D and B Leptonic Decays from the Lattice

Lattice QCD Meets Experiment Workshop

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Fermilab

presented by :

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- Experimental determinations of f_D, f_{D_s} are becoming very precise. Good for testing theory versus experiment.

$$\Gamma(D_q \to l\nu(\gamma)) = \frac{G_F^2|V_{cq}|^2}{8\pi} f_P^2 m_l^2 m_P \left(1 - \frac{m_l^2}{m_P^2}\right)^2 [1 + \mathcal{O}(\alpha_{QED})]$$

This requires $|V_{cq}|$ from other processes (or unitarity).

- Precision $f_{D_q}^{theory} + \Gamma^{exp} \longrightarrow |V_{cq}|$

  Similarly $f_B^{theory} + \Gamma^{exp}(B \to l\nu) \longrightarrow |V_{ub}|$

- f_{B_s}/f_B important part of $\xi = \frac{f_{B_s}}{f_B}\sqrt{\frac{B_{B_s}}{B_B}}$ relevant for B-mixing $\longrightarrow |V_{td}|/|V_{ts}|$.

- Combination $f_{B_q}|V_{tq}|$ also relevant for $B_q \to \mu^+\mu^-$.

- $\frac{\Gamma(D\to l\nu)}{\Gamma(D\to\pi,l\nu)}$ independent of $|V_{cq}|$ $\longrightarrow$ another consistency check

Unquenched Lattice Calculations

of f_D, f_{D_s}, f_B and f_{B_s}

$(N_f = 2 + 1)$

MILC Configurations

Improved staggered (AsqTad) sea quarks + improved glue.

Used for heavy quark physics by

- Fermilab/MILC
- HPQCD

Dynamical Domain Wall

- RBC/UKQCD : heavy flavor program recently initiated

D Meson Decay Constants

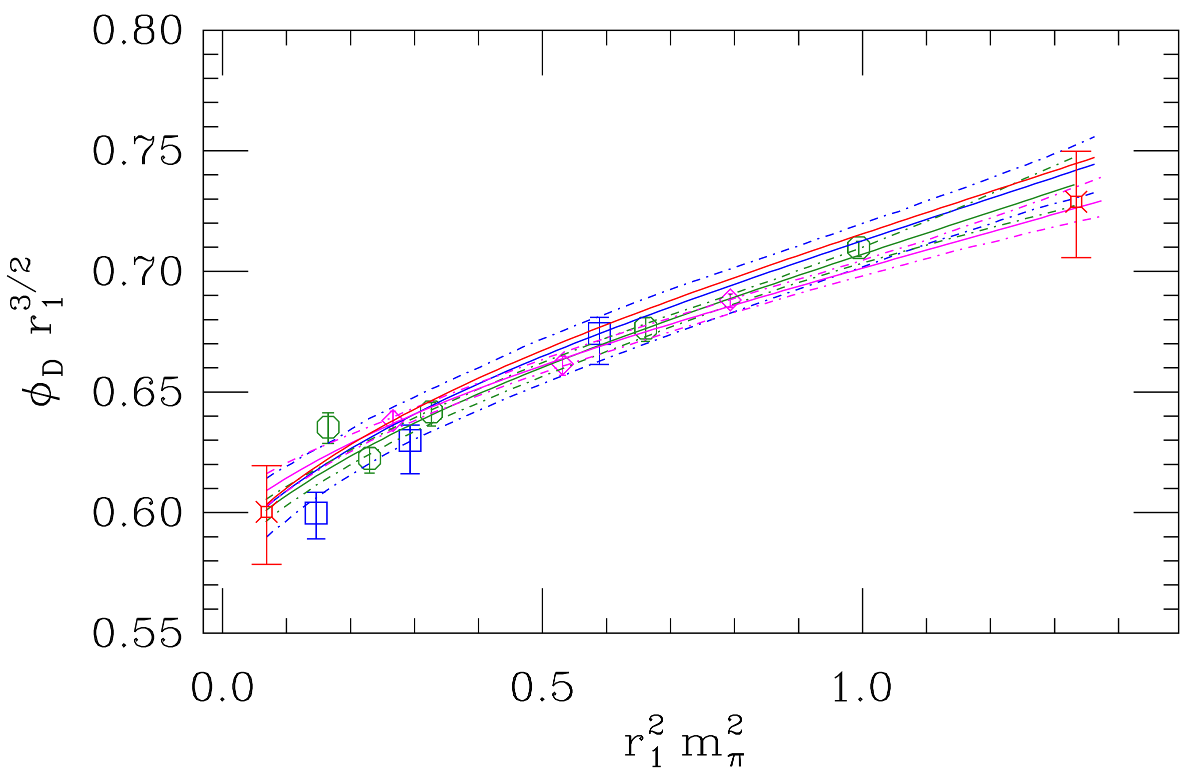
	Fermilab/MILC (2007)	HPQCD (2007)
light valence	AsqTad (improved staggered)	HISQ (highly imp. stagg.)
heavy valence	heavy clover	HISQ
lattice spacing "a" # $m_l < m_s$ (sea) # $m_l < m_s$ (val) chiral extrapol.	(0.15, 0.12, 0.09) fm (3,5,3) (29,60,27) Staggered ChPT	(0.15, 0.12, 0.09) fm (2,3,2) (2,3,2) Continuum ChPT + "a" dependent and taste breaking terms
fixing "a" value for r_1 fixing $m_{u,d}$ and m_s fixing m_c	$\Upsilon(2S-1S)$ & r_1 $r_1 = 0.318(7) fm$ Pion and Kaon D_s	$\Upsilon(2S-1S)$ & r_1 $r_1 = 0.321(5) fm$ Pion and Kaon η_c
matrix element	$\langle 0 \vert A_\mu \vert D \rangle = p_\mu f_D$	$f_D = \frac{m_c+m_l}{M_D} \langle 0 \vert \mathcal{P} \vert D \rangle$ PCAC / Ward ID
	blind analysis	

Results from Fermilab/MILC

source of error	f_D	f_{D_s}	f_{D_s}/f_D
statistics	3.8	3.1	1.0
scale (r_1), m_s, m_l	2.0	1.4	0.5
input m_c	2.7	2.7	< 0.1
matching (nonpert.)	1.4	1.4	0.0
matching (perturb.)	0.3	0.3	< 0.2
discret. (heavy quark)	2.7	2.7	0.3
discret. (light quark)	2.7	1.0	1.8
finite volume	0.6	0.2	0.6
total systematic	5.3	4.4	2.0
Total % error	6.5	5.4	2.2

Results from Fermilab/MILC (cont'd)

Fits are to all data, but only $m_{val} = m_{sea}$ points shown.

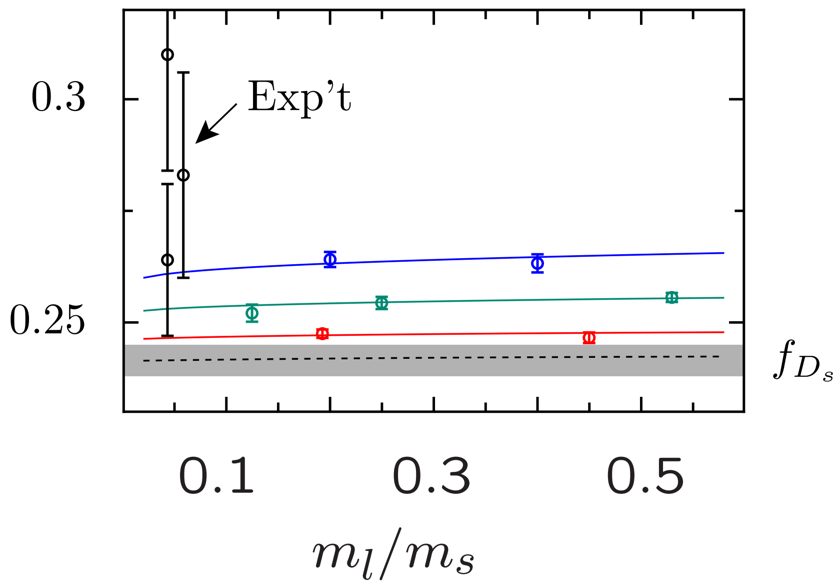


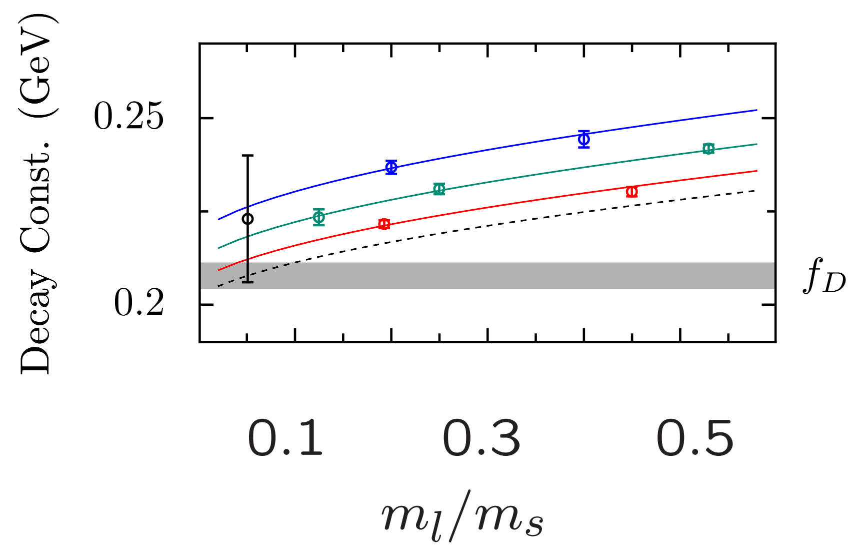
0.15fm (magenta), 0.12fm (green), 0.09fm (blue)

Extrapolated results in red. $[\Phi_D \equiv f_D\sqrt{M_D}]$

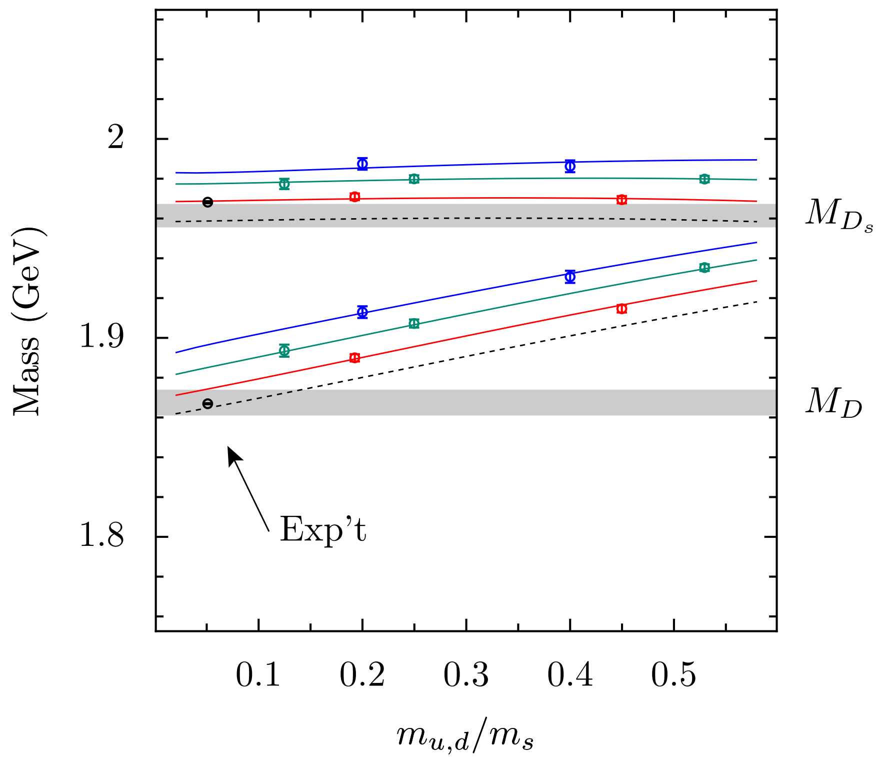
Results from HPQCD

source of error	f_D	f_{D_s}	f_{D_s}/f_D
statistics	0.7	0.6	0.5
scale (r_1) uncerty.	1.4	1.0	0.4
continuum extrap.	0.6	0.5	0.4
chiral extrap.	0.4	0.3	0.2
finite volume	0.3	0.1	0.3
m_s evolution	0.3	0.3	0.3
Total % error	1.8	1.3	0.9





Results from HPQCD (cont'd)



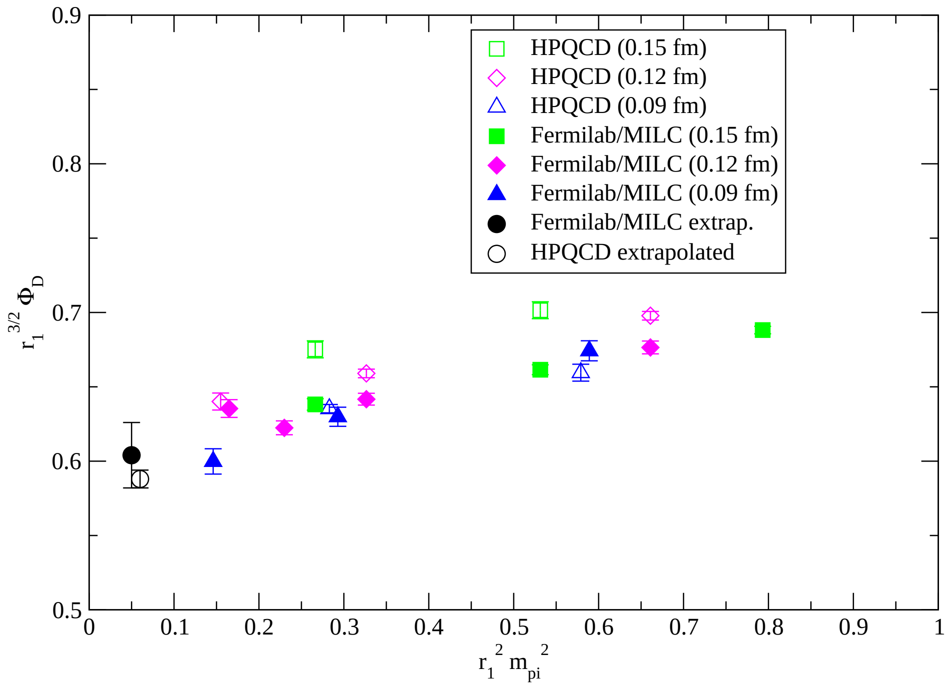
Simultaneous fit to M_{D_s},M_D, f_{D_s} and f_D.

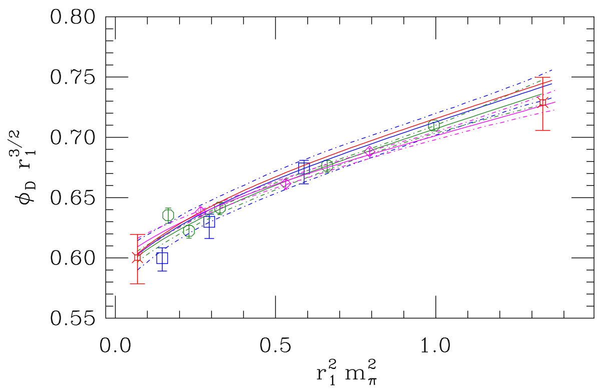
Can check tuning of m_c and m_s (which was done using η_c and m_K).

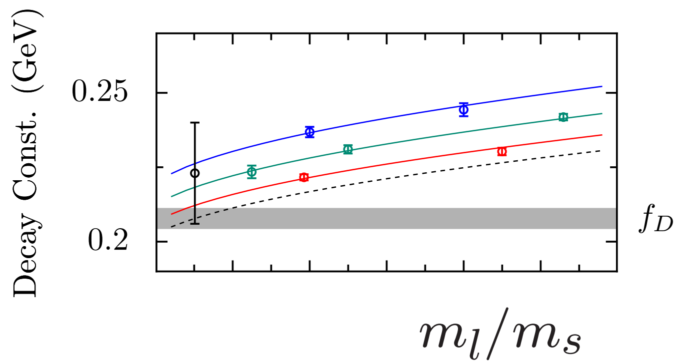
Same analysis applied here as used previously for M_π, M_K, f_K and f_π.

Errors in the D system not too much larger than in the light quark system.

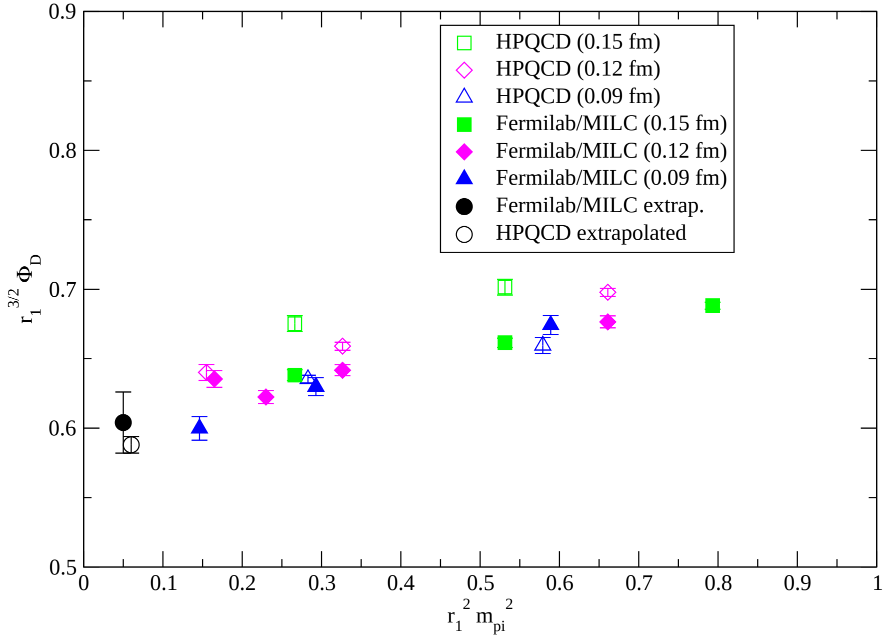
Comparing Data for D Mesons

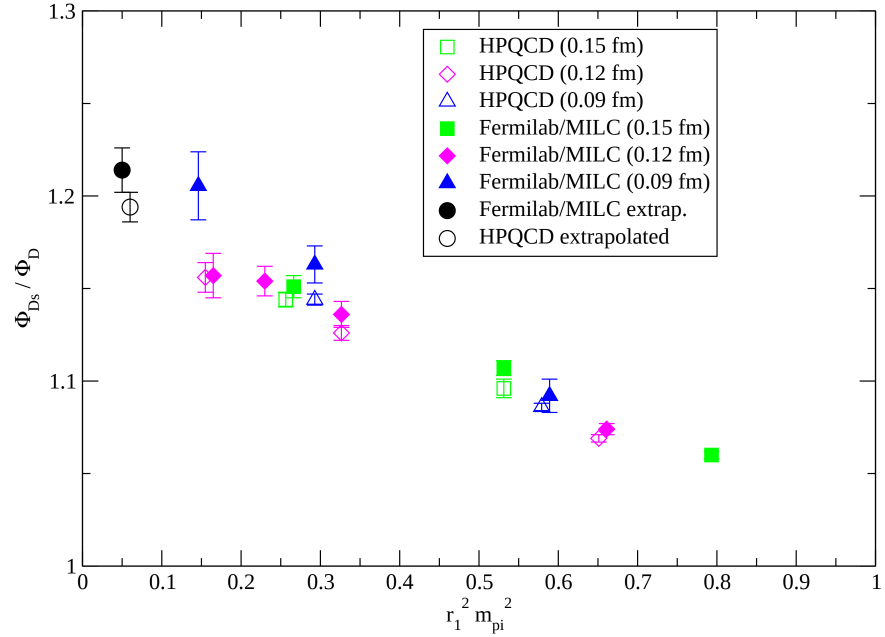






Comparing Data for D Mesons (cont'd)

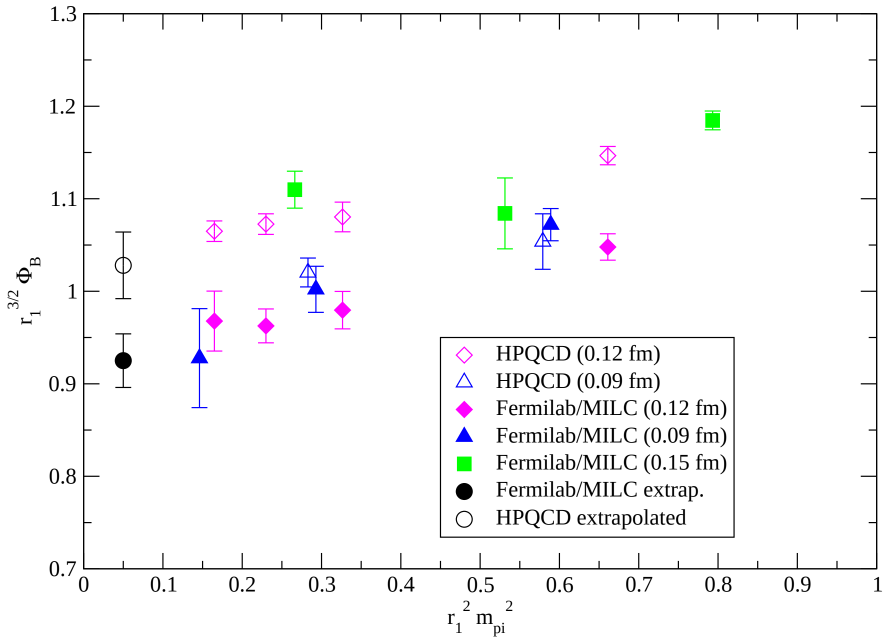


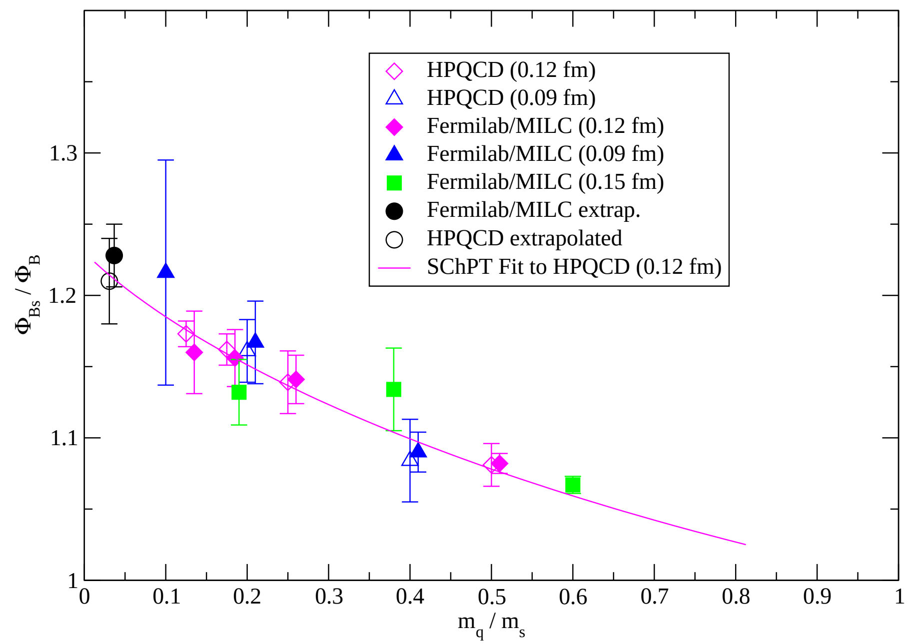


B Meson Decay Constants

	Fermilab/MILC (2007)	HPQCD (2005)
light valence	AsqTad (improved staggered)	AsqTad
heavy valence	heavy clover	NRQCD
lattice spacing "a" # $m_l < m_s$ (sea) # $m_l < m_s$ (val) chiral extrapol.	(0.15, 0.12, 0.09) fm (3,5,3) (29,60,27) Staggered ChPT	(0.12, 0.09) fm (4,2) (6,2) Staggered ChPT fit to 0.12fm data
fixing "a" value for r_1 fixing $m_{u,d}$ and m_s fixing m_b	$\Upsilon(2S-1S)$ & r_1 $r_1 = 0.318(7) fm$ Pion and Kaon B_s	$\Upsilon(2S-1S)$ & r_1 $r_1 = 0.321(5) fm$ Pion and Kaon $\Upsilon(^3S_1)$
matrix element	$\langle 0 \vert A_\mu \vert B \rangle = p_\mu f_B$	$\langle 0 \vert A_\mu \vert B \rangle = p_\mu f_B$
	blind analysis	

Comparing Data for B Mesons

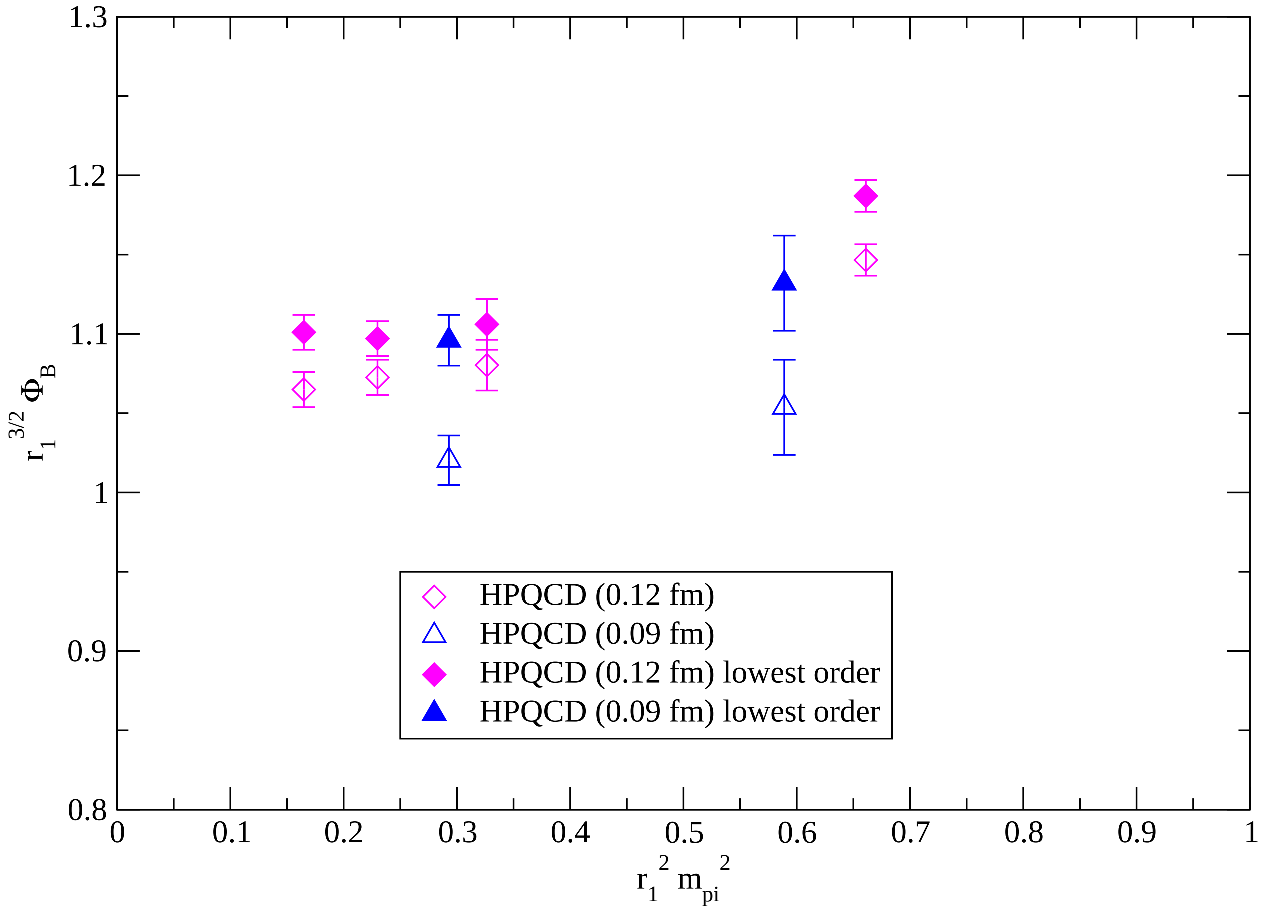




Fermilab/MILC	f_B	f_{B_s}	f_{B_s}/f_B
statistics	3.1	2.1	1.8
scale (r_1), m_s, m_l	3.8	3.1	0.6
input m_b	1.1	1.1	< 0.1
matching (nonpert.)	1.4	1.4	0.0
matching (perturb.)	1.1	1.3	< 0.2
discret. (heavy quark)	1.9	1.9	0.2
discret. (light quark)	3.8	2.0	1.8
finite volume	0.6	0.2	0.6
total systematic	6.1	4.7	2.0
Total % error	6.8	5.1	2.7

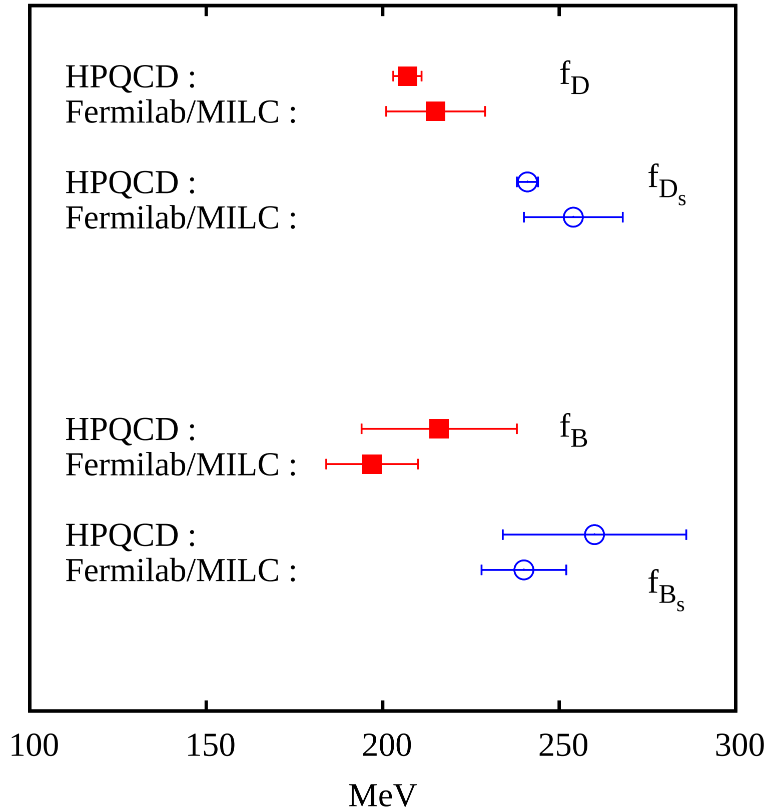
HPQCD	f_B	f_{B_s}	f_{B_s}/f_B
statistics (+ fit. + chiral extrap.)	3.5	2.7	2.5
scale (r_1) uncerty.	2.1	2.1	0.4
matching + discret.	9.0	9.0	0.3
relativistic	3.0	3.0	< 0.1
Total % error	10.3	10.1	2.6

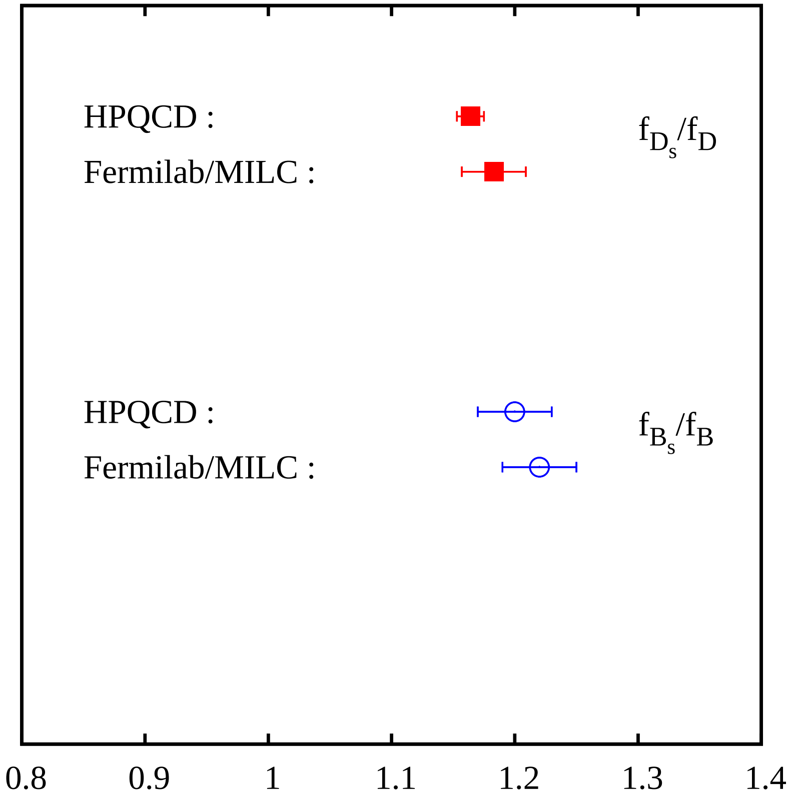
Discretization and Matching Corrections



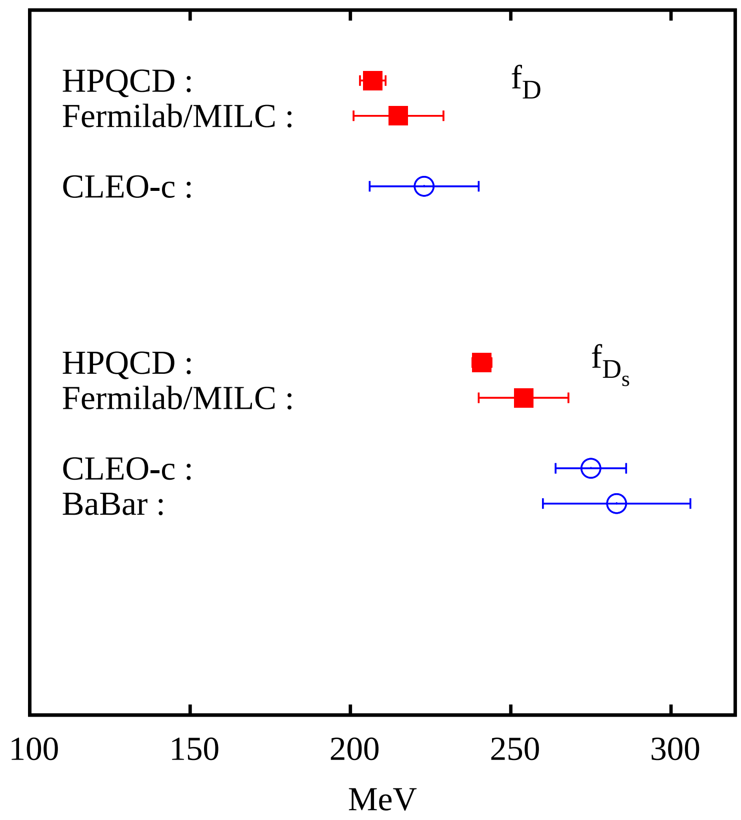
Difficult to distinguish between higher order matching and discretization errors. Need more data and two-loop (or nonperturbative) matching.

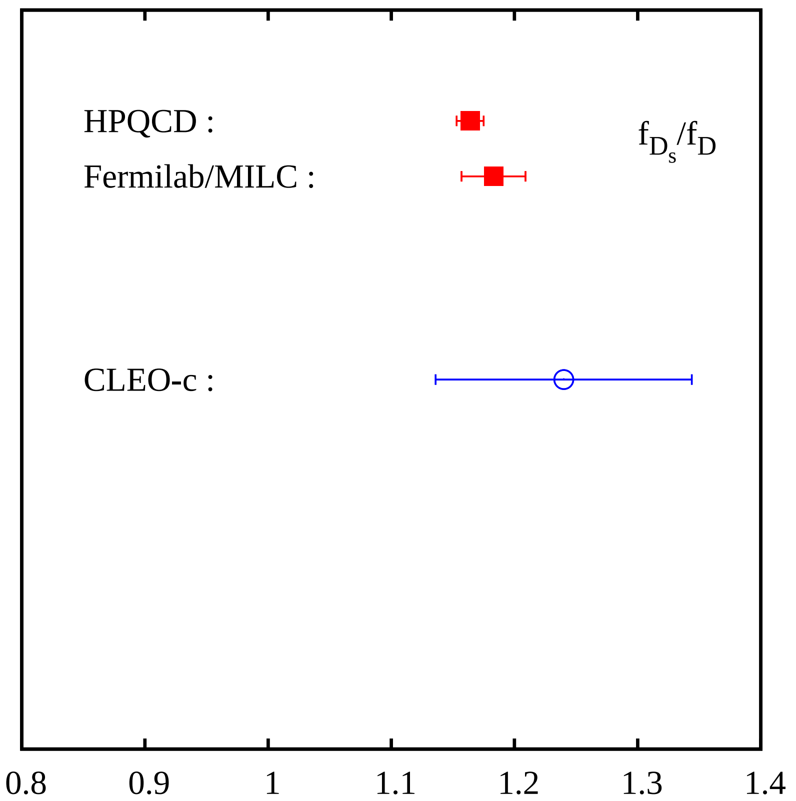
Final Numbers for Decay Constants





Final Numbers and Comparison with Experiment





Final Numbers (cont'd)

	Fermilab/MILC (2005)	Fermilab/MILC (2007)	HPQCD (2007)
f_D [MeV]	201 ± 17	215 ± 14	207 ± 4
f_{D_s} [MeV]	249 ± 16	254 ± 14	241 ± 3
f_{D_s}/f_D	1.24 ± 0.07	1.183 ± 0.026	1.164 ± 0.011

	Fermilab/MILC (2007)	HPQCD (2005)
f_B [MeV]	197 ± 13	216 ± 22
f_{B_s} [MeV]	240 ± 12	260 ± 26
f_{B_s}/f_B	1.22 ± 0.03	1.20 ± 0.03

How Can We Improve ?

Statistics : smearings, random wall sources, more ensembles, better chiral/continuum extrapolations.

Actions : AsqTad $\longrightarrow$ HISQ, Improved heavy clover, dynamical HISQ.

Tuning : more accurate Υ simulations $\rightarrow r_1$, accurate fixing of m_c and m_b (and m_s).

Matching : two-loop, nonperturbative approaches

Double ratios : e.g. $[f_{B_s}/f_B]/[f_K/f_\pi]$

Tentative Goals for Next 2 - 5 Years

Fermilab/ MILC	current → future error [%]	main areas where improvement needed
f_D	6.5 → 4.0	quark actions, fixing m_c, statistics more ensembles
f_{D_s}	5.4 → 3.5	same as for f_D
f_{D_s}/f_D	2.2 → 2.0	statistics, quark actions
f_B	6.8 → 4.0	statistics, quark actions, ensembles
f_{B_s}	5.1 → 3.5	same as for f_B
f_{B_s}/f_B	2.7 → 2.0	statistics, quark actions

HPQCD	current → future	improvements needed
f_D	1.8 → 1.6	scale (r_1), ensembles, statistics
f_{D_s}	1.3 → 1.2	same as for f_D
f_{D_s}/f_D	0.9 → 0.6	statistics, more ensembles
f_B	10.3 → 4.0	matching, statistics, quark action more ensembles
f_{B_s}	10.1 → 3.5	same as for f_B
f_{B_s}/f_B	2.6 → 2.0	statistics, quark action double ratios $[f_{B_s}/f_B]/[f_K/f_\pi]$

Summary

- In parallel with the impressive progress in experiment, lattice QCD is also making steady progress in determinations of D and B meson decay constants, with current errors ranging from 2 - 6.5% for f_D and 7 - 10% for f_B (smaller for ratios).

- Within errors results from Fermilab/MILC and HPQCD are in agreement.

- Errors appear to come in quite differently in the calculations by the two groups.

- Further improvements expected with errors reduced to 1.6 - 4% for decay constants and 0.6 - 2.0% for ratios. Will the $\sim 3\sigma$ descrepancy with CLEO-c in f_{D_S} persist ?

- Calculations using other dynamical quarks (e.g. Domain Wall) are very important.