## B -meson mixing on the lattice

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## Lattice QCD Meets Experiment Workshop 2010

- Fermilab, April 26 (2010) .


## 1. Impact of $B^{0}$ mixing on the flavour physics program

\# Determination of fundamental parameters of the SM

* CKM matrix elements: $\left|V_{t d}\right|,\left|V_{t s}\right|$


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In the Standard Model


$$
\left.\Delta M_{q}\right|_{\text {theor. }}=\frac{G_{F}^{2} M_{W}^{2}}{6 \pi^{2}}\left|V_{t q}^{*} V_{t b}\right|^{2} \eta_{2}^{B} S_{0}\left(x_{t}\right) M_{B_{s}} f_{B_{q}}^{2} \hat{B}_{B_{q}}
$$

** Non-perturbative input

$$
\frac{8}{3} f_{B_{q}}^{2} B_{B_{q}}(\mu) M_{B_{q}}^{2}=\left\langle\overline{B_{q}^{0}}\right| O_{1}\left|B_{q}^{0}\right\rangle(\mu) \text { with } \quad O_{1} \equiv\left[\overline{b^{i}} q^{i}\right]_{V-A}\left[\overline{b^{j}} q^{j}\right]_{V-A}
$$

## 1. Impact of $B^{0}$ mixing on the flavour physics program

\# Determination of fundamental parameters of the SM

* CKM matrix elements: $\left|V_{t d}\right|,\left|V_{t s}\right|$
\# Unveiling New Physics effects.
* Hints of discrepancies between SM expectations and some flavour observables
A. Buras, talk at EPS-HEP 2009 or R. Van de Water, plenary talk at Lat09
** $B_{s}$ mixing phase $\beta_{s}$ as extracted from experiment ( $S_{J / \psi \phi}$ ) and in the SM.


## 1. Impact of $B^{0}$ mixing on the flavour physics program

UT fit: Global fit to the CKM unitarity triangle using experimental and theoretical constraints. talk by E. Lunghi

$$
2-3 \sigma \text { tension in the CKM description }
$$

* Tension is between the three most precise constraints: the $K^{0}-\bar{K}^{0}$ mixing parameter $\epsilon_{K}$, the ratio of mass differences $\Delta M_{B_{s}} / \Delta M_{B_{d}}$ describing $B^{0}-\bar{B}^{0}$ mixing and $\sin (2 \beta)$.



Laiho, Van de Water and Lunghi, Phys.Rev.D81:034503(2010)

* Constraints from $\Delta M_{d} / \Delta M_{s}$ limited by lattice errors for $\xi=\frac{f_{B_{s}} \sqrt{B_{B_{s}}}}{f_{B_{d}} \sqrt{B_{B_{d}}}}$.


## 1. Impact of $B^{0}$ mixing on the flavour physics program

\# Constraining NP models.

* Comparison of $\Delta M$ and $\Delta \Gamma$ with experiment also provides bounds for NP effects


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\# Bag parameters $B_{B_{s}}$ and $B_{B_{d}}$ can be used for theoretical predictions of, for example, $\mathcal{B} r\left(B \rightarrow \mu^{+} \mu^{-}\right)$.

$$
\frac{\mathcal{B} r\left(B_{q} \rightarrow \mu^{+} \mu^{-}\right)}{\Delta M_{q}}=\tau\left(B_{q}\right) 6 \pi \frac{\eta_{Y}}{\eta_{B}}\left(\frac{\alpha}{4 \pi M_{W} \sin ^{2} \theta_{W}}\right)^{2} m_{\mu}^{2} \frac{Y^{2}\left(x_{t}\right)}{S\left(x_{t}\right)} \frac{1}{\hat{B}_{q}}
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$$

* Using lattice determinations of $\hat{B}_{q}$ HPQCD, PRD80 (2009) 014503

$$
\begin{aligned}
\rightarrow & \mathcal{B} r\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right)=(3.19 \pm 0.19) \times 10^{-9} \text { and } \\
& \mathcal{B} r\left(B_{d} \rightarrow \mu^{+} \mu^{-}\right)=(1.02 \pm 0.09) \times 10^{-10}
\end{aligned}
$$

* CDF ( $D \varnothing$ ) bounds $\mathcal{B} r\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right) \leq 3.3(5.3) \times 10^{-8}$,

$$
\mathcal{B} r\left(B_{d} \rightarrow \mu^{+} \mu^{-}\right) \leq 1 \times 10^{-8}
$$

## 1. Impact of $B^{0}$ mixing on the flavour physics program

\# In conjunction with experimental measurements ...

$$
\begin{array}{cc}
\text { HFAG 10 CDF (Run II) } \\
\left.\Delta M_{d}\right|_{\text {exp. }}=(0.507 \pm 0.005) p s^{-1} & \left.\Delta M_{s}\right|_{\text {exp. }}=(17.77 \pm 0.12) p s^{-1}
\end{array}
$$

HFAG 10

$$
\left(\frac{\Delta \Gamma}{\Gamma}\right)_{d}=0.010 \pm 0.037 \quad\left(\frac{\Delta \Gamma}{\Gamma}\right)_{s}=0.09 \pm 0.05
$$

## 2. $N_{f}=2+1$ unquenched lattice calculation of $B^{0}$ mixing parameters

## Quenched approximation: neglect vacaum polarization effects

 $\rightarrow$ uncontrolled and neducible errors talk by $\mathbf{A}$. Kronfeld- HPQCD: E. Gámiz et al., Phys.Rev.D80:014503,2009
* Configurations: MILC staggered.
* Light quarks: Improved staggered (Asqtad)
* Heavy quarks: NRQCD
- Fermilab lattice/MILC: R.T. Evans et al. ,PoS(LAT2009)245; R.T. Evans et
al., PoS(LAT2008)052 preliminary
* Configurations: MILC staggered.
* Light quarks: Improved staggered (Asqtad)
* Heavy quarks: Fermilab $\rightarrow$ it can also be used for $c$ quarks.
- RBC/UKQCD: C. Albertus et al., arXiv:1001.2023 exploratory
* Configurations: RBC/UKQCD domain wall.
* Light quarks: Domain wall.
* Heavy quarks: Static.


### 2.1. Some details of the simulations

|  | HPQCD | FNAL/MILC | RBC/UKQCD |
| :---: | :---: | :---: | :---: |
| $a$ | 0.12 fm | 0.12 fm | 0.11 fm |
|  | 0.09 fm | 0.09 fm |  |
| $\# m_{\text {light }}^{\text {sea }} / m_{s}^{\text {sea }}$ | 4 | 4 | 3 |
| $\# m^{\text {valence }}$ | 2 | 2 |  |
| renormalization | full QCD | 6 (include full QCD) | full QCD |
| lightest $\pi(M e V)$ | $\sim 230$ | one-loop | one-loop |

See talk by C. Bernard
2.2. Results: $f_{B_{q}} \sqrt{B_{B_{q}}}$

HPQCD, PRD80 (2009) 014503



$$
f_{B_{s}} \sqrt{\hat{B}_{B_{s}}}=266(6)(17) \mathrm{MeV}
$$

$$
f_{B_{d}} \sqrt{\hat{B}_{B_{d}}}=216(9)(12) \mathrm{MeV}
$$

## Chiral+continuum extrapolations: NLO Staggered CHPT.

* accounts for NLO quark mass dependence.
* accounts for light quark discretization effects through $\mathcal{O}\left(\alpha_{s}^{2} a^{2} \Lambda_{Q C D}^{2}\right)$
$\rightarrow$ remove the dominant light discretization errors

$$
\text { 2.2. Results: } \xi=\frac{f_{B_{s}} \sqrt{B_{B_{s}}}}{f_{B_{d}} \sqrt{B_{B_{d}}}}
$$

RBC/UKQCD: No extrapolation
to the continuum

FNAL/MILC: No
renormalization included, but we expect a large cancellation between $B_{s}^{0}$ and $B_{d}^{0}$ renor. corrections.

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HPQCD result $\Longrightarrow\left|\frac{V_{t d}}{V_{t s}}\right|=0.214(1)(5)$

### 2.3. Error budget for $f_{B} \sqrt{\hat{B}_{B}}$

| Source (\%) | HPQCD <br> (final) | FNAL/MILC <br> (preliminary) |
| :--- | :---: | :---: |
| stat. + chiral extrap. <br> $\chi$ PT + light quark disc. <br> residual $a^{2}$ extrap. $2^{2.3-4.1}$ | $2.7-4.0$ |  |
| $\quad$ (heavy quark disc.) | $3.0-2.0$ | $0.4-2.5$ |
| $r_{1}^{3 / 2}$ uncertainty | 2.3 | 2.0 |
| $g_{B^{*} B \pi}$ uncertainty | 1.0 | $3.0-3.1$ |
| quark masses tuning | $1.5-1.0$ | $0.3-0.6$ |
| operator matching | 4.0 | $0.6-0.5$ |
| relativistic corr. | 2.5 | 4.0 |
| Finite volume | $\leq 0.5$ | - |
| Total | $6.7-7.1$ | $6.1-7.3$ |

* Ranges indicate $B_{s}^{0}-B_{d}^{0}$ values.


### 2.3. Error budget for $\xi$

| Source (\%) | HPQCD <br> (final) | FNAL/MILC <br> (preliminary) | RBC/UKQCD <br> (exploratory) |
| :--- | :---: | :---: | :---: |
| stat. + chiral extrap. <br> $\chi$ PT + light quark disc. <br> residual $a^{2}$ extrap. $2^{2.0}$ | 3.1 | $6-5$ |  |
| $\quad$ (heavy quark disc.) | 0.3 | 2.8 | 7 |
| $r_{1}^{3 / 2}$ uncertainty | 0. | 0.2 | 4 |
| $g_{B * B \pi}$ uncertainty | 1.0 | 0.2 | 2.3 |
| quark masses tuning | 1.0 | 0.7 | 2 |
| operator matching | 0.7 | $\leq 0.5$ | 2 |
| relativistic corr. | 0.4 | - | 2 |
| Finite volume | $\leq 0.1$ | $\leq 0.1$ | 1 |
| Total | 2.6 | $\sim 4.3$ | 9 |

### 2.4. Improvements of lattice calculations of <br> $\xi$ in 2 years

| Source (\%) | HPQCD | FNAL/MILC | improvement <br> (factor of) |
| :--- | :---: | :---: | :---: |
| stat. + chiral extrap. | 2.0 | 3.1 |  |
| $\chi$ PT + light quark disc. <br> residual $a^{2}$ extrap. | - | 2.8 |  |
| $\quad$ (heavy quark disc.) | 0.3 | 0.2 |  |
| $r_{1}^{3 / 2}$ uncertainty | 0. | 0.2 |  |
| $g_{B^{*} B \pi}$ uncertainty | 1.0 | 0.3 |  |
| quark masses tuning | 1.0 | 0.7 |  |
| operator matching | 0.7 | $\leq 0.5$ |  |
| relativistic corr. | 0.4 | - |  |
| Finite volume | $\leq 0.1$ | $\leq 0.1$ |  |

2.4. Improvements of lattice calculations of $\xi$ in 2 years

| Source (\%) | HPQCD | FNAL/MILC | improvement |
| :--- | :---: | :---: | :---: |
| stat. + chiral extrap. | 1.0 | 1.5 | 2 V |
| $\chi^{\mathrm{PT}}+$ light quark disc. | - | 2.8 |  |
| residual $a^{2}$ extrap. | 0.3 | 0.2 |  |
| (heavy quark disc.) |  |  |  |
| $r_{1}^{3 / 2}$ uncertainty | 0. | 0.2 |  |
| $g_{B^{*} B \pi}$ uncertainty | 1.0 | 0.3 |  |
| quark masses tuning | 1.0 | 0.7 |  |
| operator matching | 0.7 | $\leq 0.5$ |  |
| relativistic corr. | 0.4 | - |  |
| Finite volume | $\leq 0.1$ | $\leq 0.1$ |  |

* Better statistics: More configurations (MILC multiplied by 4 $N_{\text {configurations }}$ ), improved techniques for correlation fits (smearing, random wall sources, ...)
$\sqrt{ }$ checked for one coarse ensemble (C. Bouchard for FNAL/MILC)
2.4. Improvements of lattice calculations of $\xi$ in 2 years

| Source | HPQCD | FNAL/MILC | improvement |
| :--- | :---: | :---: | :---: |
| stat. + chiral extrap. | 1.0 | 1.5 | $\mathbf{2}$ |
| $\chi$ PT + light quark disc. | - | 1.6 | $\mathbf{1 . 5 - 2}$ |
| residual $a^{2}$ extrap. | 0.2 | 0.1 | $\mathbf{1 . 5}$ |
| $\quad$ (heavy quark disc.) | 0. | 0.2 |  |
| $r_{1}^{3 / 2}$ uncertainty | 1.0 | 0.3 |  |
| $g_{B^{*} B \pi}$ uncertainty | 1.0 | 0.7 |  |
| quark masses tuning | $\leq 0.5$ | $\leq 0.5$ |  |
| operator matching | 0.4 | - |  |
| relativistic corr. | $\leq 0.1$ | $\leq 0.1$ |  |
| Finite volume |  |  |  |

* Smaller values of lattice spacing ( FNAL/MILC and HPQCD)
$a=0.09 \mathrm{fm}$ (fine) $\rightarrow a=0.06 \mathrm{fm}$ (superfine) (eventually $a=0.045 \mathrm{fm}$ ) ** Matching $\left(f_{B} \sqrt{B_{B}}\right): 4 \% \rightarrow 2.5 \%$
2.4. Improvements of lattice calculations of $\xi$ in 2 years

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| :---: | :---: | :---: | :---: |
| stat. + chiral extrap. | 1.0 | 1.5 | $\mathbf{2}$ |
| $\chi$ PT + light quark disc. | - | 1.6 | $\mathbf{1 . 5 - 2}$ |
| residual $a^{2}$ extrap. | 0.2 | 0.1 | $\mathbf{1 . 5}$ |
| (heavy quark disc.) | 0. | 0.2 | $\mathbf{2}$ |
| $r_{1}^{3 / 2}$ uncertainty | 0.5 | 0.2 | $\mathbf{1 . 5}$ |
| $g_{B^{*} B \pi}$ uncertainty | 0.5 | 0.3 |  |
| quark masses tuning | $\leq 0.5$ | $\leq 0.5$ |  |
| operator matching | 0.4 | 0.1 |  |
| relativistic corr. | $\leq 0.1$ |  |  |
| Finite volume |  |  |  |

* Better determination of inputs
* Improving the actions: HISQ, heavy formulations (improved Fermilab action, improved NRQCD)


### 2.4. Improvements of lattice calculations of $\xi$ in 2 years

| Source | HPQCD | FNAL/MILC | improvement |
| :--- | :---: | :---: | :---: |
| stat. + chiral extrap. | 1.0 | 1.5 | $\mathbf{2}$ |
| $\chi^{\text {PT }+ \text { light quark disc. }}$ | - | 1.6 | $\mathbf{1 . 5 - 2}$ |
| residual $a^{2}$ extrap. | 0.2 | 0.1 | $\mathbf{1 . 5}$ |
| (heavy quark disc.) |  | 0.2 |  |
| $r_{1}^{3 / 2}$ uncertainty | 0. | 0.2 | $\mathbf{2}$ |
| $g_{B^{*} B \pi}$ uncertainty | 0.5 | 0.3 | $\mathbf{1 . 5}$ |
| quark masses tuning | 0.5 | $\leq 0.5$ | $\sim$ |
| operator matching | $\leq 0.5$ | - |  |
| relativistic corr. | 0.4 | $\leq 0.1$ | $\mathbf{1 . 5 - 2}$ |
| Finite volume | $\leq 0.1$ | $\sim 2.3$ |  |
| Total (2 years) | $\mathbf{1 . 4}$ |  |  |

### 2.4. Improvements of lattice calculations of $\xi$ in 2 years

| Source | RBC/UKQCD (now) | RBC/UKQCD <br> (in two years) |
| :---: | :---: | :---: |
| stat. + chiral extrap. <br> $\chi$ PT + light quark disc. residual $a^{2}$ extrap. <br> (heavy quark disc.) | $\begin{gathered} 5-6 \\ 7 \\ 3 \end{gathered}$ | $\begin{gathered} \leq 3 \\ \sim 2 \\ \leq 1 \end{gathered}$ |
| scale and quark masses uncertainty $g_{B^{*} B \pi}$ uncertainty operator matching Finite volume $1 / m_{b}$ corrections | $\begin{gathered} 1 \\ 3 \\ 0-2 \\ \leq 1 \\ 2 \end{gathered}$ | $\begin{aligned} & \leq 1 \\ & \leq 1 \\ & \leq 2 \\ & \leq 0.5 \end{aligned}$ |
| Total | 9 | $\leq 4$ |

O. Witzel at All Hands' Meeting 2010: USQCD Collaboration Meeting

### 2.5. Summary of expected lattice errors

|  | $f_{B} \sqrt{B_{B}}$ | $\xi$ |
| :---: | :---: | :---: |
| current | $6-7 \%$ | $3-4 \%$ |
| 2 years | $\sim 4-5 \%$ | $\sim 1.5-2 \%$ |
| 5 years* | $\sim 2 \%$ | $\sim 1 \%$ |

\# Several high precision determinations of $B_{s}^{0}$ and $B_{d}^{0}$ mixing parameters with different heavy and light formulations.

* From FNAL/MILC estimates (talk by C. Bernard)


## 3. $B_{0}$ mixing beyond the SM

\# Effects of heavy new particles seen in the form of effective operators built with SM degrees of freedom

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\# Effects of heavy new particles seen in the form of effective operators built with SM degrees of freedom
\# The most general Effective Hamiltonian describing $\Delta B=2$ processes is

$$
\begin{gathered}
\mathcal{H}_{e f f}^{\Delta B=2}=\sum_{i=1}^{5} C_{i} Q_{i}+\sum_{i=1}^{3} \widetilde{C}_{i} \widetilde{Q}_{i} \quad \text { with } \\
Q_{1}^{q}=\left(\bar{\psi}_{b}^{i} \gamma^{\nu}\left(\mathrm{I}-\gamma_{5}\right) \psi_{q}^{i}\right)\left(\bar{\psi}_{b}^{j} \gamma^{\nu}\left(\mathrm{I}-\gamma_{5}\right) \psi_{q}^{j}\right) \quad \text { SM } \\
Q_{2}^{q}=\left(\bar{\psi}_{b}^{i}\left(\mathrm{I}-\gamma_{5}\right) \psi_{q}^{i}\right)\left(\bar{\psi}_{b}^{j}\left(\mathrm{I}-\gamma_{5}\right) \psi_{q}^{j}\right) \quad Q_{3}^{q}=\left(\bar{\psi}_{b}^{i}\left(\mathrm{I}-\gamma_{5}\right) \psi_{q}^{j}\right)\left(\bar{\psi}_{b}^{j}\left(\mathrm{I}-\gamma_{5}\right) \psi_{q}^{i}\right) \\
Q_{4}^{q}=\left(\bar{\psi}_{b}^{i}\left(\mathrm{I}-\gamma_{5}\right) \psi_{q}^{i}\right)\left(\bar{\psi}_{b}^{j}\left(\mathrm{I}+\gamma_{5}\right) \psi_{q}^{j}\right) \quad Q_{5}^{q}=\left(\bar{\psi}_{b}^{i}\left(\mathrm{I}-\gamma_{5}\right) \psi_{q}^{j}\right)\left(\bar{\psi}_{b}^{j}\left(\mathrm{I}+\gamma_{5}\right) \psi_{q}^{i}\right) \\
\tilde{Q}_{1,2,3}^{q}=Q_{1,2,3}^{q} \text { with the replacement }\left(\mathrm{I} \pm \gamma_{5}\right) \rightarrow\left(\mathrm{I} \mp \gamma_{5}\right)
\end{gathered}
$$

where $\psi_{b}$ is a heavy b-fermion field and $\psi_{q}$ a light ( $q=d, s$ ) fermion field.

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where $\psi_{b}$ is a heavy b-fermion field and $\psi_{q}$ a light ( $q=d, s$ ) fermion field.

- $C_{i}, \widetilde{C}_{i}$ Wilson coeff. calculated for a particular BSM theory
- $\left\langle\overline{B^{0}}\right| Q_{i}\left|B^{0}\right\rangle$ calculated on the lattice


## 3. $B_{0}$ mixing beyond the SM

## \# Some examples:

F. Gabbiani et al, Nucl.Phys.B477 (1996), D. Bećirević et al, Nucl.Phys.B634 (2002); general SUSY models
M. Ciuchini and L. Silvestrini, PRL 97 (2006) 021803; SUSY

Constraints on the mass insertions $\left(\left|\operatorname{Re}\left(\delta_{23}^{d}\right)_{R R}\right|<0.4,\left|\left(\delta_{23}^{d}\right)_{L L}\right|<0.1, \ldots\right)$
M. Blanke et al, JHEP 12(2006) 003; Little Higgs model with T-parity
$\Delta M_{q}$ can be used to test viability of the model. To constrain and test the model in detail $\Delta M_{s} / \Delta M_{d}$ and $\Delta \Gamma_{q}$.

Lunghi and Soní, JHEPO709(2007)053; Top Two Higgs Doublet Model Constraints on $\beta_{H}$ (ratio of vev's of the two Higgs) and $m_{H^{+}}$
M. Blanke et al, JHEP0903(2009)001; Warped Extra Dimensional Models Constraints on the KK mass scale: anarchic approach seems implausible, generally $M_{K K}>20 \mathrm{TeV}$ but can be as low as $M_{K K} \simeq 3 \mathrm{TeV}$ (moderate fine tunning).

## 3. $B_{0}$ mixing beyond the SM

\# Some examples:
W. Altmannshofer et all, 0909.1333; SUSY flavor models

Identify useful flavour observables ( $S_{\psi_{\phi},}, B_{s} \rightarrow \mu^{+} \mu^{-}, \ldots$ ) to exclude some SUSY models and/or distinguish them from LHT and RS models. Updated analysis of bound on flavor violating terms in the SUSY soft sector.
A. Soni et al, 1002.0595; SM with four generations

$$
m_{t^{\prime}} \sim 400-600 \mathrm{GeV},\left|V_{t^{\prime} b}^{*} V_{t^{\prime} s}\right|=(0.05-1.4) \times 10^{-2}, \ldots
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$$

* Only quenched calculation available Becirevic et al, JHEP 04 (2002) 025
* Straightforward extension of previous calculations
$\rightarrow$ FNAL/MILC: work in progress


## 4. $D_{0}$ mixing beyond the SM

\# SM short-distance description alone can not successfully describe $D^{0}$ mixing.
\# Neither short-distance nor long-distance SM predictions can be calculated accurately.
\# SM contribution of the order of experiment and dominated by long-distance effects.

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\# SM contribution of the order of experiment and dominated by long-distance effects.

## What can we calculate on the lattice?

* Long distance: Current lattice techniques are inefficient for calculating non-local operators* Short distance: High precision calculation on the lattice
** Same effective hamiltonian as for $\Delta B=2$ processes.
** Comparison with experiment can exclude large regions of parameters in many models, constraining BSM building. E. Golowich, J. Hewett, S. Pakvasa and A. Petrov, PRD 76 (2007)


## 4. $D_{0}$ mixing beyond the SM

** A consistent unquenched determination of all matrix elements involved, free of the uncontrolled uncertainties associated to quenching is needed

Latest SM calculations (quenched): L. Lellouch, C.-J. D Lin
Phys.Rev.D64 (2001); Huey-Wen Lin et al, Phys.Rev.D74 (2006)
Latest BSM calculation (quenched): R. Gupta et al., Phys.Rev.D55

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Latest BSM calculation (quenched): R. Gupta et al., Phys.Rev.D55 (1997)
** Work in progress: (goal: 10\% errors) FNAL/MILC

## 5. Future prospects and goals

\# Reduction of errors for $f_{B_{q}} \sqrt{B_{B_{q}}}$ and $\xi$
$\rightarrow$ high precision tests of the SM.

|  | $f_{B} \sqrt{B_{B}}$ | $\xi$ |
| :---: | :---: | :---: |
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| current | $6-7 \%$ | $3-4 \%$ |
| 2 years | $\sim 4-5 \%$ | $\sim 1.5-2 \%$ |
| 5 years* | $\sim 2 \%$ | $\sim 1 \%$ |

\# Calculation of matrix elements needed for $\Delta \Gamma_{q}$ Lenz and Nierste, JHEPO706 (2007) 072

$$
\left(\frac{\Delta \Gamma}{\Gamma}\right)=\left(\frac{1}{245 \mathrm{MeV}}\right)^{2}\left[0.170\left(f_{B_{q}}^{2} B_{B_{q}}\right)+0.059 R^{2}\left(f_{B_{q}}^{2} \tilde{B}_{S} R^{2}\right)-0.044 f_{B_{q}}^{2}\right]
$$

* Useful to impose constraints on BSM building, M. Blanke et al, LHT


## 5. Future prospects and goals

\# Unquenched calculation of matrix elements corresponding to operators that only appear in BSM theories for $B^{0}-\bar{B}^{0}$ and $D^{0}-\bar{D}^{0}$ mixing ( $10 \%$ ).

* Work in progress by fNAL/MILC
$\times$


### 3.1. Tension in the CKM unitarity triangle

CKMfitter: $\left\langle B_{q}^{0}\right| M_{12}^{S M+N P}\left|\bar{B}_{q}^{0}\right\rangle=\Delta_{q}^{N P}\left\langle B_{q}^{0}\right| M_{12}^{S M}\left|\bar{B}_{q}^{0}\right\rangle \quad$ V. Tisserand, 0905.1572

1.9 : Tension driven by the exp. measurement $\left(2 \beta_{s}, \Delta \Gamma_{s}\right)$.

2.1 $\sigma$ : Tension between $\sin (2 \beta)$

$$
\text { and }\left|V_{u b}\right|_{\tau \nu}
$$

* Tree-level mediated decays through a Four Flavor Change $\left(b \rightarrow q_{i} \bar{q}_{j} q_{k}\right)$ are SM
* NP effects in oscillation parameters, weak phases, semi-leptonic asymmetries and $B$ lifetime differences parametrized through $\Delta$
3.2. Measurement of $\operatorname{Br}\left(B_{s, d} \rightarrow \mu^{+} \mu^{-}\right)$
* Scalar operators in the effective hamiltonian can enhance branching ratios to current experimental bounds (example: Higgs penguin).
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* Scalar operators in the effective hamiltonian can enhance branching ratios to current experimental bounds (example: Higgs penguin).
* In some models there is a strong correlation between $\mathcal{B} r\left(B_{q} \rightarrow \mu^{+} \mu^{-}\right)$ and $\Delta M_{B_{q}^{0}}$ (example: some MSSM models.)
** Testing the correlation predicted by those kind of models needs a reduction of errors in the theoretical prediction for $\Delta M_{s}^{S M}$ $\rightarrow$ need smaller lattice errors for the non-perturbative inputs.
3.2. Measurement of $\operatorname{Br}\left(B_{s, d} \rightarrow \mu^{+} \mu^{-}\right)$
\# Tests of MFV: In the SM model and CMFV models, the following model independent relation hold with $r=1$ Buras, PLB566 (2003) 115

$$
\frac{\mathcal{B} r\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right)}{\mathcal{B} r\left(B_{d} \rightarrow \mu^{+} \mu^{-}\right)}=\frac{\hat{B}_{d}}{\hat{B}_{s}} \frac{\tau\left(B_{s}\right)}{\tau\left(B_{d}\right)} \frac{\Delta M_{s}}{\Delta M_{d}} r
$$

Any deviation from this relation $(r \neq 1)$ would indicate NP effects.
Supersymmetry, little Higgs models, extra space dimensions ... discussed in Buras, arxiv:0910.1032

$$
\text { LHT: } 0.3 \leq r \leq 1.6, \mathrm{RSC}: 0.6 \leq r \leq 1.3
$$

* LHCb can reach the SM level for this branching ratio.


# 4.2. Results: $\xi \sqrt{\frac{M_{B_{s}}}{M_{B_{d}}}}$ (exploratory) <br> RBC/UKQCD, arXiv:1001.2023 



* No extrapolation to the continuum

$$
\xi=\frac{f_{B_{s}} \sqrt{B_{B_{s}}}}{f_{B_{d}} \sqrt{B_{B_{d}}}}=1.13(12)
$$

4.2. Results: $\xi \sqrt{\frac{M_{B_{s}}}{M_{B_{d}}}}$
\# Comparison of final HPQCD, PRD80 (2009) 014503 and preliminary FNAL/MILC, PoS LATTICE 2009, 245 (2009)

$\xi=\frac{f_{B_{s}} \sqrt{B_{B_{s}}}}{f_{B_{d}} \sqrt{B_{B_{d}}}}=\quad \mathrm{HPQCD} \quad 1.258(25)(21) \quad \Longrightarrow\left|\frac{V_{t d}}{V_{t s}}\right|=0.214(1)(5)$ FNAL/MILC $1.205(37)(34)$

