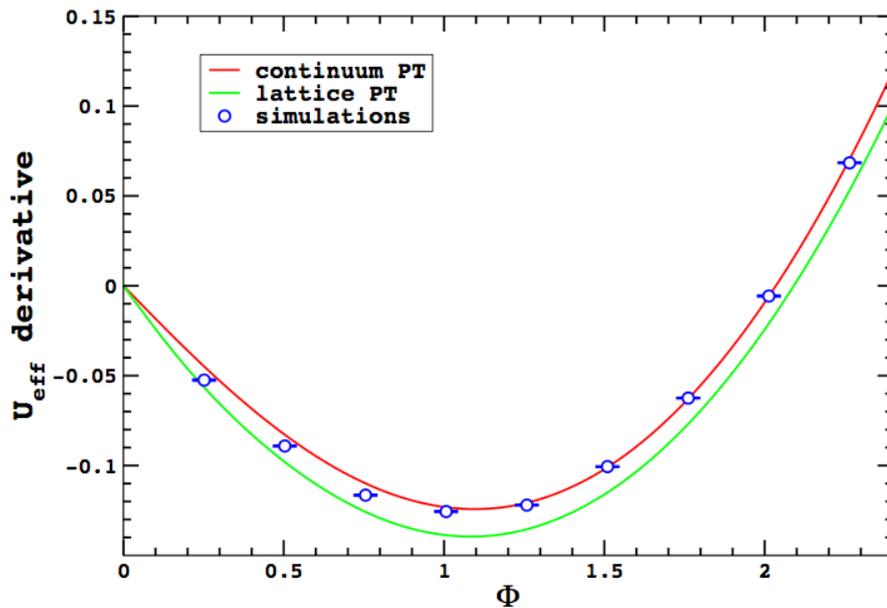
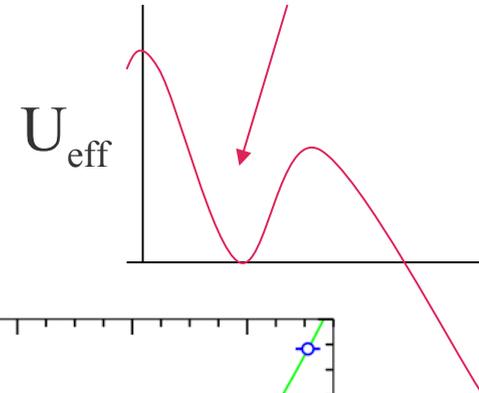
Should PDG figure be re-evaluated non-perturbatively?  
Lattice provides a Lorentz violating cut-off ( $\pi/a$ )

## Suppose

- The Higgs mass fits into a narrow band in the 140-180 GeV range, hinting at a cutoff not far from the Planck scale.
- However, a Higgs mass significantly outside this range could signal the presence of non-renormalizable operators in the effective Standard Model Higgs Lagrangian with low cutoff.
- For unexpected Higgs mass values new operators are needed because a low mass Higgs can trigger an instability due to its large Yukawa coupling to the top quark, whereas a large mass Higgs implies some new non-perturbative physics.
- Both features invoke the presence of higher dimensional non-renormalizable operators on the TeV scale, but with constraints from electroweak precision data.

# Lattice effective potential -- Fate of the false vacuum

- The lattice cut-off ( $\pi/a$ ) is not Lorentz invariant but does teach some “generic” lessons.

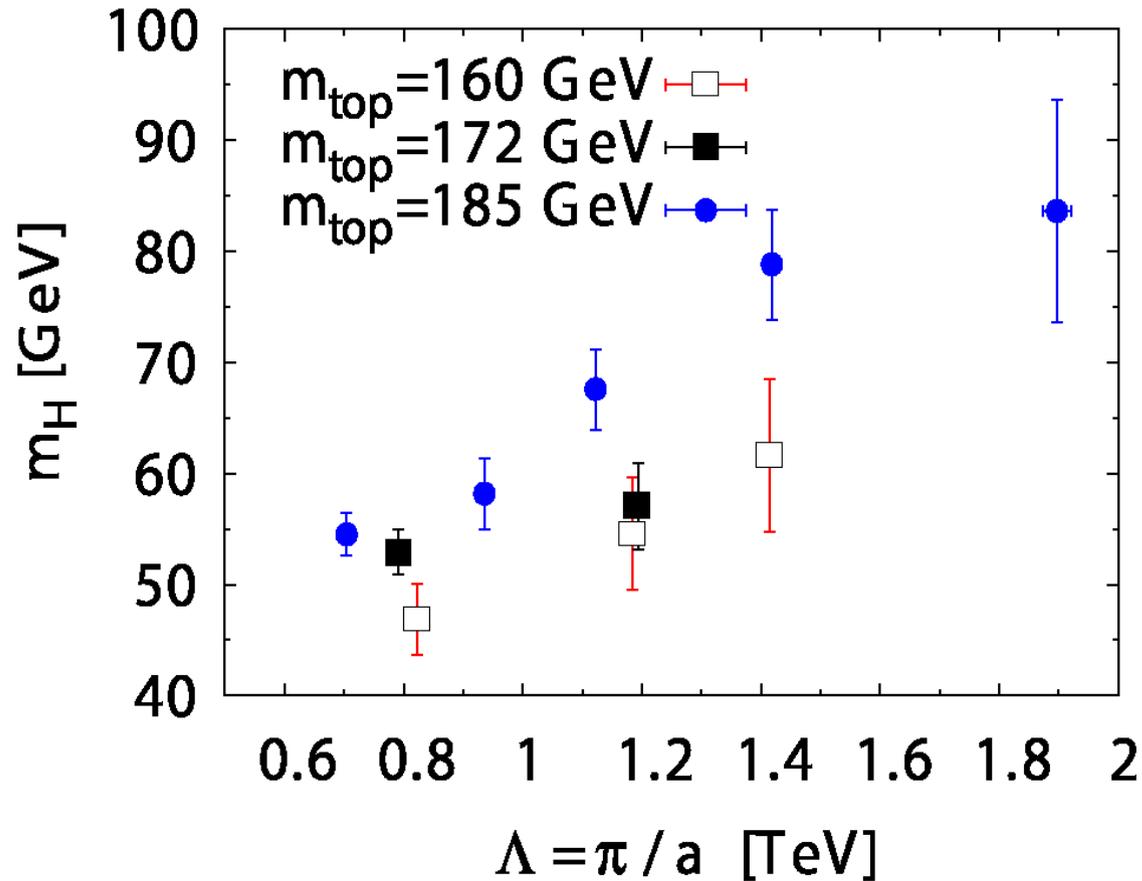


- The derivative of the effective potential  $dU_{\text{eff}}/d\Phi$  for the bare couplings. The left plot is a close-up of the behavior near the origin. The circles are the results of the simulations and the curves are given by continuum and lattice renormalized perturbation theory.
- To get continuum results of SM need chiral fermions and larger lattices: Z. Fodor, K. Holland, J. Kuti, D. Negradi, C. Schroeder, Latt 2007; Type B USQCD allocation 2007

# Higgs Project --- Future Goals

- Simulation of Higgs-Top sector with Higgs self coupling at cut-off scale.

(Higgs lower bound using overlap fermions)



- Add higher dimensional operators to give Lorentz invariant cut-off and *explore role of physics above electroweak scale.*

# Computational considerations

- To stay *close to the critical line* of spontaneous symmetry breaking requires the vacuum expectation value of the Higgs mass to be around 0.2 in lattice spacing units, which corresponds to a lattice *momentum cut-off of approximately 3 TeV*.
- To explore Higgs masses in the 100 GeV to 200 GeV range will drive down the Higgs mass to the 0.1 range, which requires *spatial lattices with 40 or 50 links*.
- To explore the heavy Higgs scenario will be similarly demanding. In addition, accurate Higgs physics requires a much *larger number of lattice configurations* than fermionic measurements in lattice QCD.

## II. Super symmetric field theories

Spin = 0 & 1/2

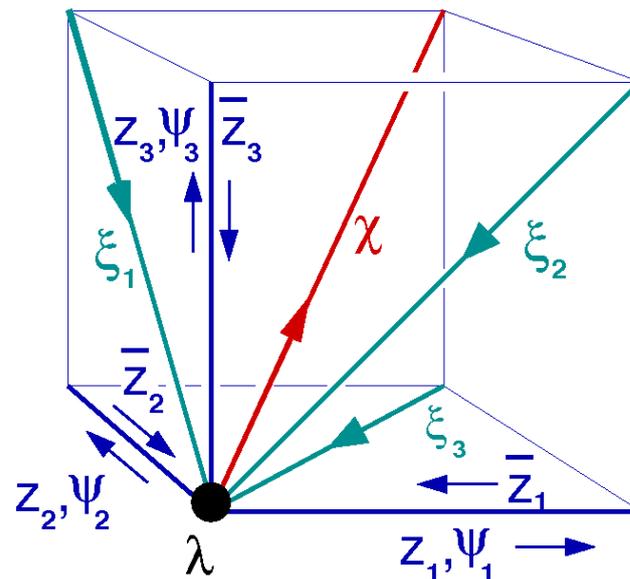
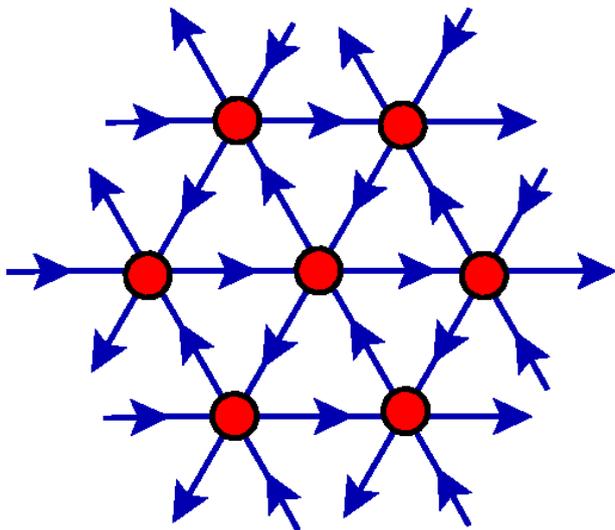
FERMIONS			BOSONS		
spin	Name	Symbols	Name	Symbols	spin
1/2	leptons	$e, \nu_{eL}$ $\mu, \nu_{\mu L}$ $\tau, \nu_{\tau L}$	sleptons	$\tilde{e}_L, \tilde{e}_R, \tilde{\nu}_{eL}$ $\tilde{\mu}_L, \tilde{\mu}_R, \tilde{\nu}_{\mu L}$ $\tilde{\tau}_L, \tilde{\tau}_R, \tilde{\nu}_{\tau L}$	0
1/2	quarks	$u, d$ $c, s$ $t, b$	squarks	$\tilde{u}_L, \tilde{d}_L, \tilde{u}_R, \tilde{d}_R$ $\tilde{c}_L, \tilde{s}_L, \tilde{c}_R, \tilde{s}_R$ $\tilde{t}_L, \tilde{b}_L, \tilde{t}_R, \tilde{b}_R$	0
1/2	gluinos	$\tilde{g}$	gluons	$g$	1
1/2	charginos	$\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm$	EW bosons	$\gamma, Z^0, W^\pm$	1
1/2	neutralinos	$\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$	higgs	$h^0, H^0, A^0, H^\pm$	0
SM particles (observed)		SM particles (not yet observed)		Super Partners (not yet observed)	

# Super symmetric field theories

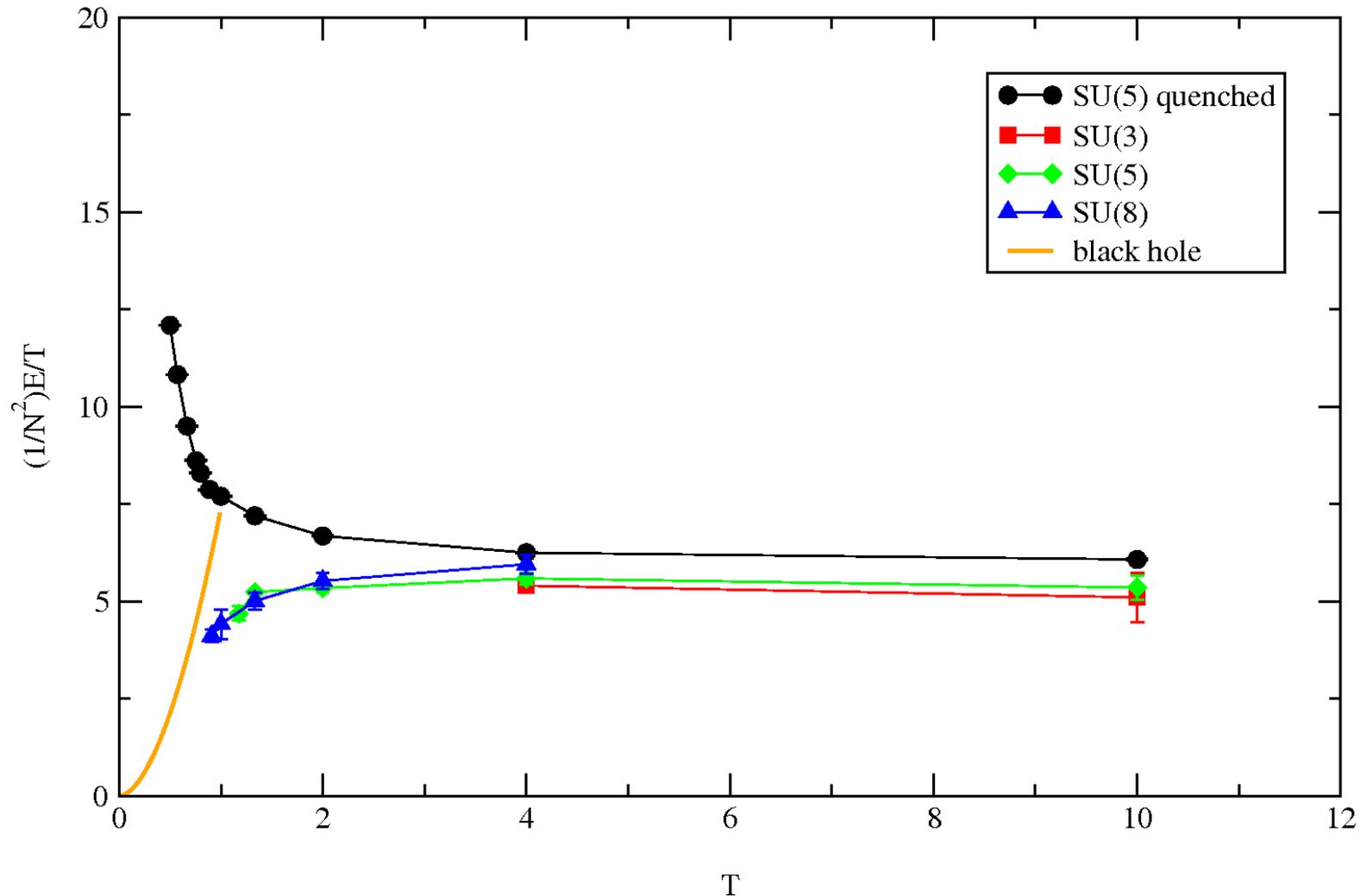
- A prime focus of initial work on lattice supersymmetric theories will be  $\mathcal{N}=1$  super Yang-Mills. It is perhaps the simplest example of a supersymmetric theory and *constitutes an important part of the minimal supersymmetric extension to the Standard Model (MSSM)*. Cancels quadratic divergence by Boson/Fermion pairing etc.
- On the lattice,  $\mathcal{N} = 1$  YM has *no fine tuning* and positive definite Pfaffian -- gold plated SUSY laboratory.
- First generation numerical simulations of the dynamical theory have already been performed using domain wall fermions (*Fleming, Kogut, Vranas*). The gluino condensate and the breaking of the  $U(1)_R$  symmetry down to  $Z_2$  was indeed observed. The presence of fractional topological charge on a toroidal lattice was also observed.

## Other SUSY theories

- A much larger range of SUSY theories are beginning to be considered on the basis of elegant lattice constructions using ideas drawn from orbifolding in string theory and the twisting procedure used in constructing topological field theories.
- These formulations retain a degree of exact supersymmetry to reduce the fine tuning required. They lead to surprising lattice geometries, where the fermionic partners are scattered on the lattice in manner reminiscent of staggered fermions, *but with no unphysical degrees of freedom*.



# $\mathcal{N} = 4$ SUSY reduced to 1-d Quantum Mechanics

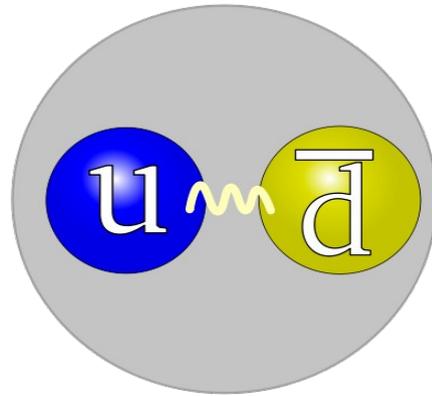


- Large  $N$  scaling of  $E/N^2 T$ . AdS/CFT duality predicts energy is equal IIA black hole mass. (S. Catterall and F. Sannino, arXiv:0706.3518, Type B USQCD allocation 2007)

# $\mathcal{N}=1$ Super Yang Mills Project

- The next steps for  $\mathcal{N}=1$  super Yang-Mills are to determine in detail the pattern of  $U(1)_R$  symmetry breaking, give accurate masses of the low lying spectrum and study mixing of the pseudoscalar glueball and the eta prime meson.
- All of these issues have been studied theoretically on the basis of various conjectures and approximation schemes. *Test SUSY effective field theory methods.*
- For  $\mathcal{N}=1$  super Yang-Mills, there are no Nambu-Goldstone bosons; so the box size  $L$  need only be larger than the lightest scale  $L \gg \mathcal{O}(1/m_{\nu})$ . The main effort is to make the gluino (fermion) mass as light as possible while taking the continuum limit and controlling finite volume effects.
- For MSSM phenomenology at LHC need to add chiral matter and study SUSY symmetry breaking --- big challenge!

### III. New strong dynamics

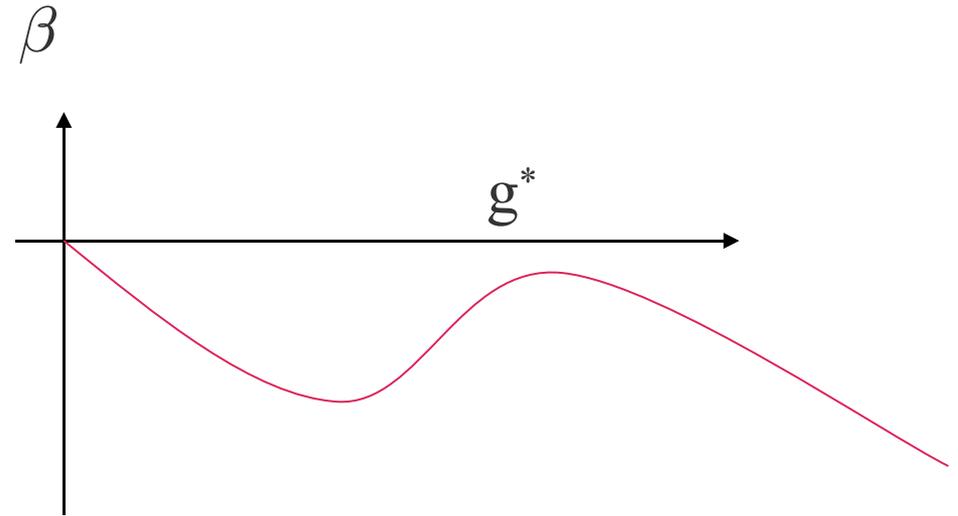
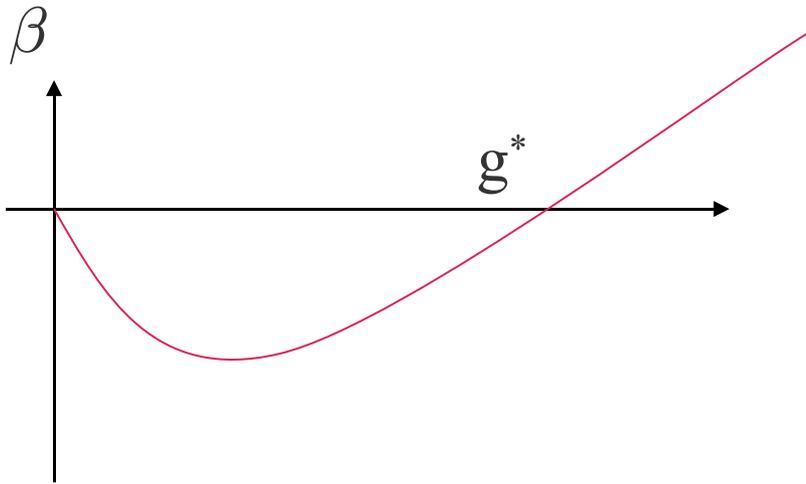


### III. New strong dynamics

- A new strong dynamics for electroweak symmetry breaking could provide an ideal application for lattice field theory. Higgs phenomena is driven by *chiral symmetry breaking* a new strongly coupled gauge field theory, such as that proposed in technicolor, Higgsless models, and extra-dimensional (Randall-Sundrum) models.
- Other non-perturbative methods, including those motivated by the AdS/CFT conjecture, can give only qualitative results, so lattice field theory offers the only *ab initio* non-perturbative method for making quantitative predictions
- Minimal technicolor has  $N_f = 2$  with Goldstone bosons (techni- $\pi$ ) giving mass electroweak  $W^+$ ,  $Z$ ,  $W^-$ . To address flavor and Fermion mass problem need  $N_f > 2$  “extended” technicolor conjectured to be a “two scales” or “walking” theory. Does this work with precision electroweak constraints?

# Adding flavors to QCD: Conformal Window

- An IR fixed point can emerge already in the two-loop  $\beta$  function as you increase the number  $N_f$  of fermions. (Gross and Wilczek, Banks and Zaks, ...)



○ **Conformal:**  $N_f > N_f^*$

- ◆ Long distance (IR) Conformal theory.
- ◆ Chiral symmetry  $SU(N_f) \times SU(N_f)$
- ◆ No Confinement
- ◆ But asymptotically free in the UV.

○ **Walking:**  $N_f < N_f^*$ , but close to  $N_f^*$

- ◆ Spontaneous breaking of chiral symmetry  $SU(N_f) \times SU(N_f)$
- ◆ Confinement
- ◆ Spontaneous breaking of an approximate (IR) conformal symmetry

## SUSY SU(N) QCD $N_f^*$ bounds

○ Degree of Freedom Inequality<sup>†</sup>:  $N_f^* \leq (3/2) N$

○ Seiberg Duality:  $N_f^* = (3/2) N$  !!

◆ Weakly coupled magnetic dual in the vicinity of this value

## SU(N) QCD $N_f^*$ bounds

○ Degree-of-Freedom Inequality:  $N_f^* \leq 4 N [1 - 1/18N^2 + \dots]$

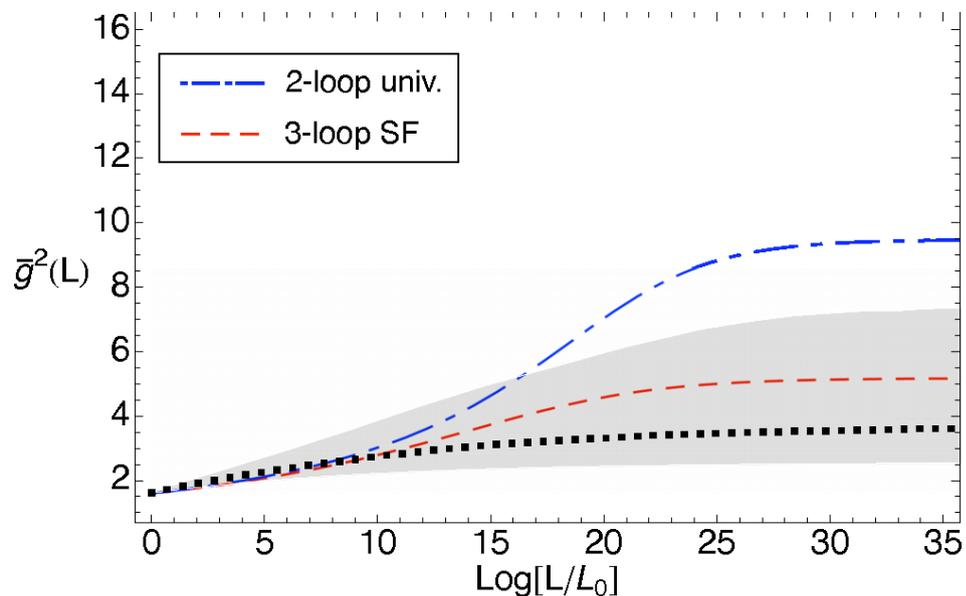
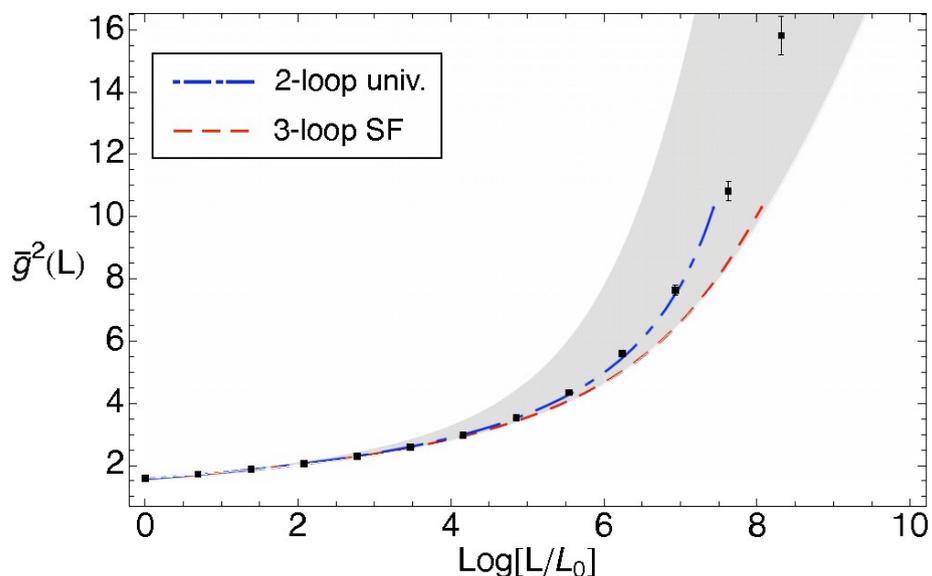
○ Gap-Equation Studies, Instantons:  $N_f^* \leq 4 N$

○ Lattice Simulation (Iwasaki et al, 2004):  $6 < N_f^* < 7$  For  $N = 3$

(<sup>†</sup> Appelquist, Cohen, Schmaltz 1999)

# Conformal Window for Lattice:

- Schrödinger Functional Results: (*coupling constant is determined by response of Action to applied E fields and the beta function by step scaling*)

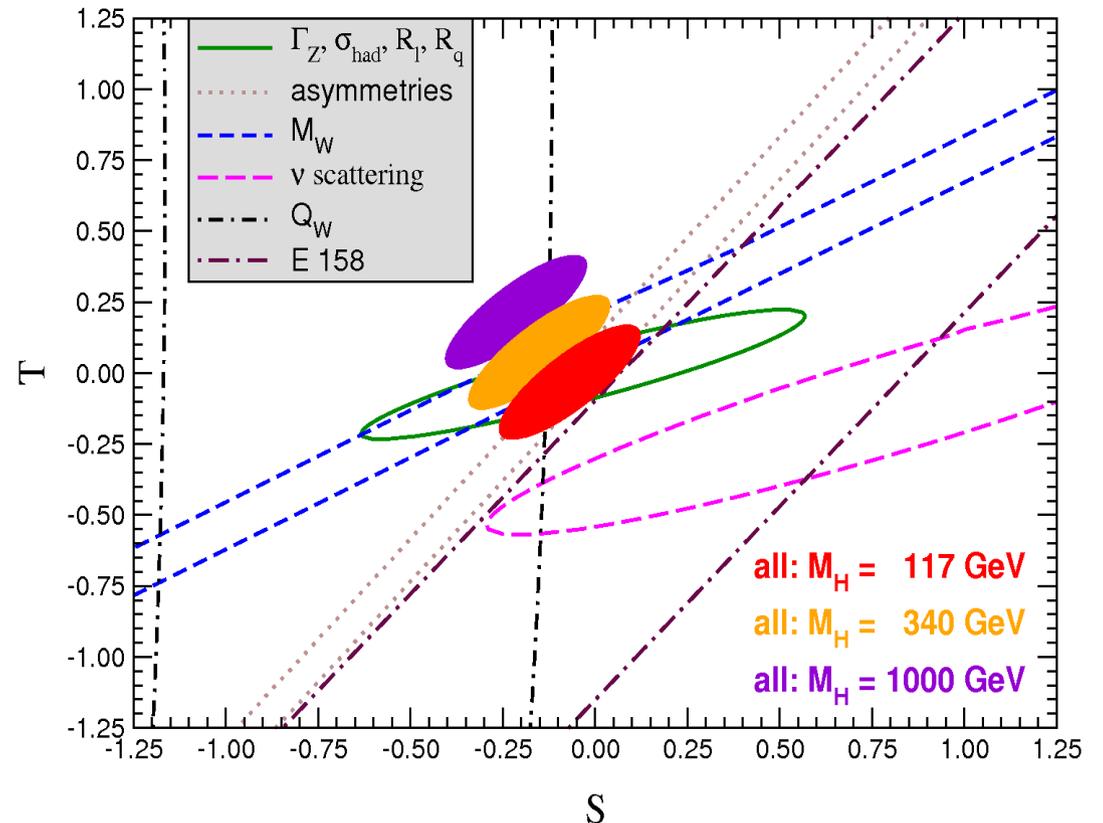


Compare  $N_f = 8$  and 12 staggered quarks

Applequist, Fleming and Neil arXiv:0712.0609 Type B USQCD allocation 2007

# Walking Multi-flavor project

- What is the value of  $N_f^*$ ? What is the order of the phase transition?
- What are the physical states below and near the transition?
- Is there “walking”? -- separation of scales:  $F_\pi / \langle \psi \psi \rangle$  small.
- The splitting of techni- $\rho$ 's and techni- $A_1$ 's are crucial
- Implications for EW precision studies? (The S parameter etc).



$$S \sim \frac{d}{dQ^2} \int dx e^{iQx} \langle J_\mu^R(x) J_\mu^L(0) \rangle, \quad Q^2 \rightarrow 0$$

# Examples of Lattice Ensemble for BSM physics

$N_f$	$am_\ell$	Size	$am_V$	$Lm_V$	“MC traj.”	TF-Yr
HIGGS	0.001	$32^3 \times 32 \times 32$	0.167	4	10000	0.66
SUSY	0.001	$24^3 \times 24 \times 32$	0.167	4	5000	0.66
$N_f = 8,9,10,11$	0.0075	$24^3 \times 64 \times 24$	0.25	6	$4 \times 5000$	2.32
TOTAL						3.64
HIGGS	0.00075	$48^3 \times 48 \times 32$	0.125	4	10000	2.11
SUSY	0.00075	$32^3 \times 32 \times 32$	0.125	4	5000	2.11
$N_f = 8,9,10,11$	0.005	$32^3 \times 64 \times 24$ u	0.25	8	$4 \times 5000$	6.80
TOTAL						11.0
HIGGS	0.0005	$64^3 \times 64 \times 32$	0.125	6	10000	23.7
SUSY	0.0005	$48^3 \times 48 \times 32$	0.125	6	5000	23.7
$N_f = 8,9,10,11$	0.0035	$48^3 \times 96 \times 32$	0.25	8	$4 \times 5000$	66.0
TOTAL						113.4

Table 1: Representative lattice ensembles for the Top-Higgs dynamics (with one fundamental fermion), for  $\mathcal{N} = 1$  SUSY (with one adjoint fermion) and for SU(3) Strong dynamics (with  $N_f = 8, 9, 10, 11$  fundamental fermions) projects.

# Smaller projects & improved methods as well.

## ○ Disconnected diagrams:

- ◆ To interpolate in  $N_f$  for “technicolor”, SUSY correlators, Taylor series in chemical potential, etc
- ◆ Strangeness content of proton,  $m_s \langle \bar{p} | s s | p \rangle / m_p$ , for detection efficiency of MSSM dark matter candidate at LHC or ILC. (Babich, Brower, Clark, Fleming, Osborn, Rebbi, Type B USQCD allocation 2007)

## ○ SUSY breaking mechanisms (Nima Arkani-Hamed )

- ◆ Driven by effective low energy theory in pure scalar sector?

## ○ Large $N_c$ studies (Narayanan and Neuberger)

- ◆ Test ideas of confinement in SUSY and non-SUSY theories.
- ◆ String/gravity duals to SUSY and non-SUSY YM theories.

## Note of caution

- We emphasize that, in contrast with QCD, the lack of experimental data for these theories makes *distinguishing lattice & finite volume artifacts from continuum predictions* more challenging and substantially raises the standards for obtaining convincing results.
- For this reason, we have recommended beginning with models that are for the most part *close variants of QCD*, enabling comparison with QCD to bring confidence to the methodology.
- Also, this makes sense from a model building perspective as a way to test the *conventional wisdom* regarding the behavior of non-perturbative QCD-like theories as the gauge or matter content is varied, which is often based on simple scaling assumptions. Validating or modifying these non-perturbative *heuristics* will have a substantial impact on comparing these models with LHC data.
- We do have the basic *algorithms, codes and multi-Teraflop/s hardware* to launch a serious lattice effort to explore physics beyond the standard model.