Charmed B semileptonic decays

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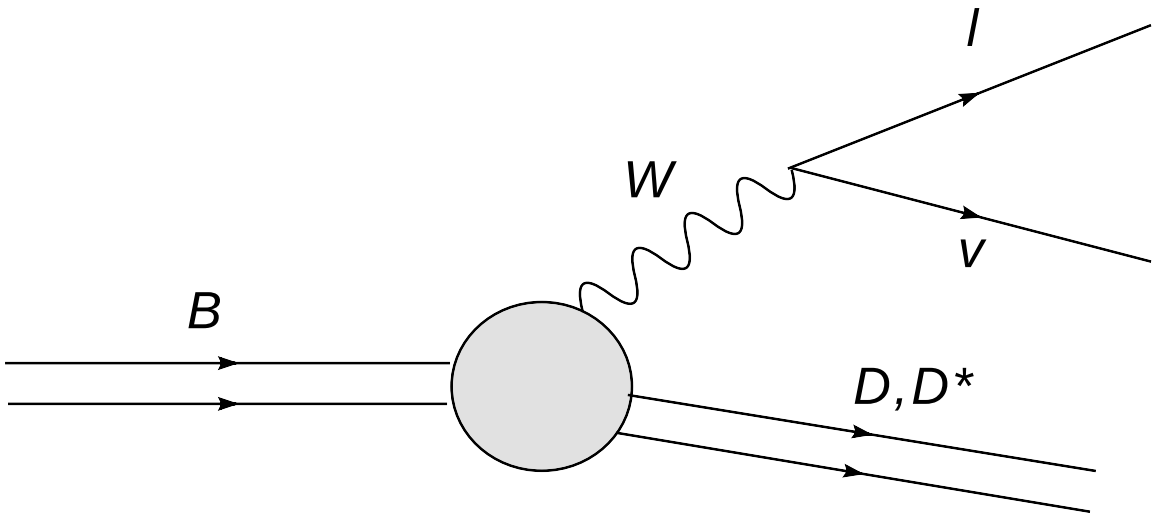
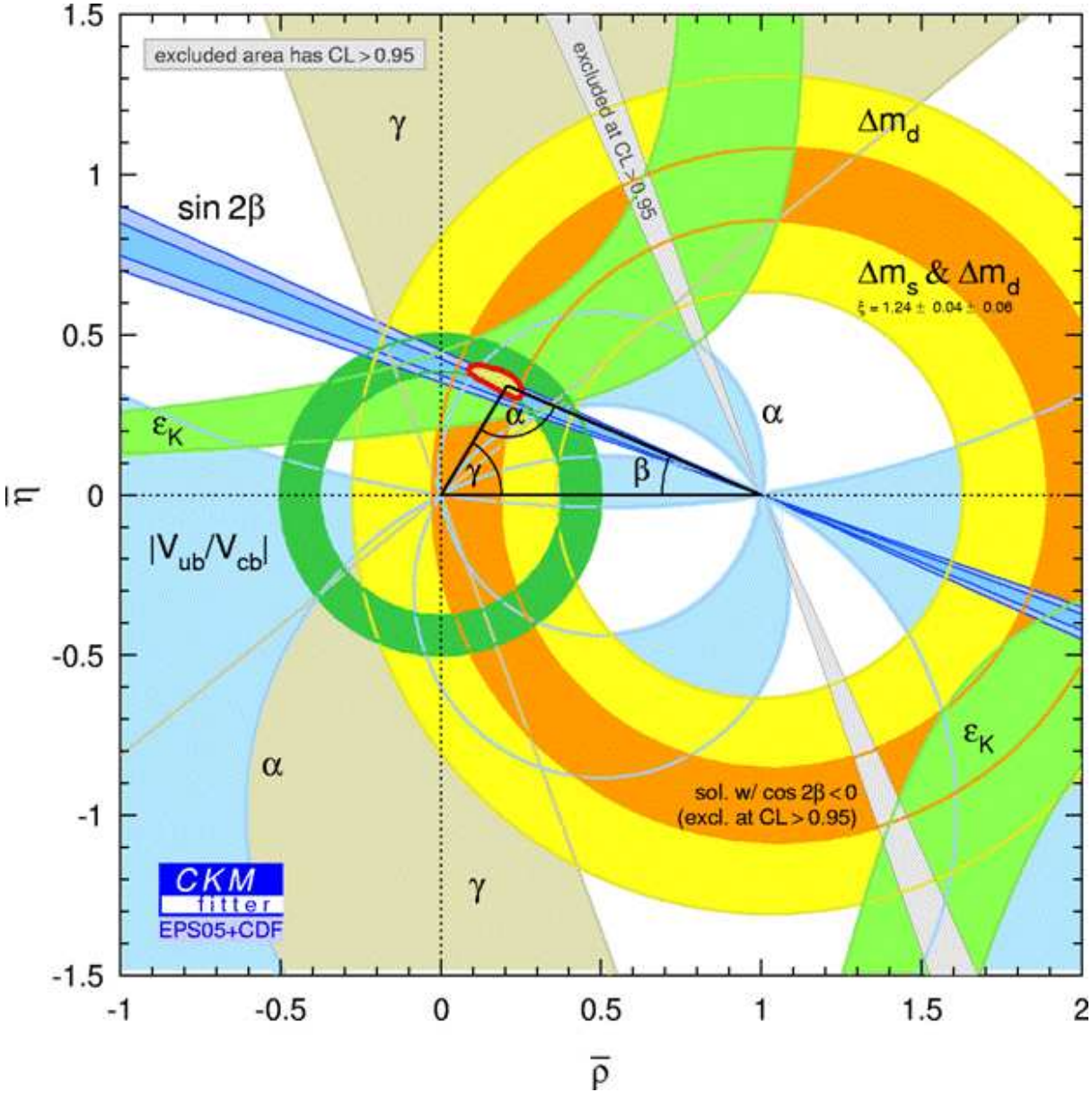


Vertex proportional to $|V_{cb}|$. In order to extract it, nonperturbative input is needed.

Constraining the Unitarity Triangle



Importance of $|V_{cb}|$

$|V_{cb}|$ is needed to constrain the apex of the unitarity triangle from kaon mixing. Given that

$$A = \frac{|V_{cb}|}{\lambda^2} \tag{1}$$

has $\approx 2\%$ error, we see that this contributes a 9% error to ϵ_K because it appears in the formula below to the fourth power.

$$|\epsilon_K| = C_\epsilon B_K A^2 \overline{\eta}\{-\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c,x_t) + \eta_2 S_0(x_t) A^2\lambda^2(1-\overline{\rho})\}$$

Given expected progress in B_K, we must lower the errors on $|V_{cb}|$. This puts pressure on the continuum perturbation theory community since the two-loop calculation of the Wilson coefficients has $\sim 7\%$ errors.

Rare kaon decays

$$\text{Br}(K^+ \to \pi^+ \nu\overline{\nu}) = \text{known factor} \times |V_{cb}|^4 X^2(x_t) \frac{1}{\sigma}[(\sigma\overline{\eta})^2 + (\rho_0 - \overline{\rho})^2], \tag{2}$$

$$\sigma = \left(\frac{1}{1 - \frac{\lambda^2}{2}}\right)^2, \quad \rho_0 = 1 + \frac{P_0(X)}{A^2 X(x_t)}. \tag{3}$$

$$\text{Br}(K_L \to \pi^0 \nu\overline{\nu}) = \text{known factor} \times |V_{cb}|^4 \eta^2 X^2(x_t), \tag{4}$$

where $X(x_t)$ and $P_0(X)$ are perturbative coefficients, and the known factor contains a non-perturbative form factor which can be obtained from $K \to \pi\ell\nu$ experiment.

(Taken from Buras, hep-ph/0101336)

Methods for extracting $|V_{cb}|$

- Inclusive $b \to c\ell\nu$ can be calculated using the OPE and perturbation theory. Requires non-perturbative input from experiment: moments of inclusive form factor $\overline{B} \to X_c \ell \overline{\nu}_\ell$ as a function of minimum electron momentum. Theoretical uncertainties from truncating the OPE and PT, and also perhaps from duality violations.

- Exclusive $B \to D\ell\nu$ has an $\sim 8\%$ experimental error in the zero-recoil point. No problem in principle of going to small recoil on the lattice.

- Exclusive $B \to D^*\ell\nu$ is experimentally cleaner ($\sim 1.7\%$ experimental error at zero-recoil).

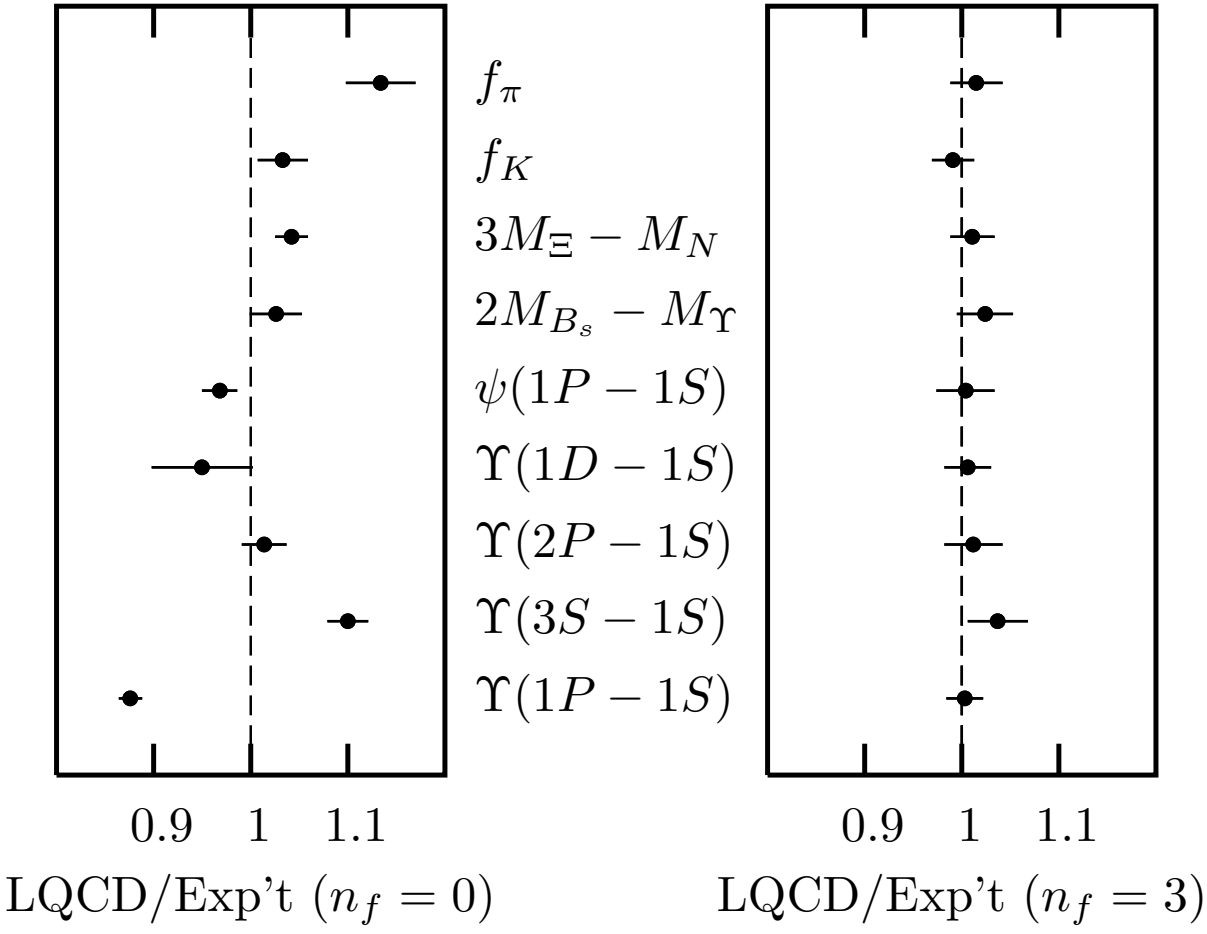
Staggered fermions

- Staggered fermions are the cheapest fermions on the market at the present time.
- The staggered action has extra unphysical species of fermions (called “tastes”) due to lattice artifacts which vanish in the continuum limit.
- This complicates the analysis with staggered fermions, as compared to “chiral” fermions such as domain-wall or overlap, which are many times more expensive.
- Staggered chiral perturbation theory gives good control over staggered discretization effects (MILC, arXiv:hep-lat/0407028).

Staggered quarks and rooting

- In the continuum limit, the four staggered tastes become degenerate
- In principle, taste breaking can be removed by taking the continuum limit, but in practice one must take the fourth root at finite lattice spacing.
- There is no rigorous proof that this procedure recovers QCD in the continuum limit, though there has been much recent progress on this issue, which is reviewed in hep-lat/0610094 by Steve Sharpe. Recent criticism has been refuted.
- It appears plausible that this procedure recovers QCD in the continuum limit, and we work under this assumption.
- There is no reason why these calculations could not be repeated with other types of lattice fermions.

Unquenching with staggered quarks



- Hadron spectroscopy – masses and decay constants
- *Good agreement for simple quantities!*

$|V_{cb}|$ from $B \to D\ell\nu$

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B + m_D)^2 (w^2-1)^{3/2} \times |V_{cb}|^2 |\mathcal{G}_{B\to D}(w)|^2 \quad (5)$$

where $w = v' \cdot v$ is the velocity transfer from initial (v) to final state (v'), and where

$$\mathcal{G}_{B\to D}(w) = h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w). \quad (6)$$

Quenched Fermilab calculation

Hashimoto et al, hep-ph/990637 computed $h_+(1)$ and $h_-(1)$ in order to construct $\mathcal{G}_{B\to D}(1)$ and extract $|V_{cb}|$. This was done using the Fermilab action for heavy quarks. Double ratios were constructed.

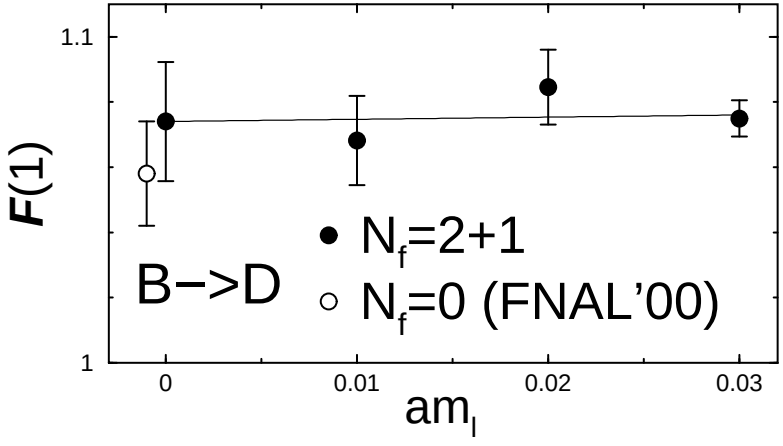
Advantages of the double ratios:

- Statistical errors cancel in the ratios
- Most of the current renormalization cancels. The remainder can be computed perturbatively.
- As shown by Kronfeld (hep-lat/0002008), heavy quark symmetry constrains the discretization errors in the double ratio for $h_+(1)$, so that for this quantity the leading corrections are of the order $\alpha_s(\overline{\Lambda}/m_Q)^2$ and $\overline{\Lambda}/m_Q^3$.
- All errors in double ratios $\mathcal{R}$ scaled as $\mathcal{R}-1$ rather than as $\mathcal{R}$, since when $m_c = m_b$ the ratio for $h_+(1)$ was one by construction. This was especially important since Hashimoto et al were working in the quenched approximation.

$\mathcal{G}_{B\to D}(1) = 1.058(^{+21}_{-17})$, $\sim 2\%$ error

Preliminary unquenched calculation

Okamoto, et al, hep-lat/0409116, for Fermilab/MILC Collaborations

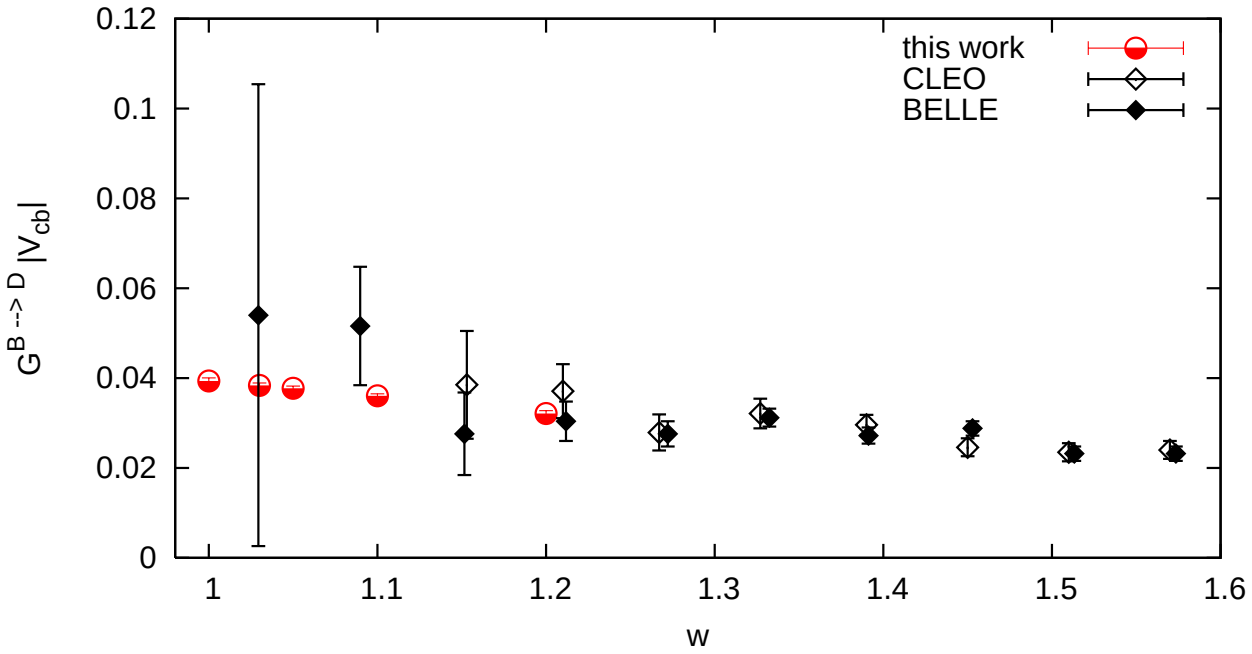


The preliminary result $\mathcal{G}(1) = 1.074(18)(16)$ was quoted, where the first error was statistical and the second was the sum of all systematic errors in quadrature.

uncertainty	$\mathcal{G}(1)$
statistical	1.7%
chiral extrapolation	$\sim 1\%$
discretization errors	$\sim 1\%$
perturbation theory	$\sim 1\%$
Total	$2-3\%$

New (quenched) result for $w \geq 1$

de Divitiis, et al, arXiv:0707.0582



New result using a step-scaling method.

A result is quoted of $\mathcal{G}_{B\to D}(1) = 1.026(17)$, with results also for $w \geq 1$. This is consistent with the quenched Hashimoto et al result of $\mathcal{G}_{B\to D}(1) = 1.058(^{+21}_{-17})$.

New (quenched) result for $w \geq 1$

A few caveats:

- Theoretical analysis of mass dependence is not fully understood. However, this appears to be unimportant because the mass dependence is so mild.
- Papers do not contain a table of the full error budget, so it is not clear if the error bar encompasses all sources of uncertainty.

Even so, the w dependence looks very promising!

Obtaining V_{cb} from $\overline{B} \to D^* l \overline{\nu}_l$

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 \sqrt{w^2 - 1} \times |V_{cb}|^2 \mathcal{G}(w) |\mathcal{F}_{B \to D^*}(w)|^2 \quad (7)$$

where $\mathcal{G}(w)|\mathcal{F}_{B \to D^*}|^2$ contains a combination of form-factors which must be computed non-perturbatively. $w = v' \cdot v$ is the velocity transfer from initial (v) to final state (v').

Calculating $B \to D^*$ form factor

$$\mathcal{F}_{B\to D^*}(1) = h_{A_1}(1), \tag{8}$$

$$\langle D^*(v)|\mathcal{A}^\mu|\overline{B}(v)\rangle \quad = \quad i\sqrt{2m_B 2m_{D^*}}\overline{\epsilon'}^\mu h_{A_1}(1). \tag{9}$$

$h_{A_1}(1)$ is constrained by heavy quark symmetry:

$$h_{A_1}(1) \quad = \quad \eta_A \left[1 - \frac{l_V}{(2m_c)^2} + \frac{2l_A}{2m_c 2m_b} - \frac{l_P}{(2m_b)^2}\right] \tag{10}$$

Quenched Fermilab calculation

Hashimoto et al, hep-ph/0110253 proposed three double ratios, one for each of the $1/m_Q^2$ coefficients on the previous slide. Fits to the three ratios using the HQET dependence on heavy quark masses yielded the $1/m_Q^2$ (and most of the $1/m_Q^3$) coefficients.
Again, the advantages of the double ratios are:

- Statistical errors cancel in the ratios
- Most of the axial current renormalization cancels with the vector current renormalization. The remainder can be computed perturbatively.
- As shown by Kronfeld (hep-lat/0002008), heavy quark symmetry constrains the discretization errors in the double ratios, so that for this quantity the leading corrections are of the order $\alpha_s(\overline{\Lambda}/m_Q)^2$ and $\overline{\Lambda}/m_Q^3$.
- All errors in double ratios $\mathcal{R}$ scaled as $\mathcal{R}-1$ rather than as $\mathcal{R}$, since when $m_c = m_b$ the ratios were one by construction. This was especially important since Hashimoto et al were working in the quenched approximation.

$\sim 4\%$ error was quoted for $\mathcal{F}_{B\to D^*}(1)$

New calculation (Fermilab/MILC)

- We still use the Fermilab method to treat heavy quarks, as in the original quenched calculation of Hashimoto et al, hep-ph/0110253.
- Now using the MILC 2+1 flavor lattices, so the calculation is unquenched, with improved staggered (asqtad) light fermions in valence and sea
- Staggered quarks allow us to go to much lighter quark masses. Staggered chiral perturbation theory (SχPT) allows us to control systematic errors from staggered quarks in heavy-light quantities. (Aubin and Bernard, arXiv:hep-lat/0510088)
- Many MILC lattice ensembles were used. This work uses three lattice spacings ($a \approx 0.15$ fm, $a \approx 0.12$ fm, $a \approx 0.09$ fm).
- New double ratio is constructed which gives the answer more directly, allowing a cleaner determination and a huge savings in computing cost ($\sim$ factor of 10)

New Method

$$\frac{\langle D^*|\overline{c}\gamma_j\gamma_5 b|\overline{B}\rangle\langle\overline{B}|\overline{b}\gamma_j\gamma_5 c|D^*\rangle}{\langle D^*|\overline{c}\gamma_4 c|D^*\rangle\langle\overline{B}|\overline{b}\gamma_4 b|\overline{B}\rangle} = |h_{A_1}(1)|^2 \,. \tag{11}$$

- Statistical errors cancel in the ratio
- Most of the axial current renormalization cancels with the vector current renormalization. The remainder can be computed perturbatively.
- This ratio gives (the lattice approximation of) h_{A_1} directly to all orders in HQET
- The ratio can then be calculated at the tuned $m_{b,c}$, so that many heavy quark mass values are not needed.
- Fewer masses and fewer ratios means a factor of ~ 10 less computer time
- Not all errors scale as $\mathcal{R}-1$, but in a full-QCD setting, it is no longer essential. One must simply compare (total error)/(computer time).

JL for the Fermilab and MILC Collaborations, arXiv:0710.1111

Fiducial point method

To disentangle heavy-quark discretization effects from chiral and continuum limit issues, I introduced suitable ratios designed for this purpose.

- Chiral fits are done on ratios, i.e. the chiral fit is normalized to a data point at a chosen fiducial mass. The fiducial point should be a compromise between being light enough for χPT to apply, but not so light as to be very noisy/expensive.

- Heavy quark discretization errors largely cancel in such a ratio, disentangling the heavy quark discretization errrors.

- Ultimately, such a ratio can be normalized by data on an ultra fine lattice. Only the fiducial point must be calculated on the ultra-fine lattice, so a very light mass (and correspondingly large lattice) is unnecessary, and can be done relatively cheaply.

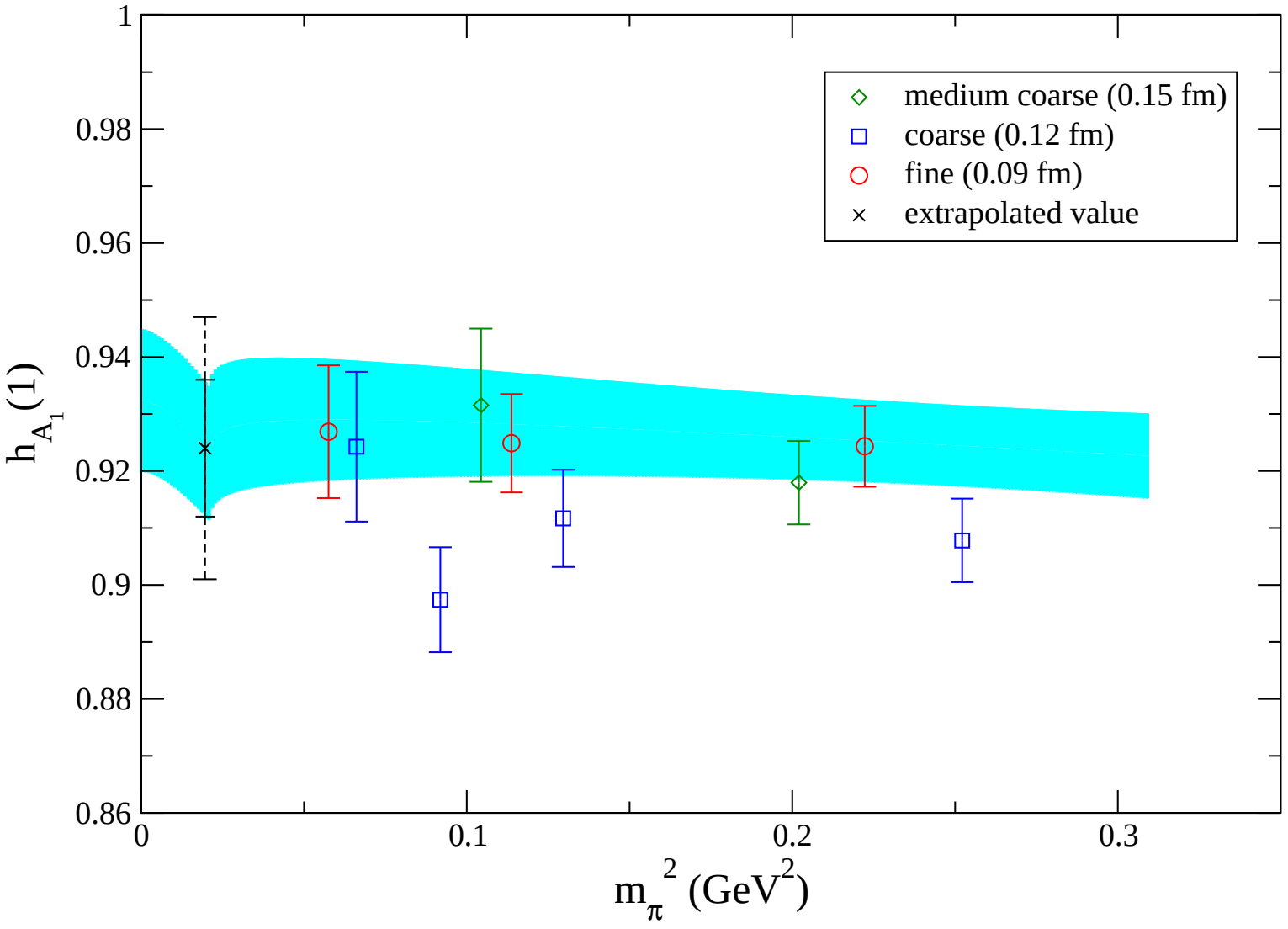
Staggered ChPT formula

[from J.L. and Van de Water, PRD74 (2006) 034510]

$$
\begin{aligned}
h_{A_1}^{2+1}(1) \quad = \quad & 1 + X_A + \frac{g_\pi^2}{48\pi^2 f^2}\left[\frac{1}{16}\sum_B (2\overline{F}_{\pi_B} + \overline{F}_{K_B}) - \frac{1}{2}\overline{F}_{\pi_I} + \frac{1}{6}\overline{F}_{\eta_I}\right. \\
& + a^2\delta'_V\left(\frac{m_{S_V}^2 - m_{\pi_V}^2}{(m_{\eta_V}^2 - m_{\pi_V}^2)(m_{\pi_V}^2 - m_{\eta'_V}^2)}\overline{F}_{\pi_V}\right. \\
& + \frac{m_{\eta_V}^2 - m_{S_V}^2}{(m_{\eta_V}^2 - m_{\eta'_V}^2)(m_{\eta_V}^2 - m_{\pi_V}^2)}\overline{F}_{\eta_V} \\
& \left.\left. + \frac{m_{S_V}^2 - m_{\eta'_V}^2}{(m_{\eta_V}^2 - m_{\eta'_V}^2)(m_{\eta'_V}^2 - m_{\pi_V}^2)}\overline{F}_{\eta'_V}\right) + (V \to A)\right],
\end{aligned}
\tag{12}
$$

where a is the lattice spacing, δ'_V, g_π and X_A are constants, and $\overline{F}$ is a complicated function involving logs.

Chiral Extrapolation



Total error budget

uncertainty	$h_{A_1}(1)$
statistical	1.3%
g_π	0.6%
NLO vs partial NNLO ChPT fits	0.9%
discretization errors	1.3%
kappa tuning	1.0%
perturbation theory	0.4%
Total	2.4%

Preliminary Result for F(1)

$$h_{A_1}(1) = 0.924(11)(19)$$

where the first is error is statistical and the second systematic. (JL for Fermilab/MILC, arXiv:0710.111).

This is consistent with the earlier quenched result of $[0.913^{+0.024+0.017}_{-0.017-0.030}]$.

Applying a QED correction of 0.7%, and taking the Heavy Flavor Averaging Group value, $F(1)|V_{cb}| = (36.0 \pm 0.6) \times 10^{-3}$, we get

$$|V_{cb}| = (38.7 \pm 0.7_{exp} \pm 0.9_{theo}) \times 10^{-3}.$$

For comparison, the inclusive number is (PDG 2006)

$$|V_{cb}| = (41.7 \pm 0.7_{exp}) \times 10^{-3}.$$

Prospects for the future

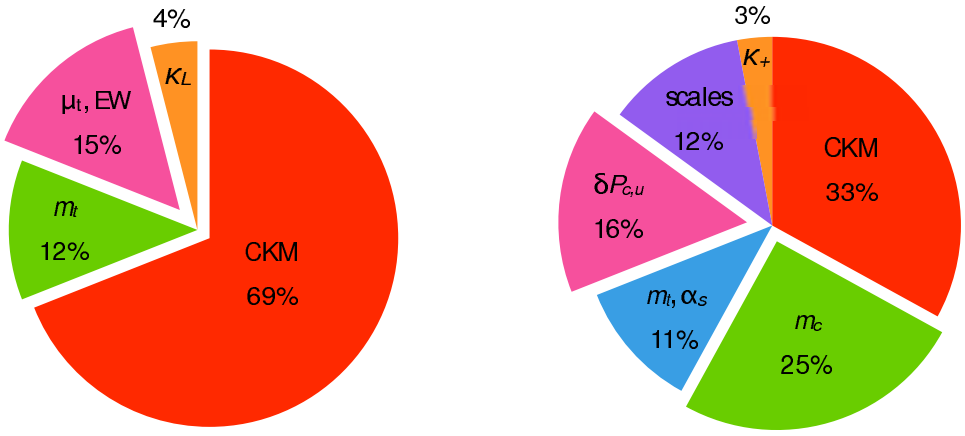
Errors in lattice calculation of $F(1)$

now	2 yrs	5 yrs	5 yrs (with higher loop corrections)
2.4%	$\sim 1.6\%$	$\sim 0.8\%$	$\sim 0.3\%$

I assume that the dominant errors, heavy quark discretization errors and statistics, will decrease in a well understood way with improved computing resources and existing methods.

The higher loop corrections require significant human time.

Rare K decays and Project X



Error budget of the SM prediction of BR($K_L \to \pi^0 \nu\overline{\nu}$) (left) and BR($K^+ \to \pi^+ \nu\overline{\nu}(\gamma)$)) (right)

$$\text{BR}(K_L \to \pi^0 \nu\overline{\nu})_{\text{SM}} = (2.54 \pm 0.35) \times 10^{-11}, \quad (13)$$

$$\text{BR}(K^+ \to \pi^+ \nu\overline{\nu}(\gamma))_{\text{SM}} = (7.96 \pm 0.86) \times 10^{-11} \quad (14)$$

CKM is mostly $|V_{cb}|$, and we have seen that the prospects for improvement are good. What about m_c?

Charts and predictions from Haisch, arXiv:0707.3098

Improving m_c

The dominant error in m_c is the two-loop matching of the lattice calculation. This calculation is in progress by the HPQCD Collaboration for HISQ (highly improved staggered quarks). They expect errors in the near future of $2-3\%$.

In six years, sub-percent errors could be possible.