Leptonic D Decays

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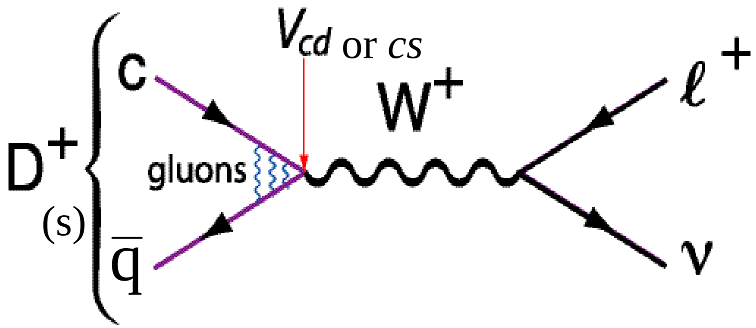
Outline: Experimental Measurements

- $D^+ \to \mu^+\nu$
- $D_s^+ \to \mu^+\nu$
- $D_s^+ \to \tau^+\nu;\ \tau^+ \to \pi^+\overline{\nu}$
- $D_s^+ \to \tau^+\nu;\ \tau^+ \to e^+\nu\overline{\nu}$ New
- $D_s^+ \to \tau^+\nu;\ \tau^+ \to \rho^+\overline{\nu}$

All CLEO-c results are updated using the final luminosity

Leptonic Decay: $D^+_{(s)} \to \ell^+ \nu$

c and $\bar{q}$ (q=*d*,*s*) can annihilate to virtual W^+, probability is proportional to wave into wave function overlap



In Standard Model (SM):

$$\Gamma(D^+_{(s)} \to \ell^+ \nu) = f^2_{D^+_{(s)}} \, |V_{cq}|^2 \frac{G_F^2}{8\pi} m_\ell^2 M_{D^+_{(s)}} \left(1 - \frac{m_\ell^2}{M^2_{D^+_{(s)}}}\right)^2$$

- *f* is decay constant, related to the overlap of the heavy and light quark wave-functions.
- V_{cq} are well known, we take $|V_{cd}| = |V_{us}| = 0.2246(12)$ & $|V_{cs}| = |V_{ud}| - |V_{cb}|^2/2 = 0.97345(22)$, where $|V_{ud}| = 0.97425(22)$.

PDG'2010

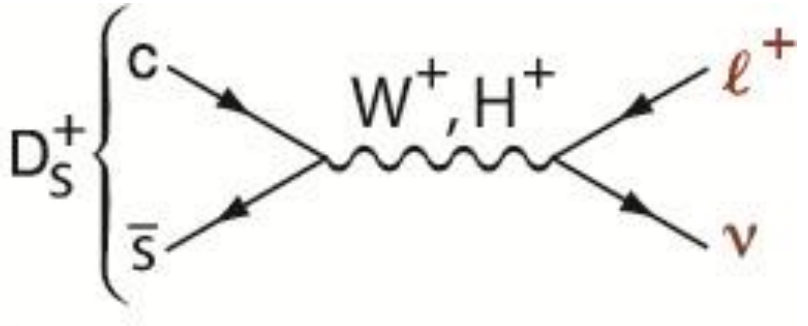
Reason to Measure

- Test of Lattice QCD calculations:
  - Lattice calculations on f_{Bd} & f_{Bs}/f_{Bd} are inputs for extracting CKM matrix elements. The analogous quantities f_D & f_{Ds}/f_D provide an experimental check.

$$\Delta m_d \propto f_{B_d}^2 \, |V_{td} V_{tb}^*|^2$$

$$\Delta m_s \propto f_{B_s}^2 \, |V_{ts} V_{tb}^*|^2$$

- Possibilities to see effects of New Physics, for example H^+



Which Channels to Measure?

$$\Gamma(D^+_{(s)} \to \ell^+\nu) = f^2_{D^+_{(s)}} |V_{cq}|^2 \frac{G_F^2}{8\pi} m_\ell^2 M_{D^+_{(s)}} \left(1 - \frac{m_\ell^2}{M^2_{D^+_{(s)}}}\right)^2$$

- CKM-factor: D_s rate / D^+ rate ≈ 20
- Helicity × Phase Space:

$D^+ \to (\tau^+\nu):(\mu^+\nu):(e^+\nu)$

rate: 2.67 : 1 : 2.4×10^{-5}

$D_s^+ \to (\tau^+\nu):(\mu^+\nu):(e^+\nu)$

rate: 9.76 : 1 : 2.4×10^{-5}

- τ modes decay fraction

11% $\pi^-\nu$ 18% $e^-\nu\bar{\nu}$ 25% $\rho^-\nu$

- τ rates largest, but experimentally most difficult (at least 2 neutrinos missing: background larger). They can be used in D_s because of its extremely large rate.
- μ is the cleanest signal, because of only missing 1 neutrino.
- e rates too small to see, except if there is new physics

CLEO-c f_{D^+} Technique

CLEO-c uses Tagging:

$e^+e^- \to \psi(3770) \to D^0\overline{D^0},\ D^+D^-$

Fully reconstruct D^- as tag, then examine the other D^+

- Can then infer neutrinos from

$$MM^2 = (\mathbf{p}_{D^+} - \mathbf{p}_{\ell^+})^2$$

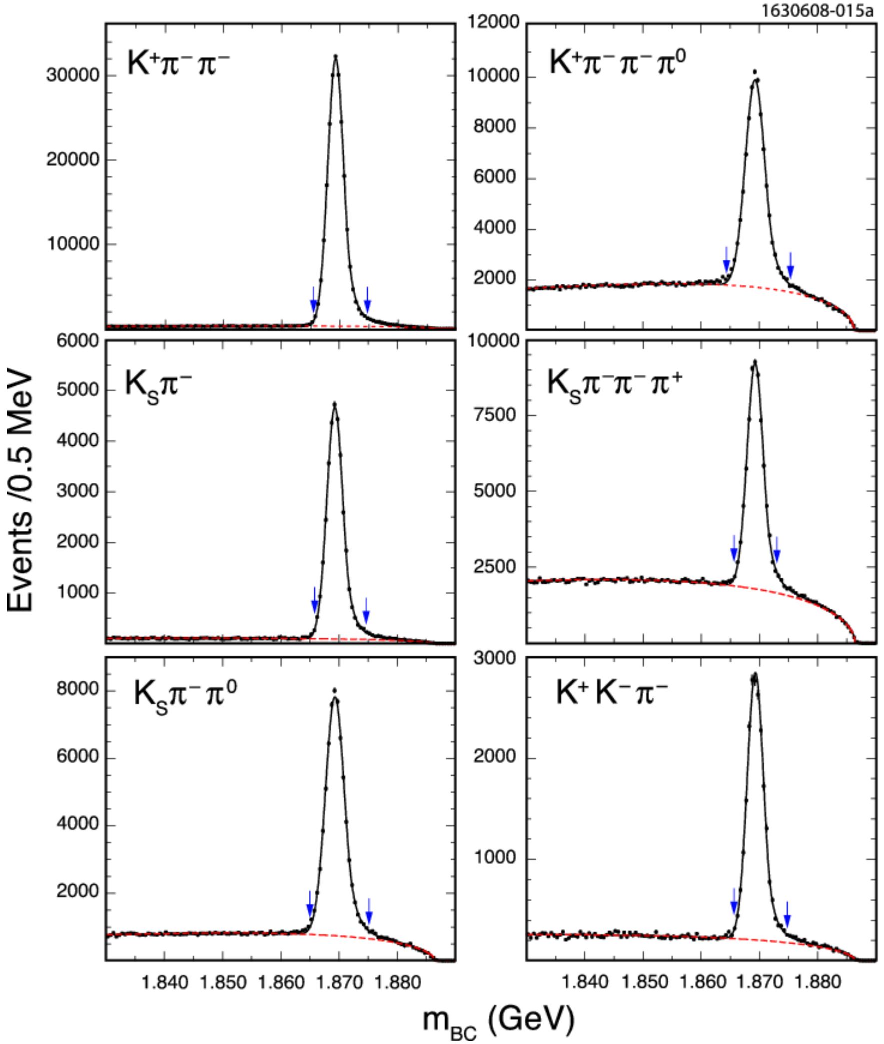
we know $E_{D^+} = E_{\text{beam}}$ and $\vec{p}_{D^+} = -\vec{p}_{D^-}$

- Can measure absolute $\mathcal{B}$

CLEO-c fully reconstructed D^- tags

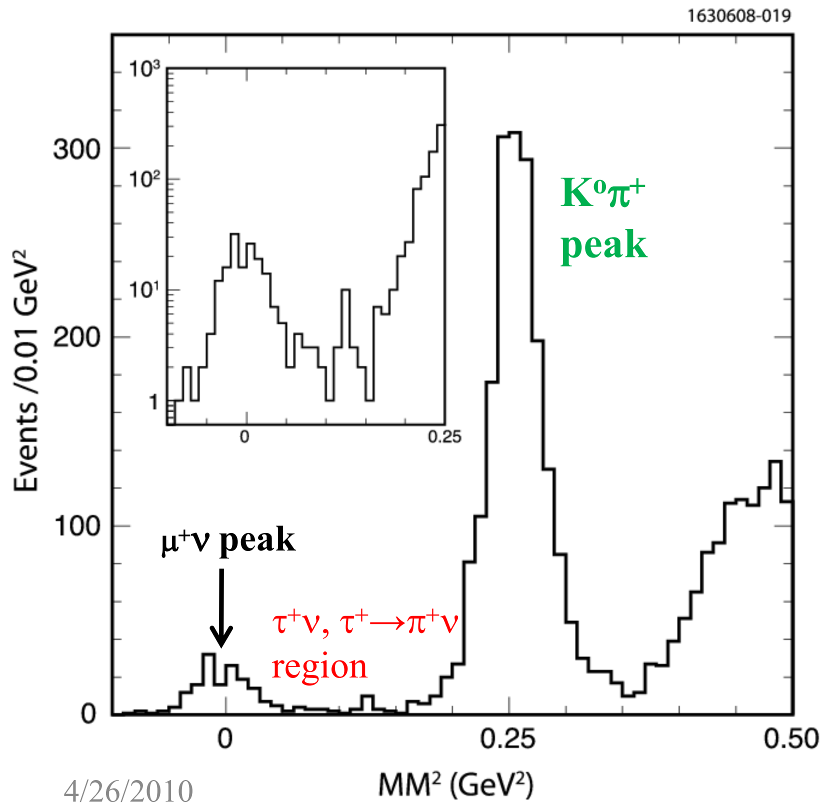
- Total of 460,000 tags
- Purity 84%

CLEO-c D^- tags



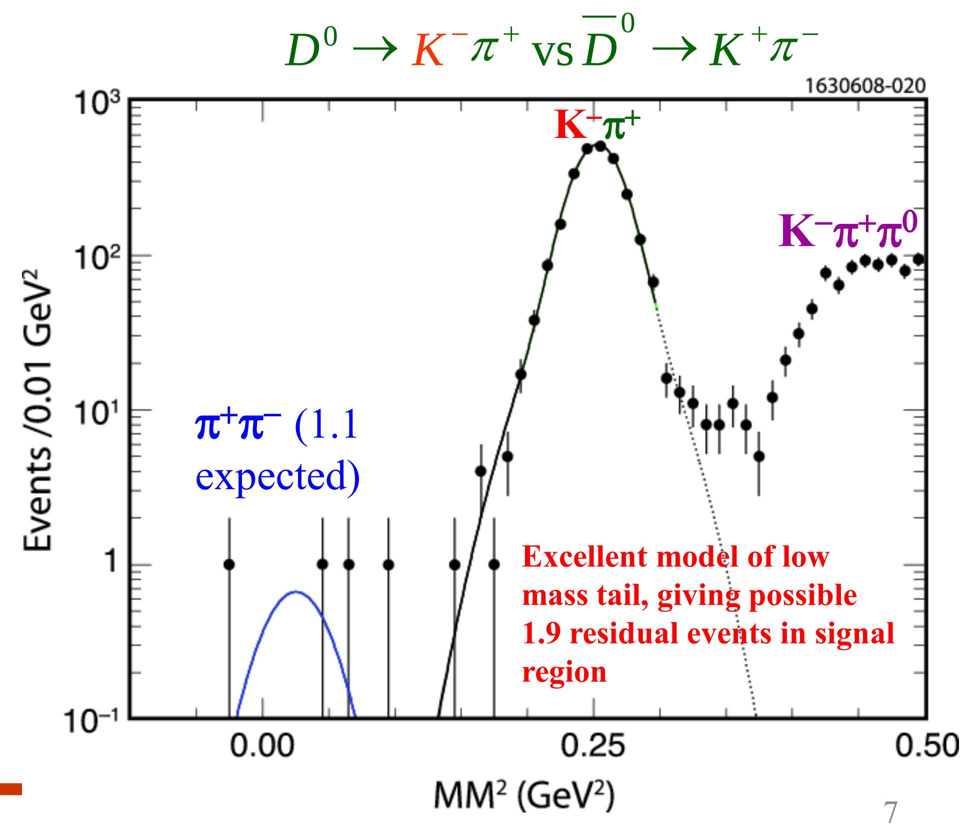
MM^2 *Distributions*

- Require only one charged track
- No additional photon > 250 MeV
- Minimum ionization μ^+ deposits E in calorimeter (E_{Cal}<300 MeV): 98.8% efficient, rejects 45% of



Model of $K^0\pi^+$ shape

- Use $D^0 \to K^-\pi^+$ vs $\overline{D}^0 \to K^+\pi^-$, with loose cut on the 2nd D^0
- Ignore the K^- to calculate MM^2



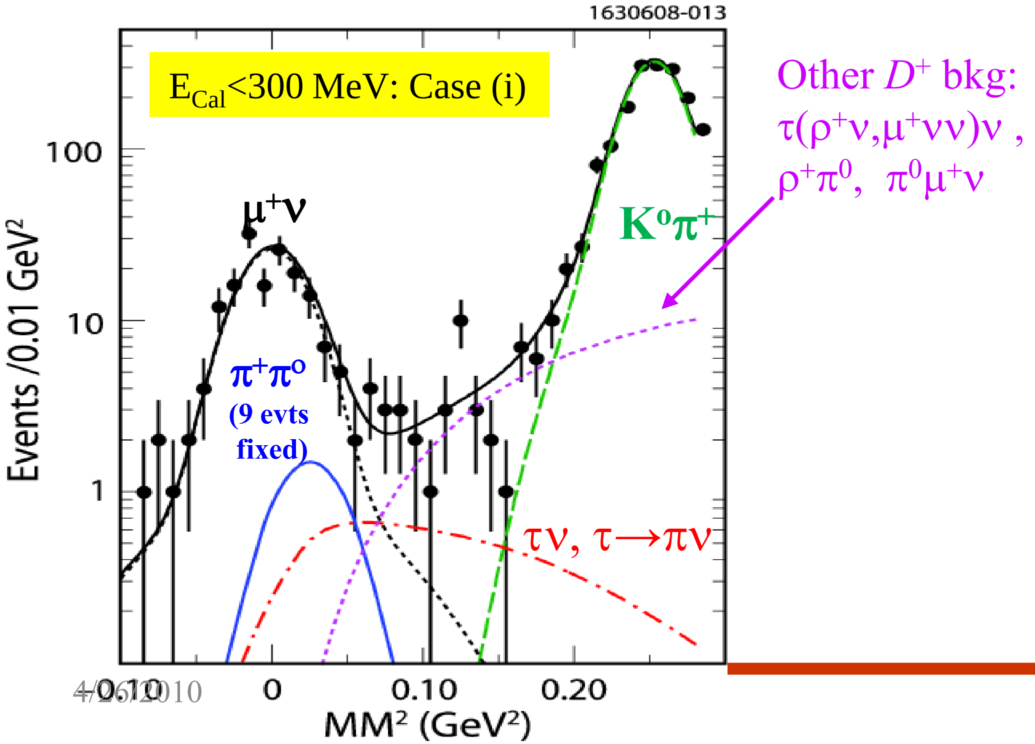
$D^+ \to \mu^+ \nu$ Fits

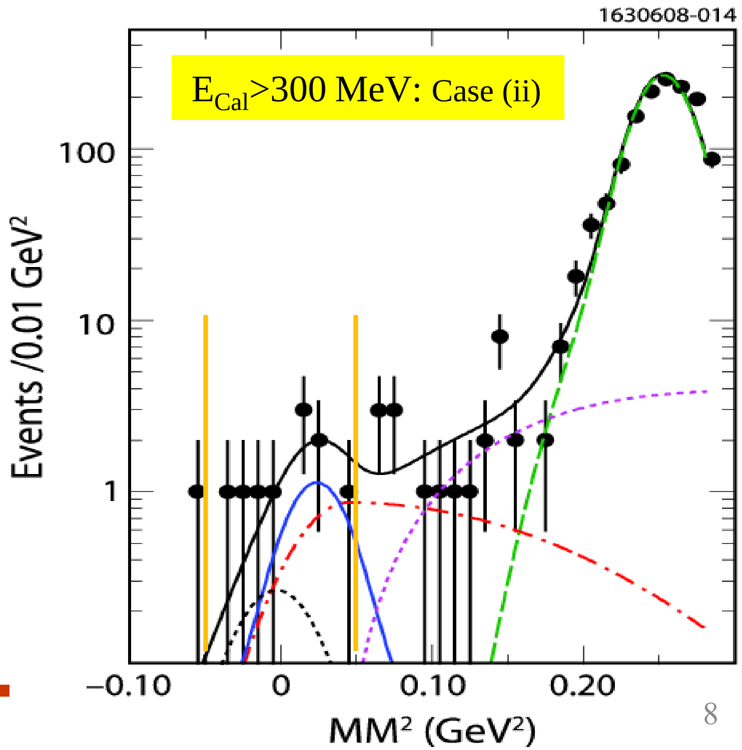
Fits \ N	$D\to\mu\nu$	$D\to\tau(\pi\nu)\nu$
Fix $\tau(\pi\nu)\nu/\mu\nu$	149.7 ± 12.0	28.5 ± 2.3
Float $\tau(\pi\nu)\nu/\mu\nu$	153.9 ± 13.5	13.5 ± 15.3

Events in E_{Cal}>300 MeV can be used for background check in signal region.

- E_{Cal}>300 MeV rejects 98.8% μ^+ and 55% π^+

Fitted $N_{\mu\nu}$ then subtracted by 2.4±1.0 for extra BKG from continuum, D^0 and residual $K^0\pi^+$





Systematic Error Summary

- No one dominant systematic error
- May be hard for BES-III to improve?

Error on f_{D^+} is 1/2 of this

TABLE III. Systematic errors on the $D^+ \to \mu^+ \nu$ branching ratio.

	Systematic errors (%)
Track finding	0.7
PID cut	1.0
MM^2 width	0.2
Minimum ionization cut	1.0
Number of tags	0.6
Extra showers cut	0.4
Radiative corrections	1.0
Background	0.7
Total	2.2

Ref: Statistic error 8.4%

Branching Fractions & f_{D^+}

PRD **78**, 052003 (2008)
818 pb^{-1}

Fix $\tau\nu/\mu\nu$ at SM ratio of 2.67

$$\mathcal{B}(D^+\to\mu^+\nu) = (3.82 \pm 0.32 \pm 0.09)\times 10^{-4}$$

$f_{D^+} = (206.7 \pm 8.5 \pm 2.5)$ MeV [$\pm$ 4.1% $\pm$ 1.2%]

This is the appropriate number in context of SM

Float $\tau\nu/\mu\nu$

$$\mathcal{B}(D^+\to\mu^+\nu) = (3.93 \pm 0.35 \pm 0.10)\times 10^{-4}$$

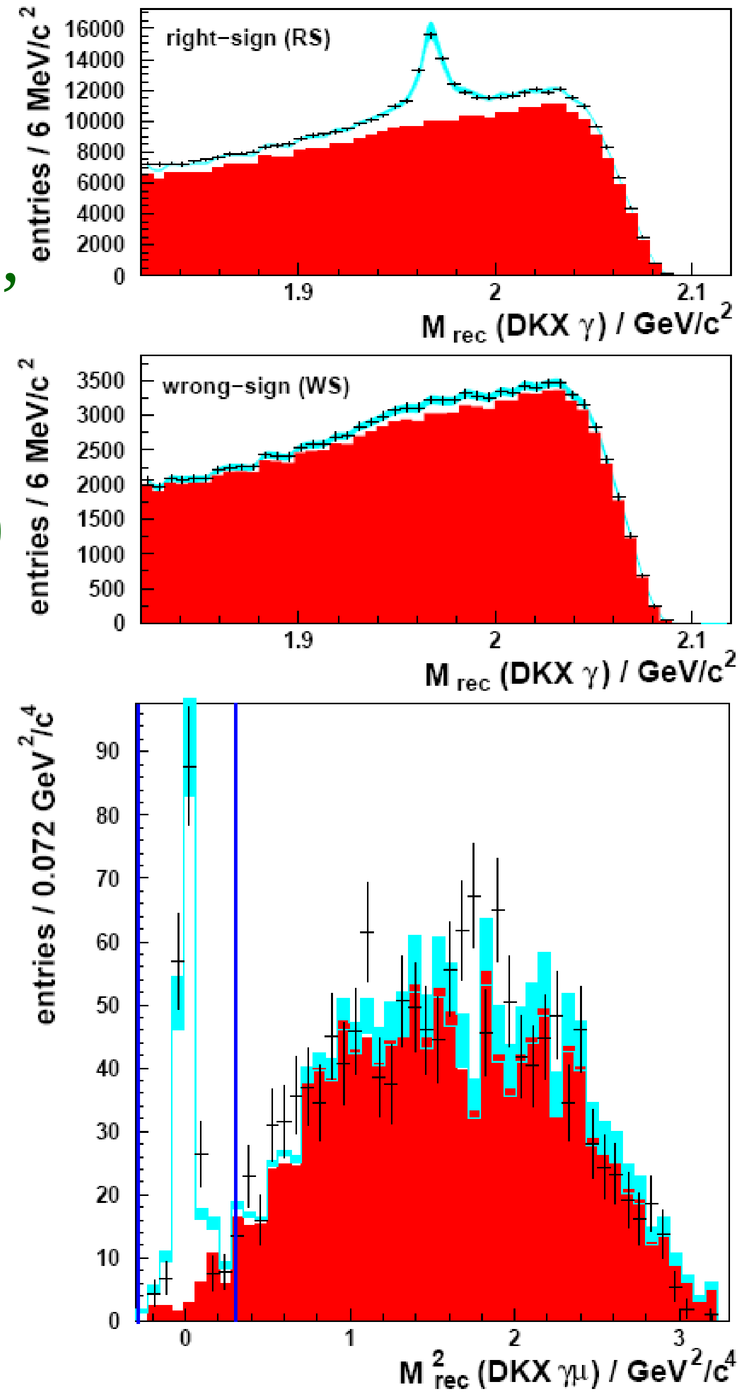
$f_{D^+} = (209.7 \pm 9.3 \pm 2.5)$ MeV

This is the appropriate number for use with Non-SM models

Radiative correction reduced $\mathcal{B}$ by -1%

Belle: $D_s^+ \to \mu^+ \nu$

- Look for $e^+e^- \to DKXD_s^*(\to \gamma D_s)$, where $X=(\gamma)n\pi$ & the D_s is not observed but inferred from calculating the MM (called M_{rec})
- Then add a candidate μ^+ and compute MM^2
- $N_{Ds} = 32100 \pm 870 \pm 1210$
- $N_{\mu\nu} = 169 \pm 16 \pm 8$



Results& Systematic Error

- $\mathcal{B}(D_s^+ \to \mu^+\nu) = (0.638 \pm 0.076 \pm 0.057)\%$
- $f_{Ds} = 274 \pm 16 \pm 12$ MeV

  [±5.8% ±4.4%]

PRL **100**, 241801 (2008)
548 fb^{-1}

Radiative correction reduced $\mathcal{B}$ by -1%

Systematic Error on $\mathcal{B}$

Source of Error	%
Background	4.5
Signal MC statistics	6.4
Muon tracking and Id	2.8
Tag simulation	2.9
Total	**8.9**

Error on f_{Ds} is 1/2 of this

CLEO: $D_s^+ \to \mu^+\nu$ & $\tau^+\nu$ ($\tau^+ \to \pi^+\bar{\nu}$)

Use $e^+e^- \to D_sD_s^*$ at 4170 MeV

$\sigma_{D_s^\pm D_s^{*\mp}} \approx 1\,\text{nb}, \sigma_{D_s^\pm D_s^\mp} \approx 0.05\,\text{nb}, \sigma_{D^*\bar{D}^*} \approx 5\,\text{nb}, \sigma_{D\bar{D}^*} \approx 2\,\text{nb}$

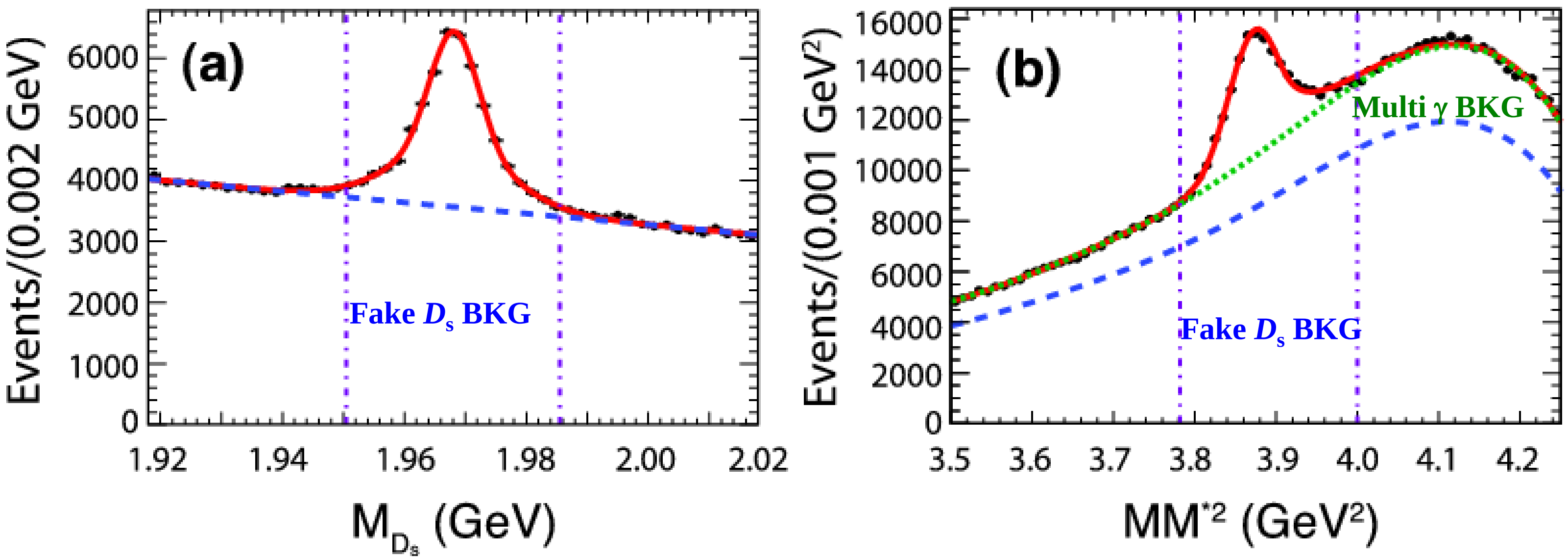
$e^+e^- \to D_s^{*+} D_s^-$ or $e^+e^- \to D_s^+ D_s^{*-}$ $\Big\} \to D_s^+ D_s^- \gamma$

Reconstruct D_s^- & γ as tag

$N_{tag} = 43859 \pm 936 \pm 877$

Sum of 9 tag modes

$MM^{*2} \equiv (\mathbf{p}_{beam} - \mathbf{p}_{D_s^-} - \mathbf{p}_\gamma)^2$

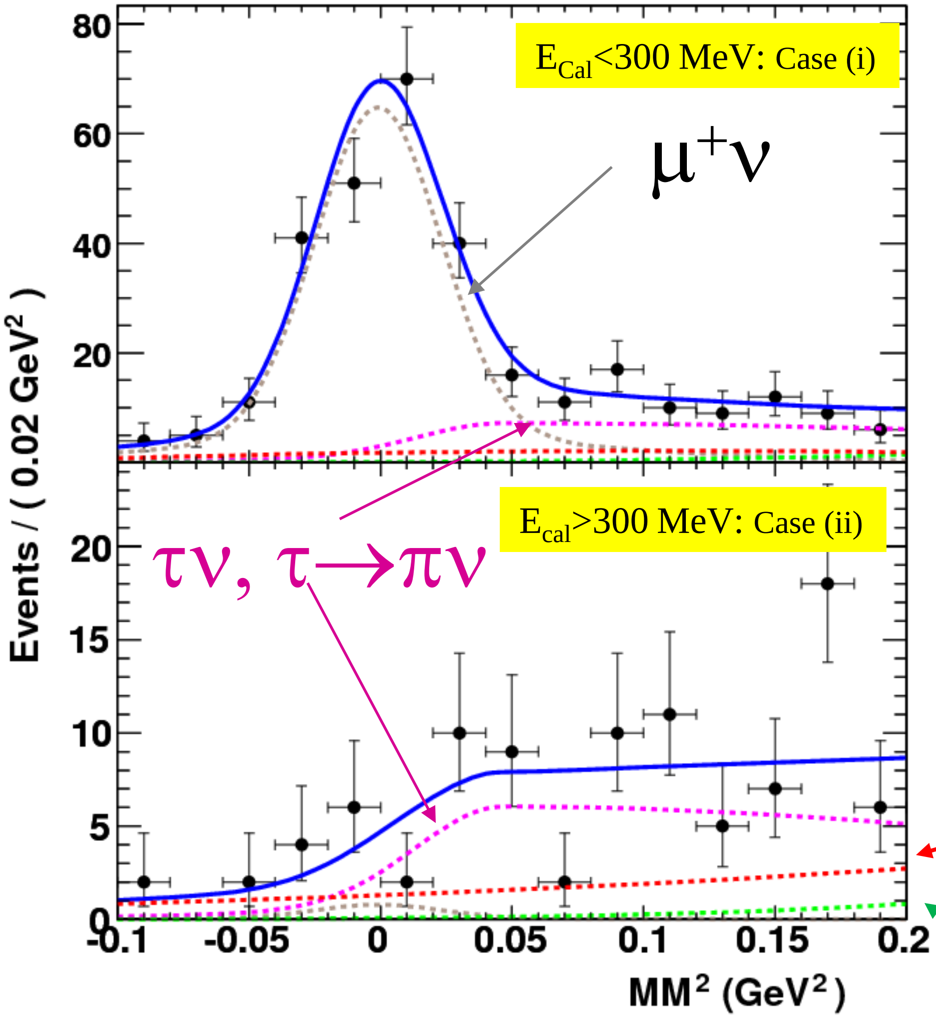


Two dimensional (2D) fit to (M_{D_s}, MM^{*2}) → Better understanding of "fake D_s background" that reduces systematic error

Fit & Results $D_s^+ \to \mu^+\nu$ & $\tau^+\nu$ ($\tau^+ \to \pi^+\overline{\nu}$)

$MM^2 \equiv (\mathbf{p}_{beam} - \mathbf{p}_{D_s^-} - \mathbf{p}_\gamma - \mathbf{p}_\mu)^2$

PRD **79**, 052001 (2009)
600 pb^{-1}



Fake D_s background

Real D_s background

Float $\tau\nu/\mu\nu$:

$N_{\mu\nu}$ = 222.4 ± 17.1 in Case(i)

$\mathcal{B}(D_s^+ \to \mu^+\nu) = (0.565 \pm 0.045 \pm 0.017)\%$

$\Rightarrow f_{Ds} = (257.6 \pm 10.3 \pm 4.3)$ MeV
[± 4.0% ± 1.7%]

Radiative correction reduced $\mathcal{B}$ by -1%

$N_{\tau\nu}$ = 125.6 ± 15.7 in (i)+(ii)

$\mathcal{B}(D_s^+ \to \tau^+\nu) = (6.42 \pm 0.81 \pm 0.18)\%$

$\Rightarrow f_{Ds} = (278.0 \pm 17.5 \pm 4.4)$ MeV

Systematic Error Summary

- Dominated by "number of tags"

Error on f_{Ds} is 1/2 of this

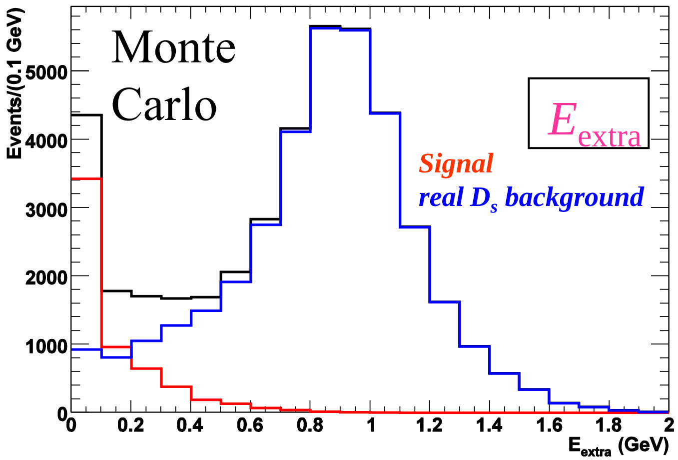
TABLE III. Systematic errors on determination of the $D_s^+ \to \mu^+ \nu$ branching fraction.

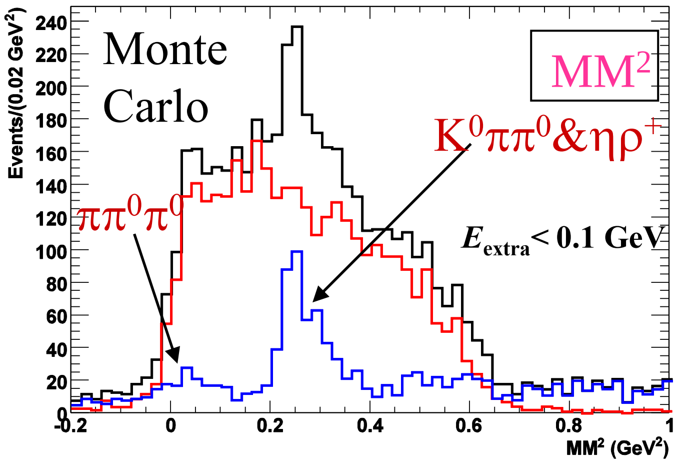
Error Source	Size (%)
Track finding	0.7
Particle identification of μ^+	1.0
MM^2 width	0.2
Photon veto	0.4
Background	1.0
Number of tags	2.0
Tag bias	1.0
Radiative Correction	1.0
Total	3.0

CLEO: $D_s^+ \to \tau^+ \nu$ ($\tau^+ \to \rho^+ \bar{\nu}$)

PRD **80**, 112004 (2009)
600 pb^{-1}

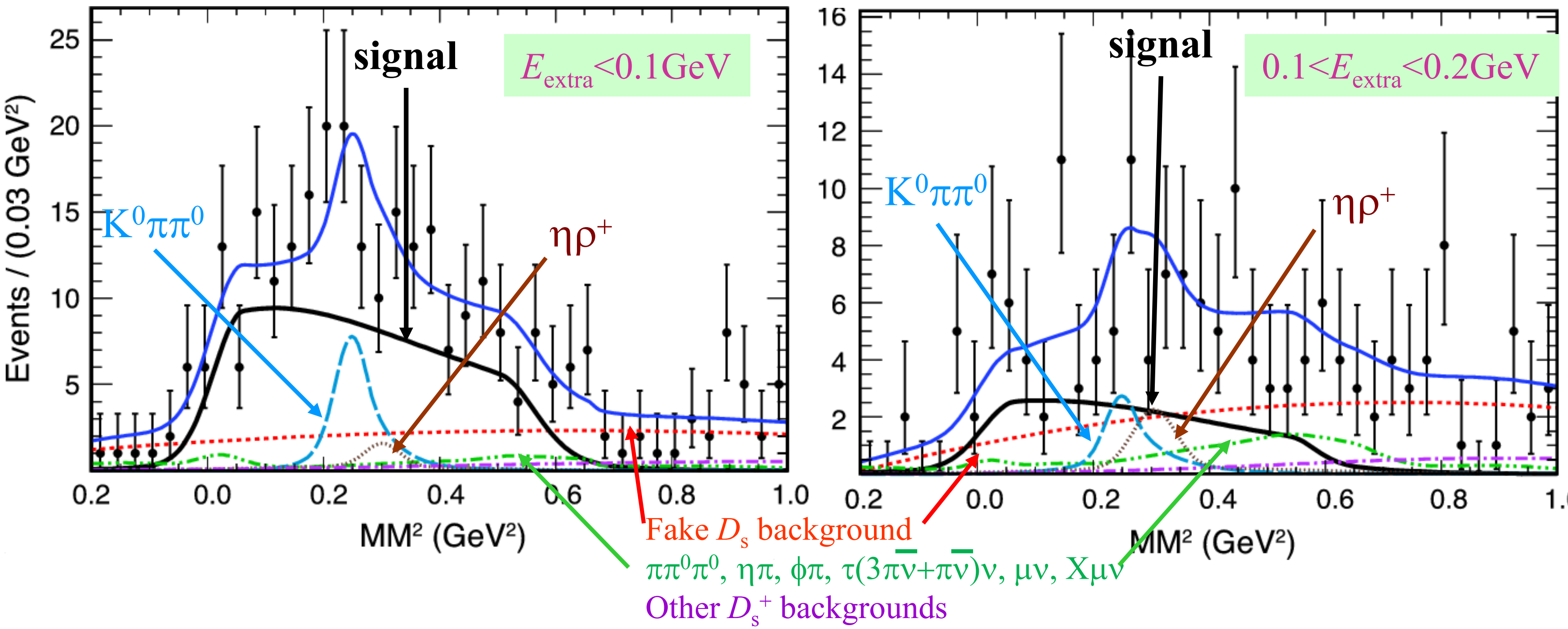
- Same tag and fit technique as $D_s \to \mu^+ \nu$
- Because of the two neutrinos, the signal does not peak in MM^2, but the most important backgrounds do
- Use sum of extra E in calorimeter (E_{extra}) to suppress background





- Fit two MM^2 distributions in first two E_{extra} bins [0,0.1) & [0.1,0.2)
  - External Gaussian constraints on the expected background yields are added in the likelihood fit: allowing them varying within the measured branching fraction error.
  - We also measure three background $\mathcal{B}$'s $D_s^+ \to \pi^+ \pi^0 \pi^0$, $K^0 \pi^+ \pi^0$ & $\eta \rho^+$
- Check: Fitting in E_{extra}>0.8 GeV shows that we well understand the other background.

Fit & Results $D_s^+ \to \tau^+ \nu$ $(\tau^+ \to \rho^+ \bar{\nu})$

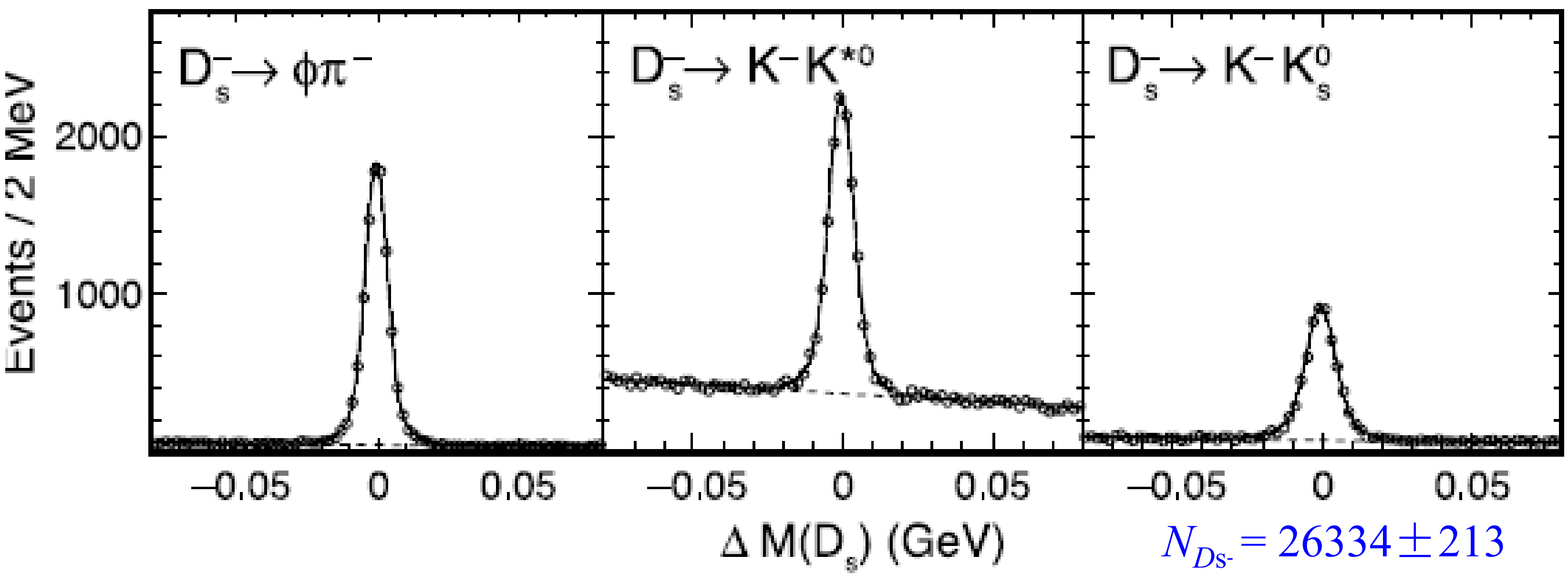


$E_{extra} \in$	Signal yields	Efficiency	$\mathcal{B}(D_s^+ \to \tau^+\nu)$
[0,100] MeV	155.2 ± 16.5	25.3%	(5.48 ± 0.59)%
[100,200] MeV	43.7 ± 11.3	6.9%	(5.65 ± 1.47)%
[0,200] MeV	198.8 ± 20.0	32.2%	$(5.52 \pm 0.57 \pm 0.21)$%

$f_{Ds} = 257.8 \pm 13.3 \pm 5.2$ MeV [$\pm 5.2\%$ $\pm 1.9\%$]

CLEO: $D_s^+ \to \tau^+ \nu$, $\tau^+ \to e^+ \nu \bar{\nu}$

- $\mathcal{B}(D_s^+ \to \tau^+ \nu) \bullet \mathcal{B}(\tau^+ \to e^+ \nu\nu) \sim 1.3\%$ is "large" compared to background $\mathcal{B}(D_s^+ \to X e^+ \nu) \sim 8\%$
- We will be searching for events opposite a tag with one electron and not much other energy
- Opt to use only a subset of the cleanest tags



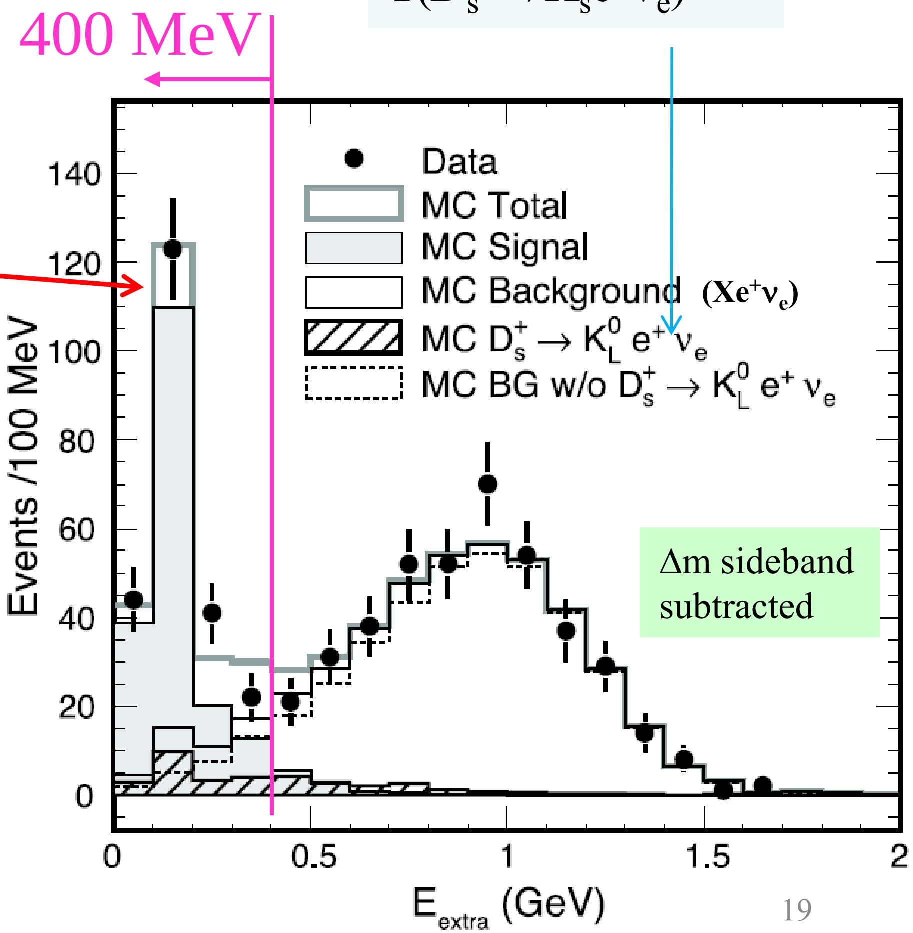
$N_{D_s^-} = 26334 \pm 213$

Measuring $D_s^+ \to \tau^+\nu$, $\tau^+ \to e^+\nu\bar{\nu}$

- Technique is to find events with an e^+ opposite D_s^- tags & no other tracks, with E_{extra} < 400 MeV
- No need to find γ from D_s^*, γ peak
- $\mathcal{B}(D_s^+ \to \tau^+\nu) = (5.30 \pm 0.47 \pm 0.22)\%$
- $f_{Ds} = 252.6 \pm 11.2 \pm 5.6$ MeV [$\pm$4.4% $\pm$2.2%]

PRD **79**, 052002 (2009)
602 pb^{-1}

Largest source of systematic error due to error on $\mathcal{B}(D_s^+ \to K_s e^+\nu_e)$

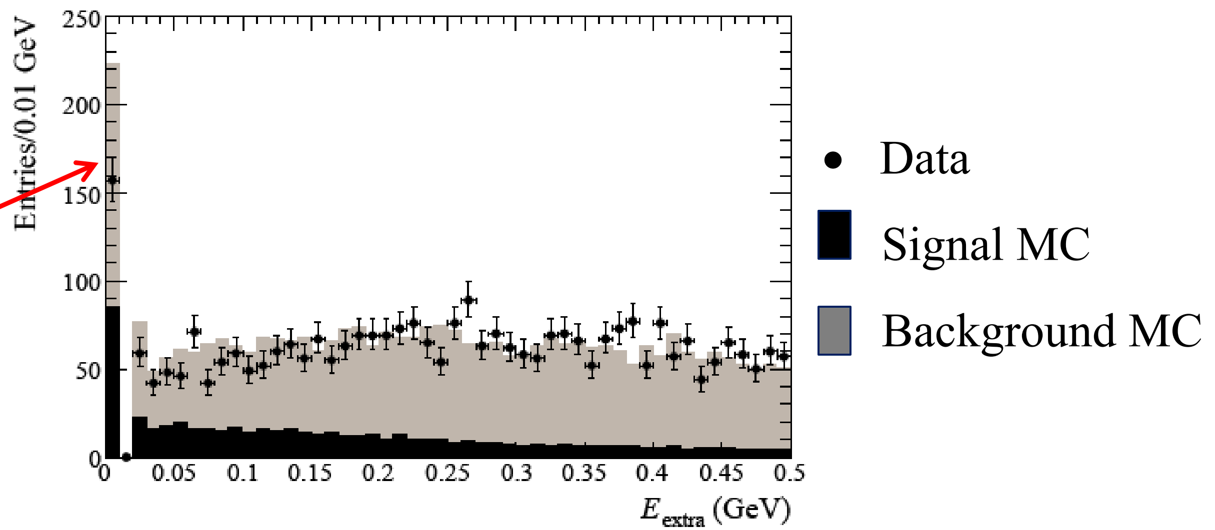


Δm sideband subtracted

BaBar: $D_s^+ \rightarrow \tau^+ \nu$, $\tau^+ \rightarrow e^+ \nu \bar{\nu}$

- Look for $e^+e^- \rightarrow DKXD_s^*(\rightarrow \gamma D_s)$, where X=nπ & the D_s is not observed but inferred from calculating the M_{Rec}
- Normalize to $D_s^+ \rightarrow K_sK^+$ $\mathcal{B}$=(1.49±0.09)%, instead of measuring N_{tag}.
- Require signal side having a single e^+ or only K_sK^+.
- E_{extra} used to separate signal & background

MC overestimates events because of underestimate of noise in calorimeter and of beam background



MC prediction is expected to be right in the whole E_{extra}<0.5 GeV

Fits of $D_s^+ \to \tau^+ \nu,\ \tau^+ \to e^+ \nu \bar{\nu}$

Divide into two samples

$E_{extra} = 0$

- Fit M_{Rec}
- Peaking background: true $D_s \to Xe\nu$ peaks at M_{Rec}

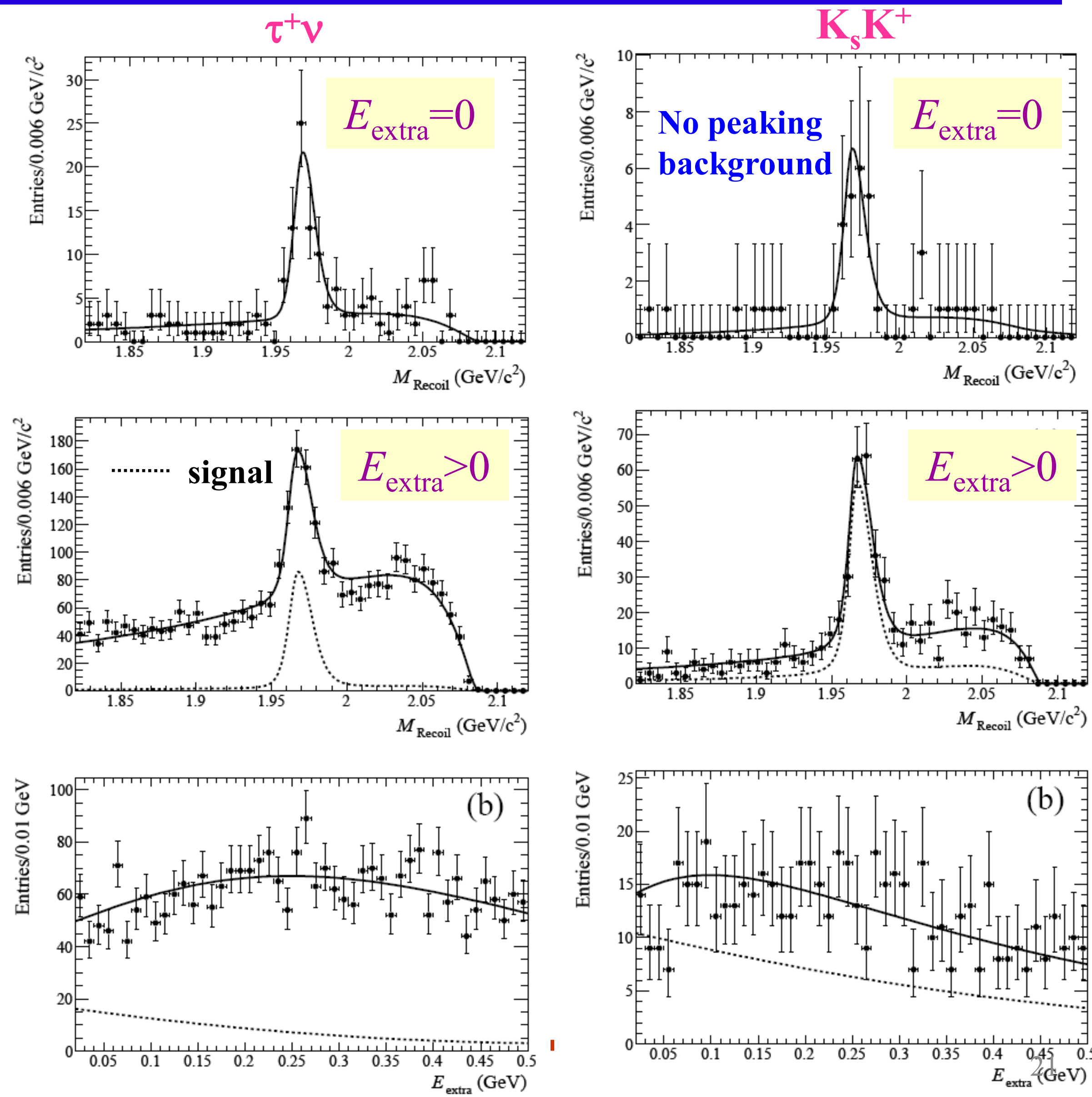
$E_{extra} > 0$

- Fit (M_{Rec}, E_{extra})
- E_{extra} has some discriminating power to separate peaking background

Then subtract # of peaking background estimated from MC

$N_{\tau\nu} = 448 \pm 36$

$N_{KsK} = 333 \pm 28$



Results & Systematic Error

$\mathcal{B}(D_s^+ \to \tau^+ \nu) = (4.5 \pm 0.5 \pm 0.4 \pm 0.3)\%$

arXiv:1003.3064
427 fb^{-1}

Last error is due to uncertainties on the $\mathcal{B}$ for $D_s^+ \to K_s K^+$, $K_s \to \pi^+\pi^-$ and $\tau \to e\nu\nu$

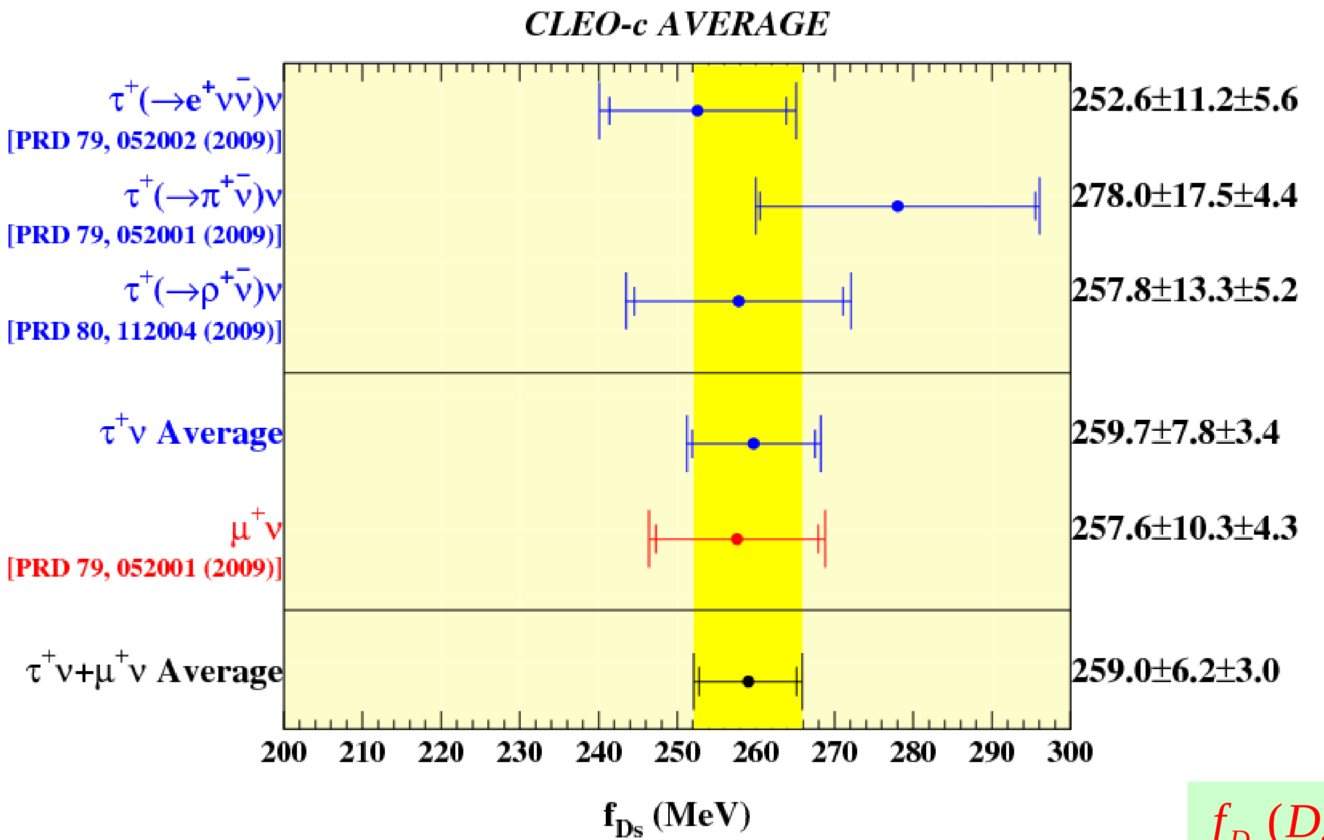
$f_{Ds} = 233 \pm 13 \pm 10 \pm 7$ MeV [$\pm 5.6\%$ $\pm 4.3\%$ $\pm 3.0\%$]

TABLE I: Relative systematic uncertainty estimates on the branching fraction.

Source	Uncertainty (%)
Event Selection	3.0
Particle Identification	0.82
Tracking	0.68
$\tau\nu_\tau$ PDF Distribution	+7.7 -4.7
$K_S^0 K$ PDF Distribution	+4.9 -0.6
Peaking Background	+4.5 -4.3

Varying $\mathcal{B}$'s of peaking backgrounds

CLEO-c f_{Ds} Average



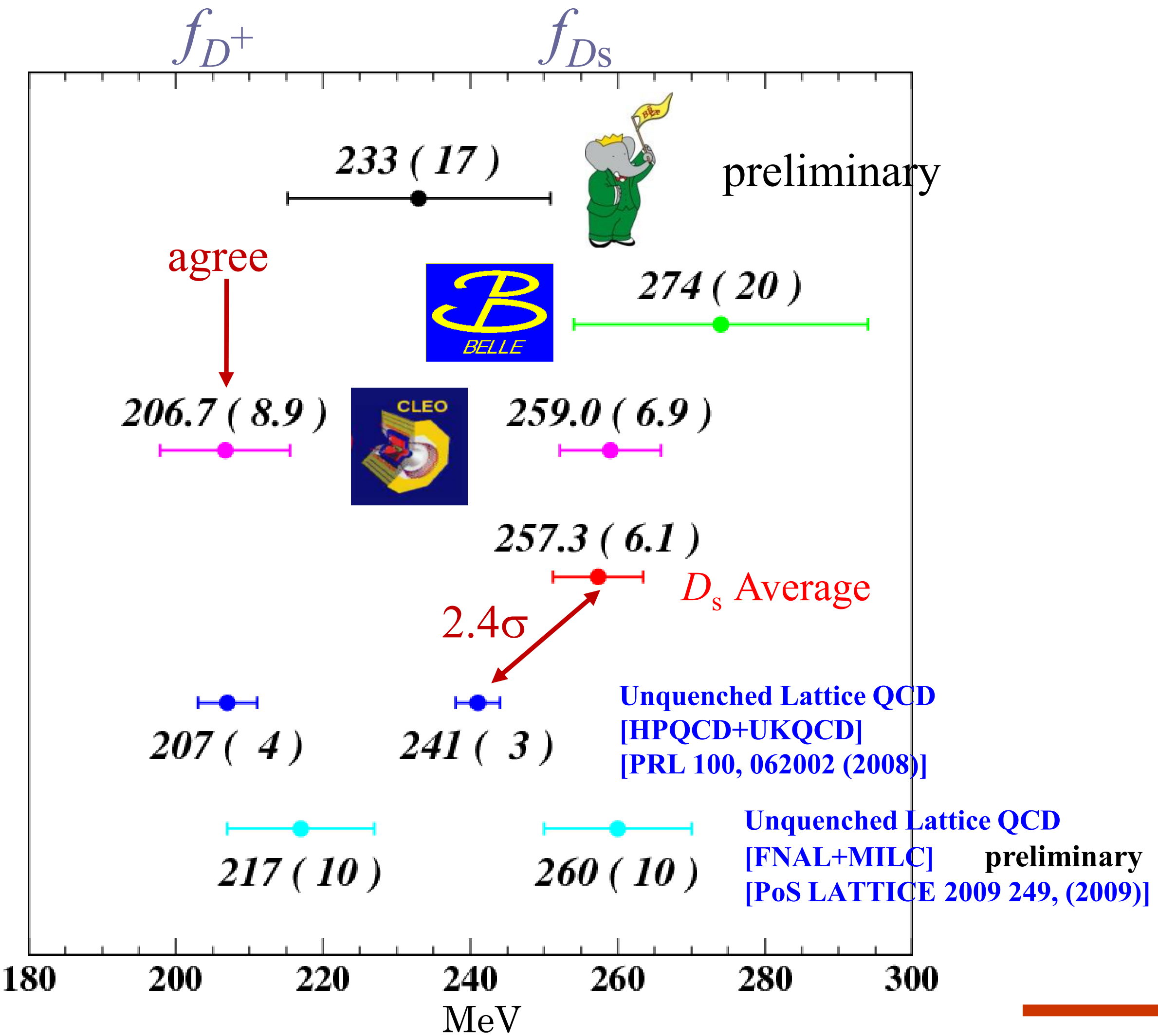
$f_{Ds} = 259.0 \pm 6.2 \pm 3.0$ MeV
$[\pm 2.4\% \ \pm 1.1\%]$

$$\frac{f_{D_s}(D_s^+ \to \tau^+ \nu)}{f_{D_s}(D_s^+ \to \mu^+ \nu)} = 1.01 \pm 0.05$$

All systematic errors include ± 1.8 MeV due to uncertainties on τ_{Ds} (dominant contribution), V_{cs} & Masses.

Summary

Experiments have achieved errors 4.3% on f_{D^+} and 2.4% on f_{Ds}



Prospect from BES-III

- BES-III plans to take 20fb^{-1} each at ψ(3770) and 4170 MeV
  - So far 500 pb^{-1} taken at ψ(3770) from Jan. 2010.
  - Muon detector works in the appropriate momentum region with 90% efficiency and 5% fake rate.

$$\frac{\delta f_D}{f_D} = \sqrt{\left(\frac{\delta B}{2B}\right)^2 + \left(\frac{\delta \tau_D}{2\tau_D}\right)^2 + \left(\frac{\delta V_{cq}}{V_{cq}}\right)^2}$$

	$\delta\mathcal{B}/\mathcal{B}$	$\delta\tau_D/\tau_D$	$\delta V_{cq}/V_{cq}$	$\delta f_D/f_D$
$D^+\to\mu^+\nu$	2%(stat.) 2%(syst.)?	0.6%	0.53%	1.5%
$D_s^+\to\mu^+\nu$	2%(stat.)* 2%(syst.)?	1.4%	0.03%	1.6%

*With clean tags

BES-III can achieve 1-2% errors on f_{D^+}, f_{Ds} and f_{D^+}/f_{Ds}

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"Physics at BES-III" arXiv:0809.1869

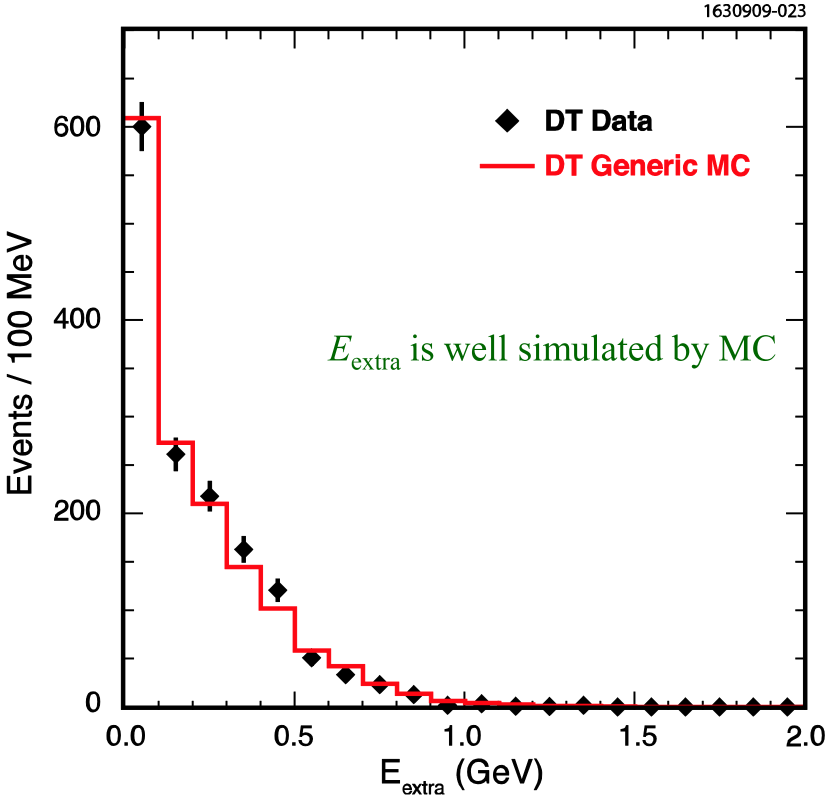
Backup

Systematic error $D_s^+ \to \tau^+ \nu$, $\tau^+ \to e^+ \nu \nu$

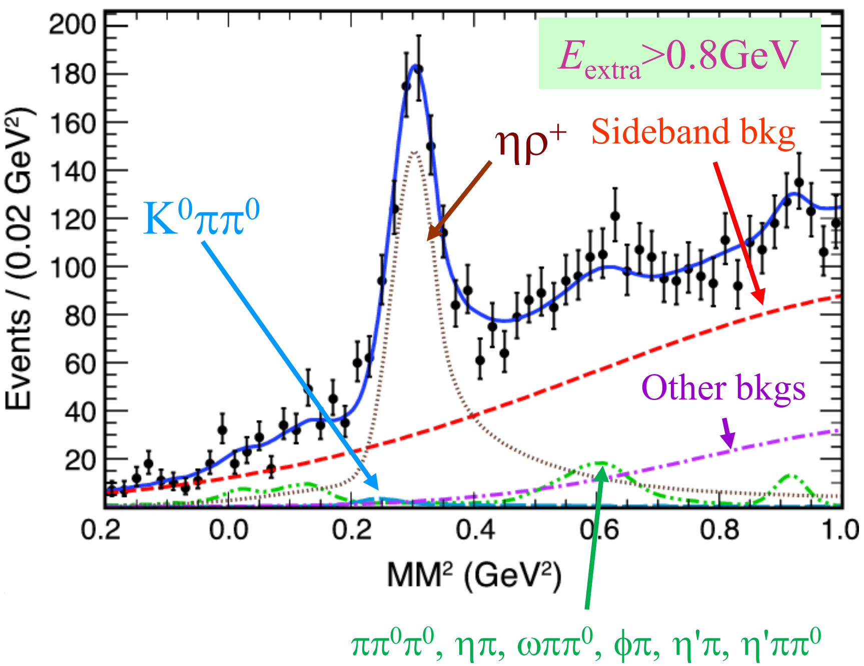
Source	Effect on $\mathcal{B}$ (%)
Background (nonpeaking)	0.7
$D_s^+ \to K_L^0 e^+ \nu_e$ (peaking)	3.2
Extra shower	1.1
Extra track	1.1
$Q_{\text{net}} = 0$	1.1
Non electron	0.1
Secondary electron	0.3
Number of tag	0.4
Tag bias	0.2
Tracking	0.3
Electron identification	1.0
FSR	1.0
Total	4.1

Systematic Checks $D_s^+ \to \tau^+ \nu$ ($\tau^+ \to \rho^+ \nu$)

E_{extra} from fully reconstructed $D_s D_s^*$ event (9×9 tag modes)

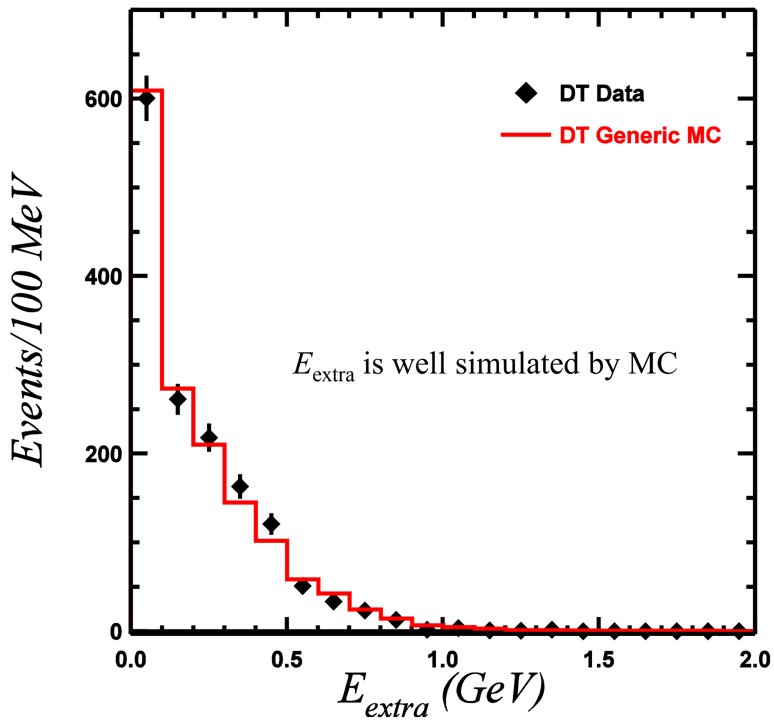


$E_{extra} > 0.8$ GeV test shows that all backgrounds are consistent with MC predictions



E_{extra}: from Fully Reconstructed $D_sD_s^*$

- Compare Data with Generic MC after background subtraction
- Numbers of tags in Generic MC are re-weighted mode-by-mode according to that in the real data
- Value at 300 MeV is chosen, because it has the similar efficiency as $\rho^+\nu$ at 200 MeV



E_{extra} (MeV)	$\epsilon_{Data}(\%)$	$\epsilon_{MC}(\%)$	$\epsilon_{Data}/\epsilon_{MC} - 1$ (%)
<100	40.24 ± 1.27	40.81 ± 0.31	-1.4 ± 3.2
<200	57.75 ± 1.28	59.12 ± 0.31	-2.3 ± 2.2
<300	72.35 ± 1.16	73.21 ± 0.28	-1.2 ± 1.6
<400	83.27 ± 0.97	82.91 ± 0.24	0.4 ± 1.2

← set $\sqrt{1.2^2+1.6^2}$ = 2.0% error

Expected (from MC) and Fit Yields

$D_s^+ \to \tau^+ \nu$ ($\tau^+ \to \rho^+ \nu$)

Component	$\mathcal{B}(\%)$	Constraint Error (%)	$E_{\rm extra} < 0.1$ GeV # MC	# Data	$0.1 < E_{\rm extra} < 0.2$ GeV # MC	# Data
Signal				155.2±16.5		43.7±11.3
$K^0\pi^+\pi^0$	1.0±0.2	20	26.1	25.2±4.8	11.0	10.5±2.1
$\eta\rho^+$	8.9 ± 0.7	4.2	7.1	7.0±0.6	10.6	10.5±0.9
$\pi^+\pi^0\pi^0$	0.65 ± 0.14	22	2.8	2.8 ± 0.6	1.5	1.6 ± 0.3
$\tau^+ \to (\pi^+ + \pi^+\pi^0\pi^0)\overline{\nu}$	1.14 ± 0.06	25†	8.5	8.4 ± 2.1	12.2	10.9 ± 3.0
$\mu^+\nu$	0.576 ± 0.045	5.4	1.0	1.0 ± 0.1	0.48	0.5 ± 0.1
$\eta\pi^+$	1.58 ± 0.21	13.3	0.9	0.9 ± 0.1	0.9	0.9 ± 0.1
$\phi\pi^+$	4.35 ± 0.35	8	1.7	1.7 ± 0.2	2.8	2.8 ± 0.3
$X\mu^+\nu$	5.9	35‡	3.4	3.4 ± 1.2	7.4	6.6 ± 2.6
Other background		30*	11.5	11.4 ± 3.3	11.8	10.5 ± 3.3
Fake D_s^- background				81.8 ± 5.0		74.8 ± 4.6

† We assign a 25% error based on the uncertainties of the resonant substructure.

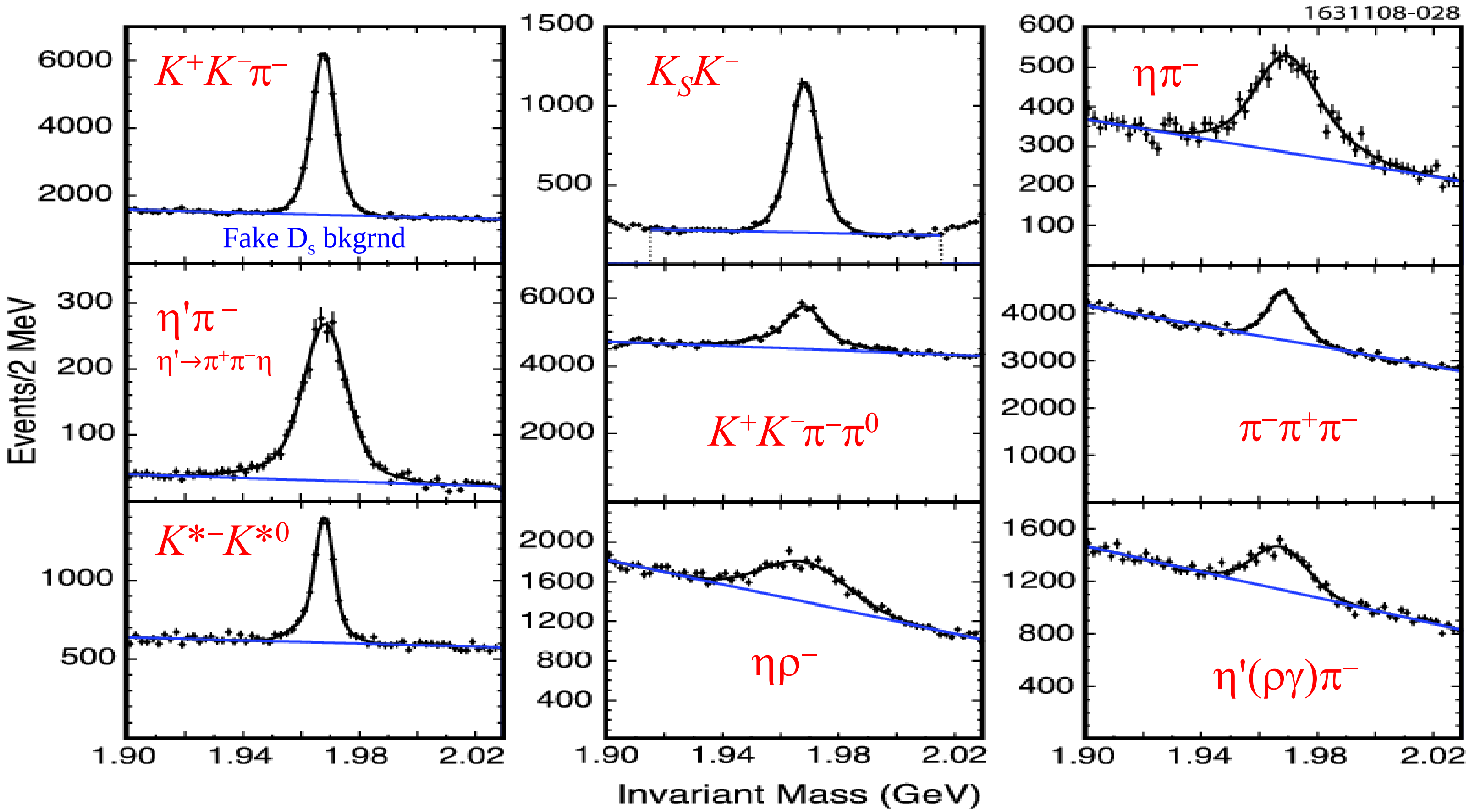
‡ We have checked the yields and assign a 35% uncertainty based on a study of $D_s^+ \to Xe^+\nu$.

$*$ We assign a 30% uncertainty based on the sample size.

Mass Distribution of D_s Tags

We fit MM*^{2} combining with D_s invariant mass to measure the single tag yield:

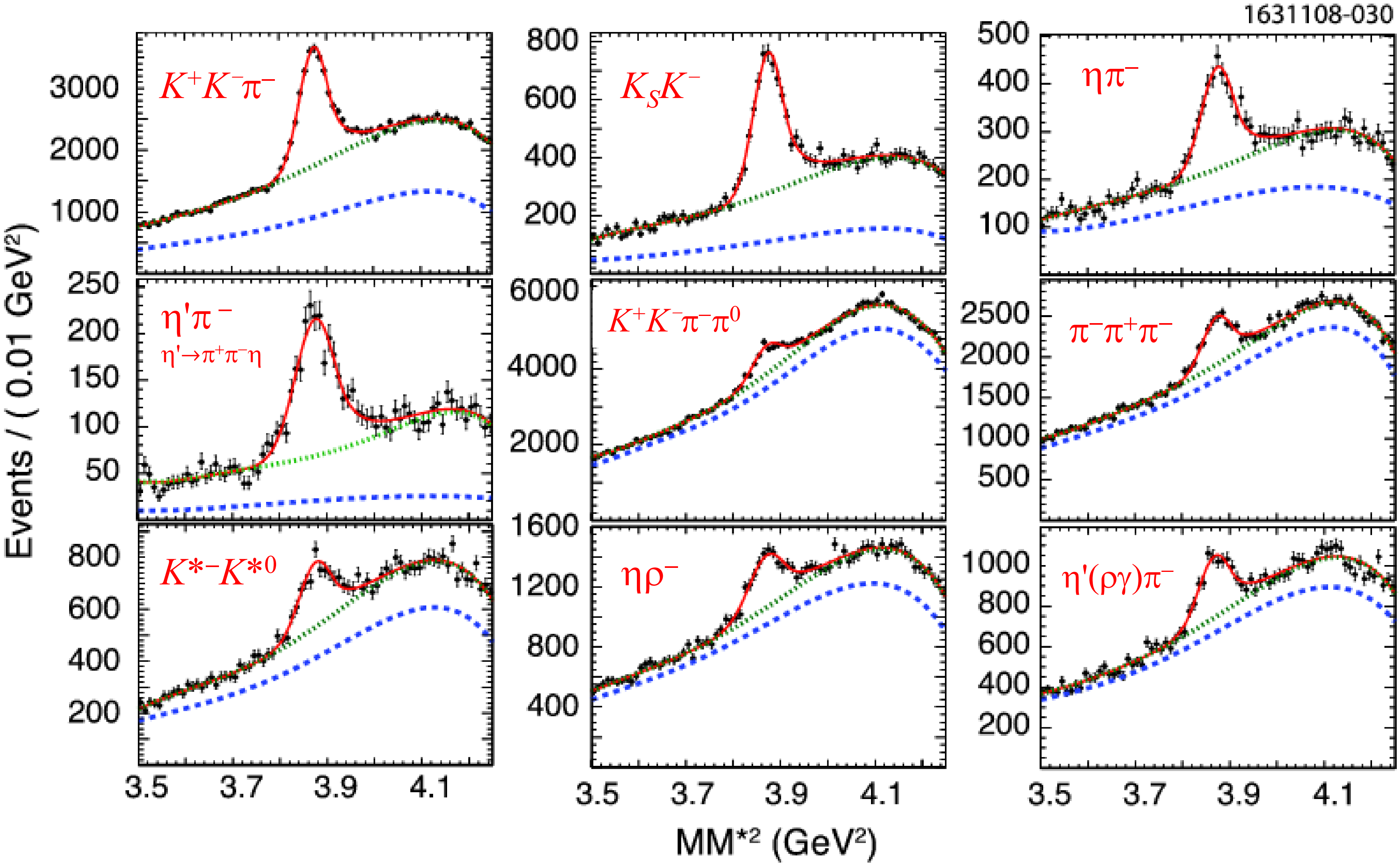
D_s invariant mass gives better control of the wrong D_s (or called "sideband") background.



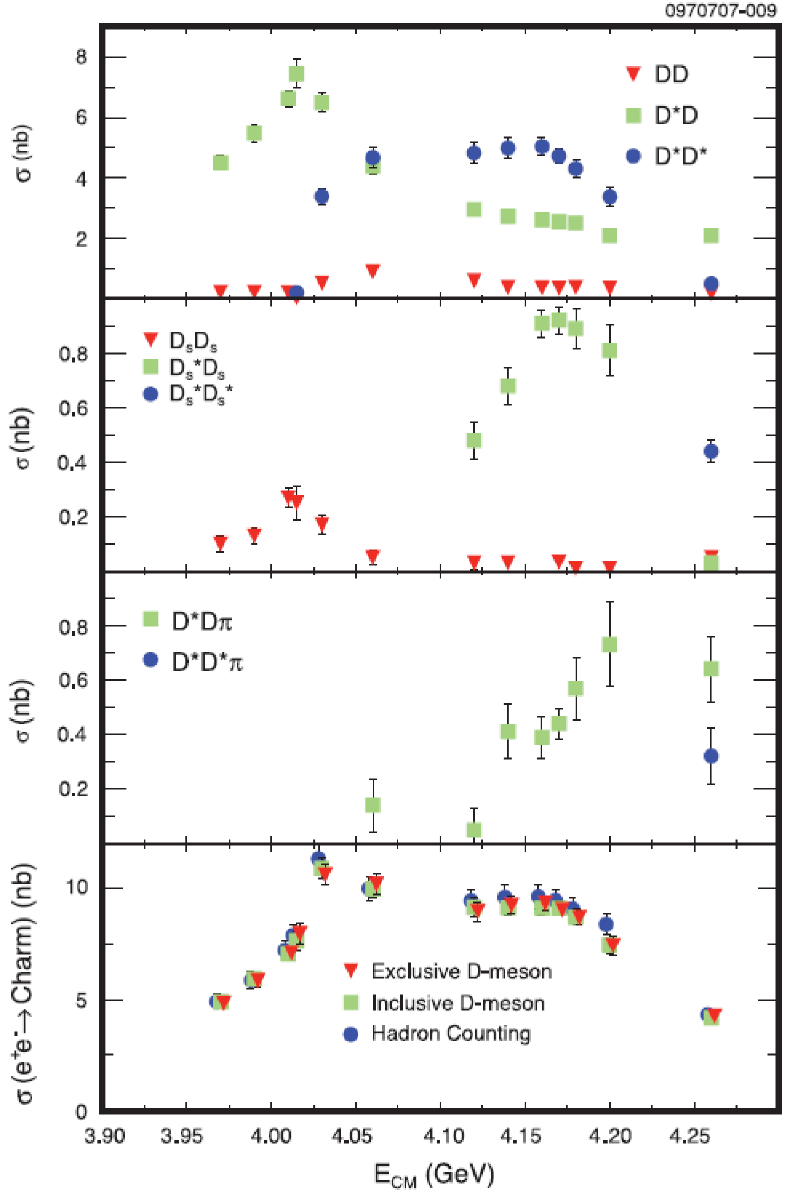
Systematic Errors $D_s^+ \to \tau^+ \nu$ ($\tau^+ \to \rho^+ \nu$)

Source of Error	%
Finding the π^+ track from ρ^+ decay	0.3
Particle identification of π^+	1.0
Finding the π^0 track from ρ^+ decay	1.3
$E_{\text{extra}} < 0.2$GeV signal efficiency	2.0
$E_{\text{extra}} < 0.2$GeV & π^0 efficiencies on background	1.1
Background modeling	1.1
Number of single tag D_s^-	2.0
Tag Bias	1.0
Total	**3.8**

Number of $D_s + \gamma$ Tags



Cross-section vs. CM



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Fit yields of $D_s^+ \to \tau^+ \nu$, $\tau^+ \to e^+ \nu \bar{\nu}$

	N_{sig}	N_{bkg}
$E_{extra}=0$	70 ± 10	87 ± 11
$E_{extra}>0$	378 ± 35	2186 ± 57
Total	448 ± 36	2273 ± 58

Peaking background in total N_{bkg}	N
$D_s^+ \to \eta e^+ \nu$	226
$D_s^+ \to \eta' e^+ \nu$	24
$D_s^+ \to \phi e^+ \nu$	75
$D_s^+ \to K_L e^+ \nu$	59
Total	384

Rediative Correction

- FSR of the muon has been corrected in MC simulation.
- However, another process where the $D^+ \to \gamma D^{*+} \to \gamma \mu^+ \nu$, where the D^{*+} is a virtual vector or axial-vector meson. The process is not helicity suppressed. With our photon energy cut, we find the contribution of such process is about 1%.

Updated Rosner & Stone Table

TABLE I: Experimental results for $\mathcal{B}(D_s^+ \to \mu^+\nu)$, $\mathcal{B}(D_s^+ \to \tau^+\nu)$, and $f_{D_s^+}$. Numbers for $f_{D_s^+}$ have been extracted using updated values for masses and $|V_{cs}|$ (see text); radiative corrections have been included. Common systematic errors in the CLEO results have been taken into account.

Experiment	Mode	$\mathcal{B}$	$f_{D_s^+}$ (MeV)
CLEO-c [12]	$\mu^+\nu$	$(5.65 \pm 0.45 \pm 0.17) \times 10^{-3}$	$257.6 \pm 10.3 \pm 4.3$
Belle [13]	$\mu^+\nu$	$(6.38 \pm 0.76 \pm 0.57) \times 10^{-3}$	$274 \pm 16 \pm 12$
Average	$\mu^+\nu$	$(5.80 \pm 0.43) \times 10^{-3}$	261.5 ± 9.7
CLEO-c [12]	$\tau^+\nu\ (\pi^+\overline{\nu})$	$(6.42 \pm 0.81 \pm 0.18) \times 10^{-2}$	$278.0 \pm 17.5 \pm 3.8$
CLEO-c [14]	$\tau^+\nu\ (\rho^+\overline{\nu})$	$(5.52 \pm 0.57 \pm 0.21) \times 10^{-2}$	$257.8 \pm 13.3 \pm 5.2$
CLEO-c [15]	$\tau^+\nu\ (e^+\nu\overline{\nu})$	$(5.30 \pm 0.47 \pm 0.22) \times 10^{-2}$	$252.6 \pm 11.2 \pm 5.6$
BaBar [16]	$\tau^+\nu\ (e^+\nu\overline{\nu})$	$(4.54 \pm 0.53 \pm 0.40 \pm 0.28) \times 10^{-2}$	$233.8 \pm 13.7 \pm 12.6$
Average	$\tau^+\nu$	$(5.58 \pm 0.35) \times 10^{-2}$	255.5 ± 7.5
Average	$\mu^+\nu + \tau^+\nu$		257.5 ± 6.1

TABLE II: Theoretical predictions of $f_{D_s^+}$, f_{D^+}, and $f_{D_s^+}/f_{D^+}$. QL indicates a quenched-lattice calculation, while PQL indicates a partially-quenched lattice calculation. (Only selected results having errors are included.)

Model	$f_{D_s^+}$(MeV)	f_{D^+}(MeV)	$f_{D_s^+}/f_{D^+}$
Experiment (our averages)	257.5 ± 6.1	206.7 ± 8.9	1.25 ± 0.06
Lattice(HPQCD+UKQCD) [26]	241 ± 3	208 ± 4	1.162 ± 0.009
Lattice (FNAL+MILC+HPQCD) [27]	260 ± 10	217 ± 10	1.20 ± 0.02
PQL [28]	244 ± 8	197 ± 9	1.24 ± 0.03
QL (QCDSF) [29]	$220 \pm 6 \pm 5 \pm 11$	$206 \pm 6 \pm 3 \pm 22$	$1.07 \pm 0.02 \pm 0.02$
QL (Taiwan) [30]	$266 \pm 10 \pm 18$	$235 \pm 8 \pm 14$	$1.13 \pm 0.03 \pm 0.05$
QL (UKQCD) [31]	$236 \pm 8^{+17}_{-14}$	$210 \pm 10^{+17}_{-16}$	$1.13 \pm 0.02^{+0.04}_{-0.02}$
QL [32]	$231 \pm 12^{+6}_{-1}$	$211 \pm 14^{+2}_{-12}$	1.10 ± 0.02
QCD Sum Rules [33]	205 ± 22	177 ± 21	$1.16 \pm 0.01 \pm 0.03$
QCD Sum Rules [34]	235 ± 24	203 ± 20	1.15 ± 0.04
Field Correlators [35]	260 ± 10	210 ± 10	1.24 ± 0.03
Light Front [36]	268.3 ± 19.1	206 (fixed)	1.30 ± 0.04