Hadronic light-by-light contribution to the muon g-2 from the lattice

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Lattice Meets Experiment Workshop, Fermilab, April 26, 2010

Outline of the talk

- Introduction
- Light-by-light (α^3)
- Summary/Outlook

Introduction to muon g-2

Classical interaction of particle with static magnetic field

$$V(\vec{x}) \;=\; -\vec{\mu} \cdot \vec{B}$$

The magnetic moment $\vec{\mu}$ is proportional to its spin

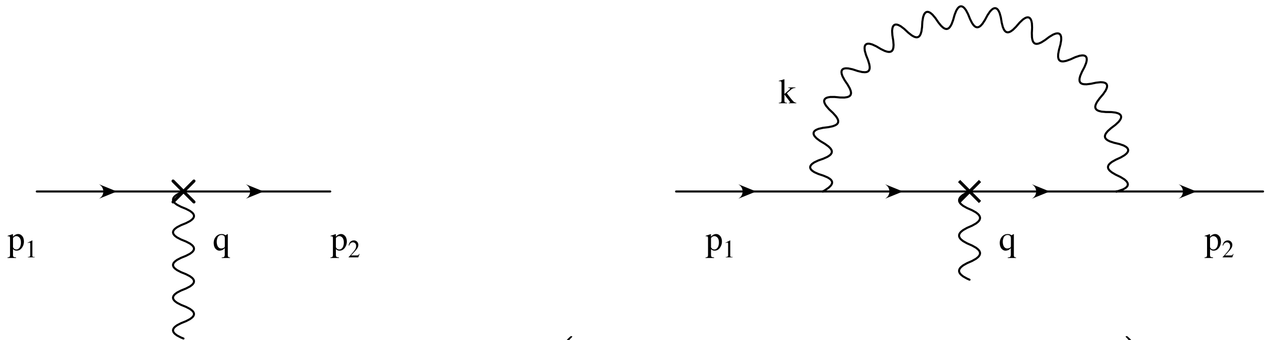
$$\vec{\mu} \;=\; g \left(\frac{e}{2m}\right) \vec{S}$$

The Landé g-factor is predicted from the free Dirac eq. to be

$$g \;=\; 2$$

for elementary fermions

In the quantum (field) theory g receives radiative corrections



$$\gamma_\mu \quad \rightarrow \quad \Gamma^\mu(q) = \left(\gamma^\mu F_1(q^2) + \frac{i\,\sigma^{\mu\nu}\, q_\nu}{2m} F_2(q^2) \right)$$

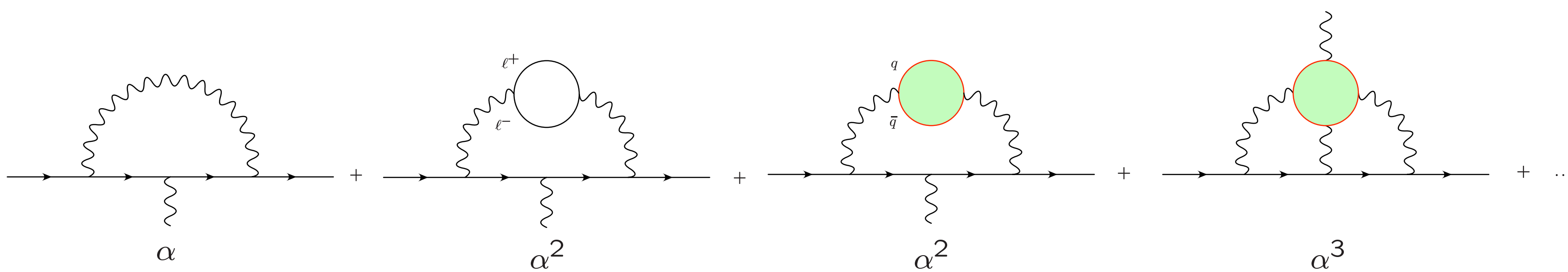
which results from Lorentz invariance and current-conservation (Ward-Takahashi identity) when the muon is on-mass-shell.

$$F_2(0) \quad = \quad \frac{g-2}{2} \equiv a_\mu$$

(the anomalous magnetic moment)

Compute these corrections order-by-order in perturbation theory by expanding $\Gamma^\mu(q^2)$ in QED coupling constant

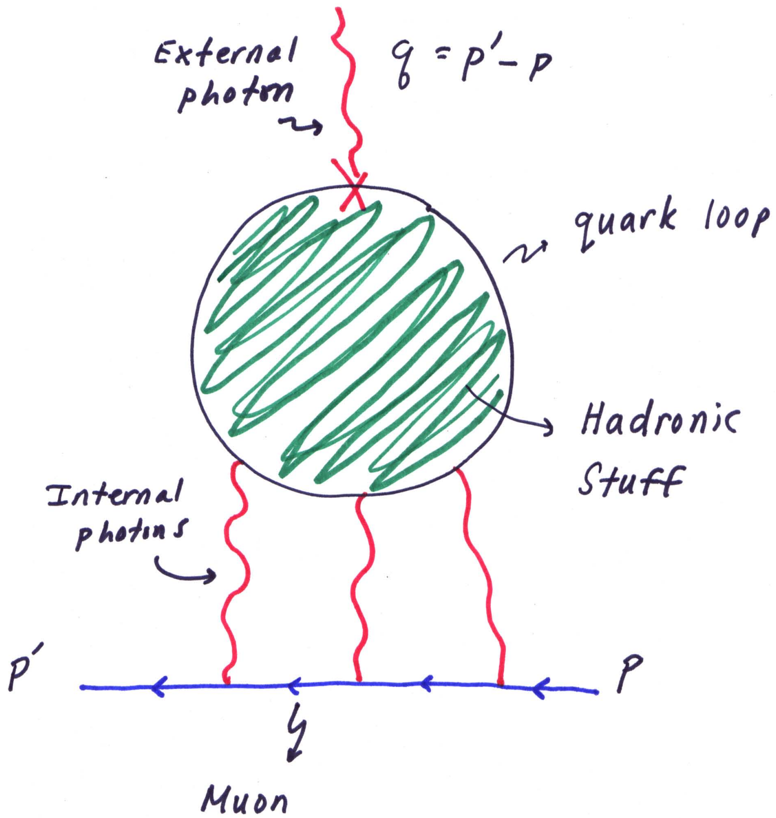
$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137} + \ldots$$



Corrections begin at $\mathcal{O}(\alpha)$; Schwinger term $= \frac{\alpha}{2\pi} = 0.0011614\ldots$

Hadronic contribution $\sim 6 \times 10^{-5}$ times smaller

The hadronic light-by-light contribution ($O(\alpha^3)$)



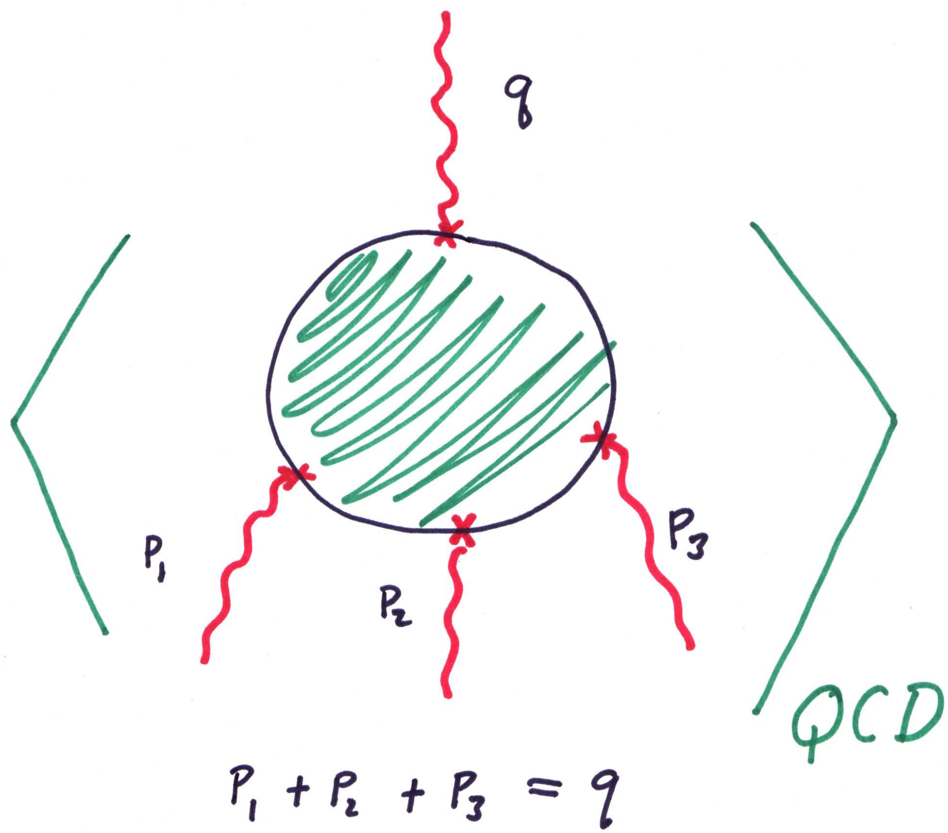
The blob contains all possible hadronic states. Can't use perturbation theory and

No dispersion relation *a'la* vacuum polarization

Model estimates put this contribution at about $(10 - 12) \times 10^{-10}$ with a 30-40% uncertainty

About 60$\times$ smaller than hadronic vac. polarization ($O(\alpha^2)$) contribution and 200$\times$ smaller than the QED LbL contribution

Conventional approach (QCD only on the lattice)



Correlation of 4 currents

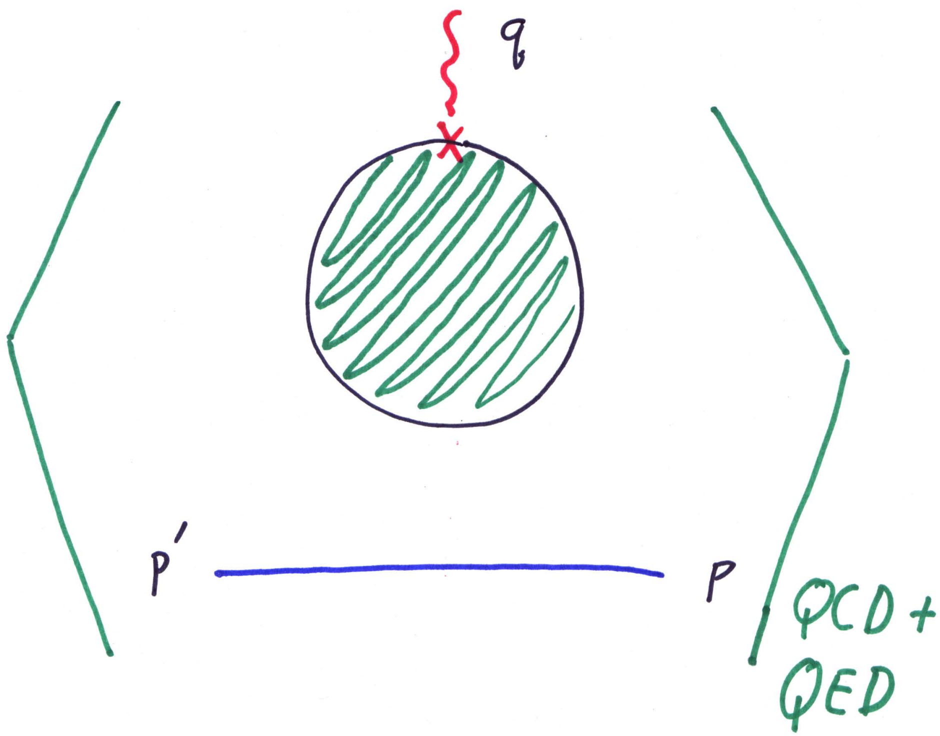
Two independent momenta
+external mom q

Compute for all possible values of p_1 and p_2, four index tensor

Average over all possible QCD gauge field configuratons

several q, (extrap. $q \to 0$), plug into perturbative QED two-loop integrals... difficult

New approach QCD and QED on the lattice



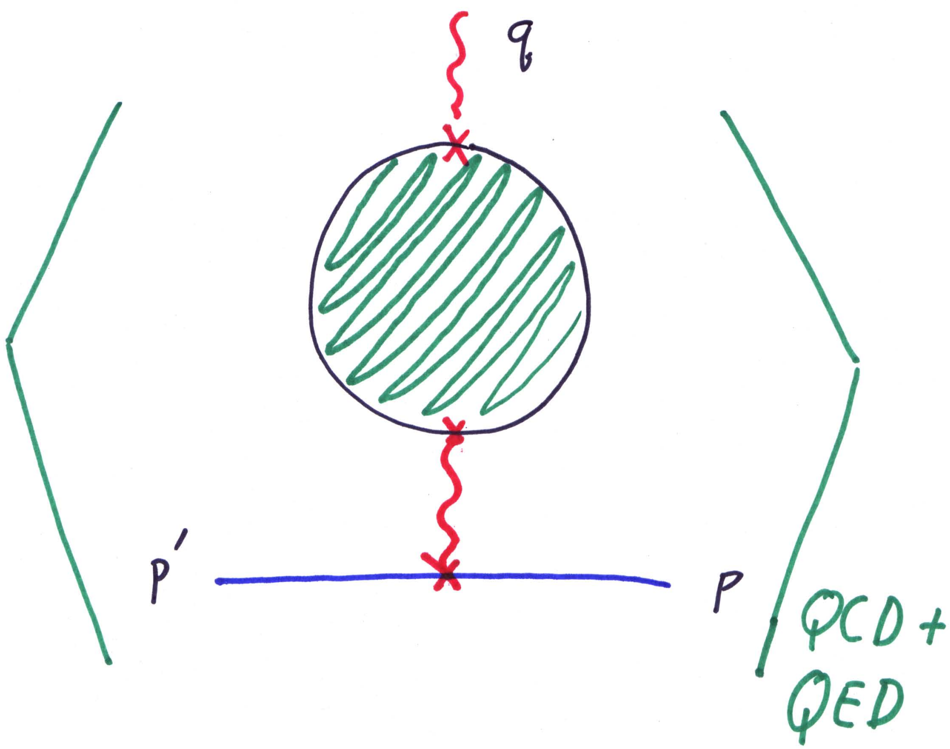
Average over combined gluon *and* photon gauge field configurations

Quarks coupled to gluons and photons

muon coupled to photons

`[hep-lat/0509016;`
`Chowdhury` *et al.* `(2008);`
`Chowdhury Ph. D. thesis (2009)]`

New approach...

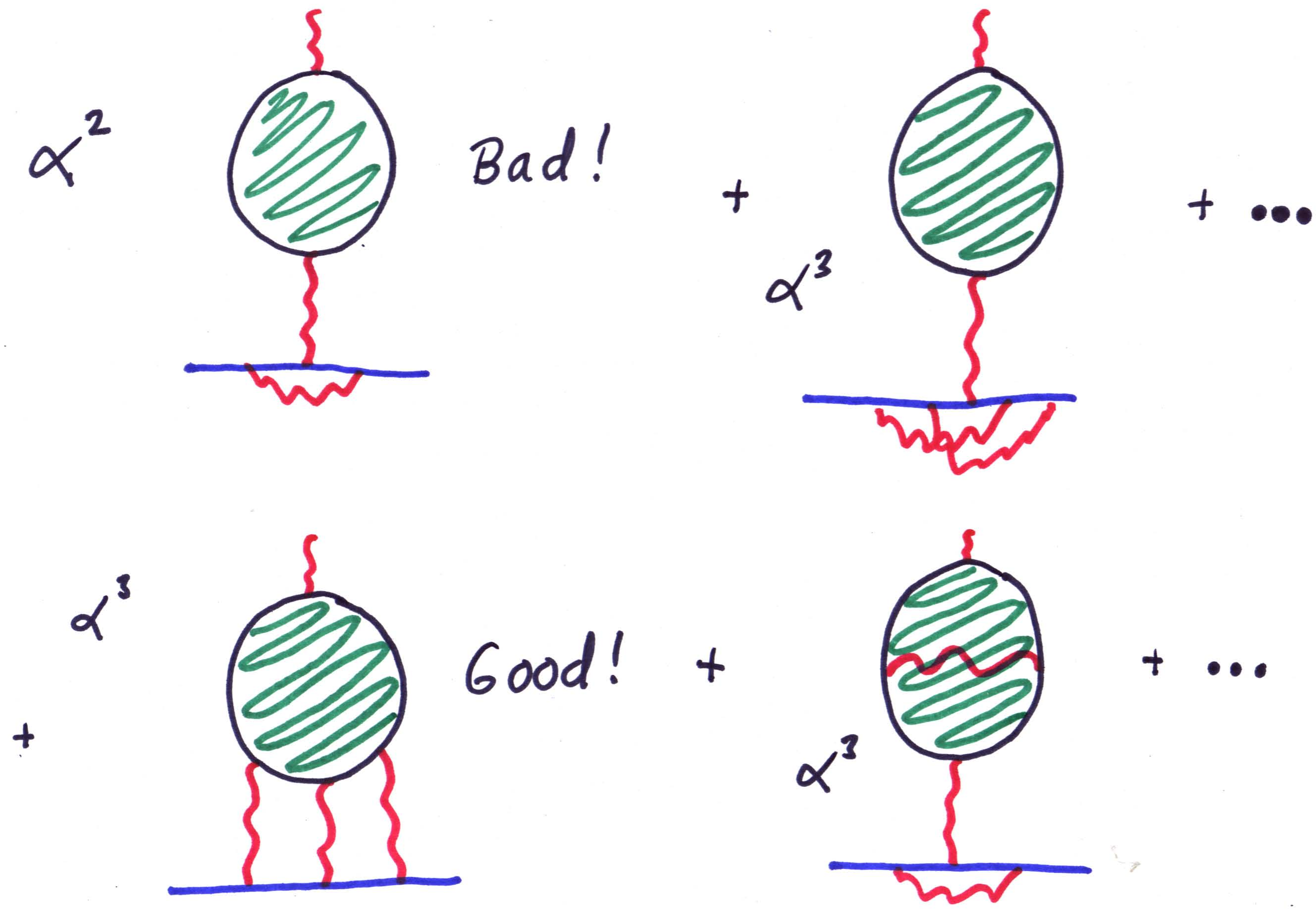


Attach one photon by hand (see why in a minute)

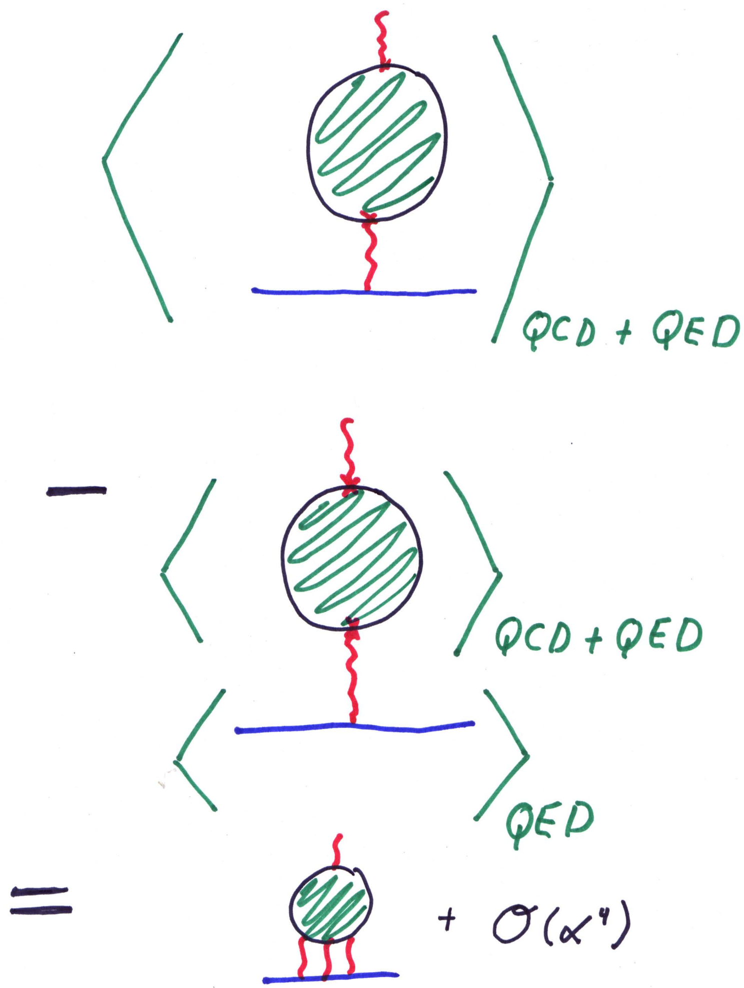
Correlation of 2pt hadronic loop and 3pt muon line

[hep-lat/0509016;
Chowdhury *et al.* (2008);
Chowdhury Ph. D. thesis (2009)]

The leading and next-leading contributions in α to magnetic part of correlation function come from



Subtraction of lowest order piece:



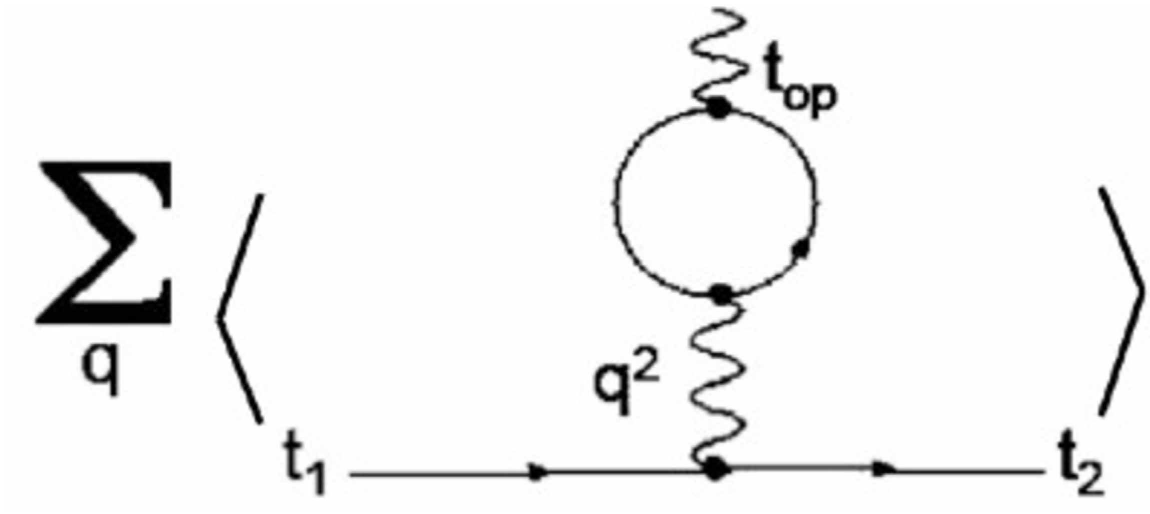
Subtraction term is product of separate averages of the loop and line

Gauge configurations identical in both, so two are highly correlated. Only way this will work.

In PT in α, correlation function and subtraction have same contributions except the light-by-light term which is absent in the subtraction

Test calculation in pure QED [Chowdhury thesis]

(Compare to well known PT result)



- Incoming muon has $\vec{p} = 0$, outgoing $\vec{p}' = -\vec{p}_{op} = (1, 0, 0)$ (and permutions)
- external muons put on shell in usual way ($t_1 \ll t_{\rm op} \ll t_2$)
- (4d) Fourier transform "loop" and "line" separately
- Average over all QED gauge field configurations
- Subtraction difficult in practice since need to average loop and line separately over gauge fields first, then sum over q^2.

more lattice details

- Domain wall fermions (match 2+1 QCD ensembles)
- loop/line masses degenerate: $m_\mu = 0.4$ (heavy), later loop mass = 0.01
- Focus on $16^3 \times 32$ lattice size (some 16^3, $24^3 \times 64$)
- Non-compact quenched QED (easy, fewer lattice artifacts)
- Fix to Feynman gauge (photon propagator is simple)
- To enhance signal, take $e = 1$, or $\alpha = 1/4\pi$ instead of 1/137
- Few hundreds to couple thousands of configurations (measurements)
- Single lowest non-zero momentum used (no extrap $q \to 0$ yet)

Extracting the form factors

$$G^\mu(t',t) \;=\; \langle \psi(t',\vec{p}')\, J^\mu(t,q)\, \psi^\dagger(0,\vec{p})\rangle .$$

Insert two complete sets of states,

$$\begin{aligned} G^\mu(t',t) &= \sum_{s,s'} \langle 0|\chi_N|p',s'\rangle\langle p',s'|J^\mu|p,s\rangle\langle p,s|\chi_N^\dagger|0\rangle \frac{1}{2E\,2E'} e^{-E'(t'-t)} e^{-Et} + \ldots \\ &= G^\mu(q^2)\times f(t,t',E,E') + \ldots , \end{aligned}$$

(like LHZ reduction, but in Euclidean space)

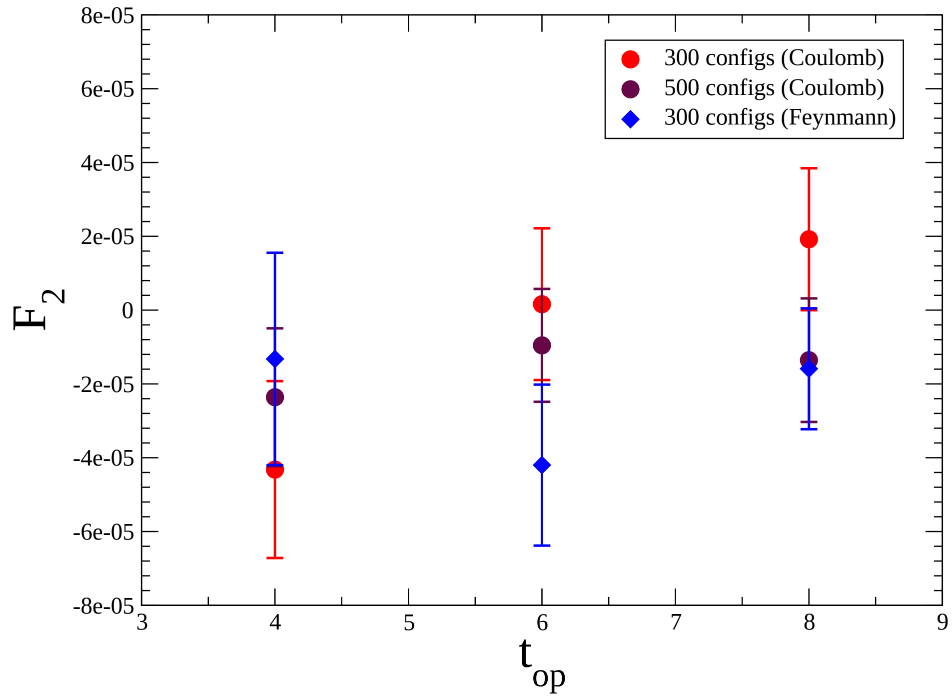
For example, $J^\mu = J^x$,

$$\begin{aligned} \mathrm{tr}\mathcal{P}^{xy}\, G^x(q^2) &= p_y\, m(F_1(q^2) + F_2(q^2)) \\ \mathcal{P}^{xy} &= \frac{i}{4}\frac{1+\gamma^t}{2}\gamma^y\gamma^x \end{aligned}$$

Similarly,

$$\begin{aligned} \mathrm{tr}\mathcal{P}^t\, G^t(q^2) &= m\,(E+m)\left(F_1(q^2) + \frac{q^2}{(2m)^2}F_2(q^2)\right), \\ \mathcal{P}^t &= \frac{1}{4}\frac{1+\gamma^t}{2}, \end{aligned}$$

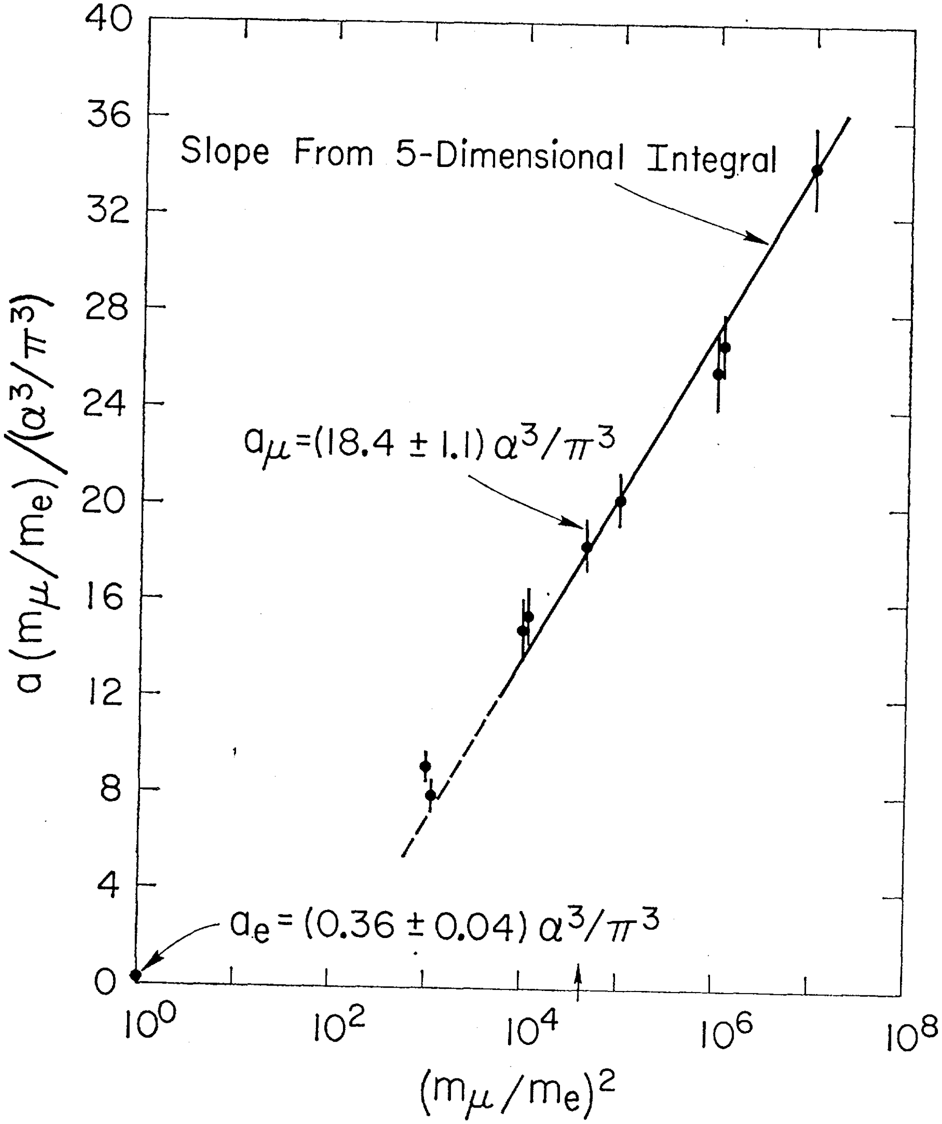
$F_2(0)$ (degenerate leptons, or g-2 of electron)



$F_2 = (-1.5 \pm 1.1) \times 10^{-5}$ (lowest non-zero momentum)

Continuum PT result: $0.36(\alpha/\pi)^3 = 0.585 \times 10^{-5}$ $(e = 1)$

Statistical error same order as PT result

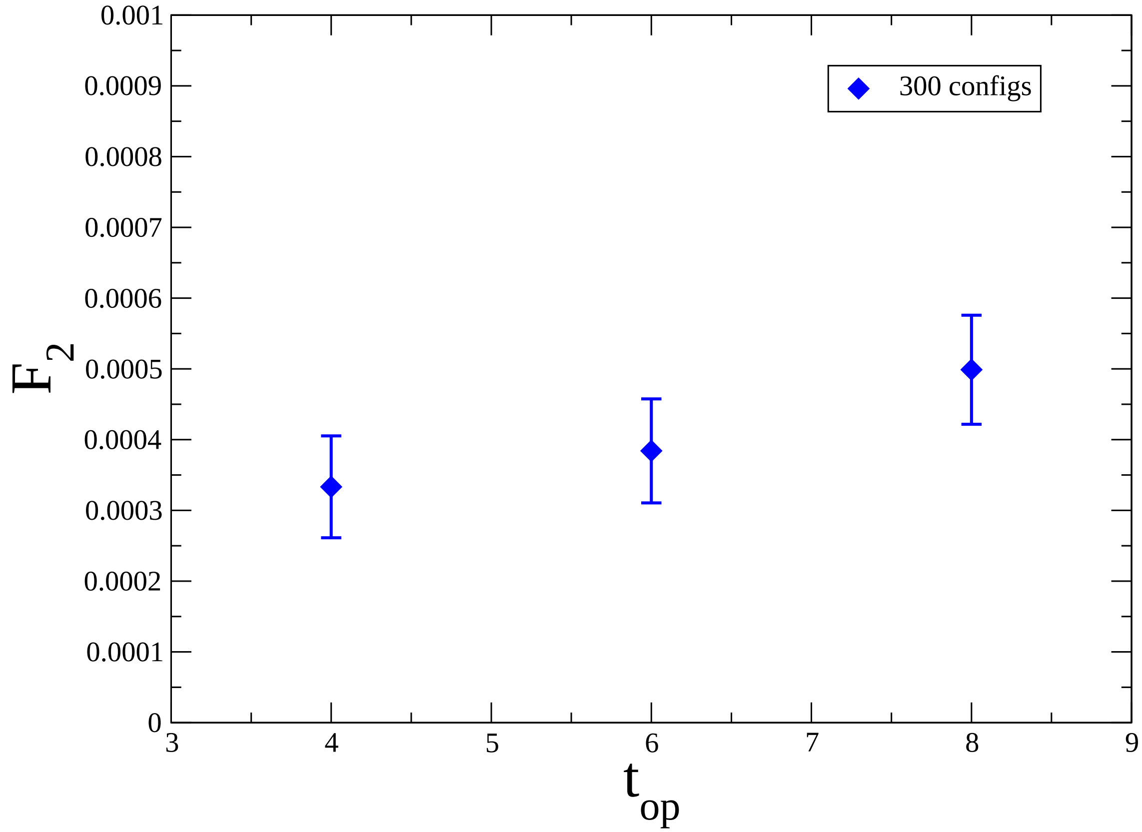


[Aldins, Brodsky, Duffner, Kinoshita (1970)]

Large m_μ/m_e enhancement seen in perturbation theory

Try $m = 0.01$ in the loop, or $m_\mu/m_e = 40$

F_2 $m_\mu/m_e = 40$



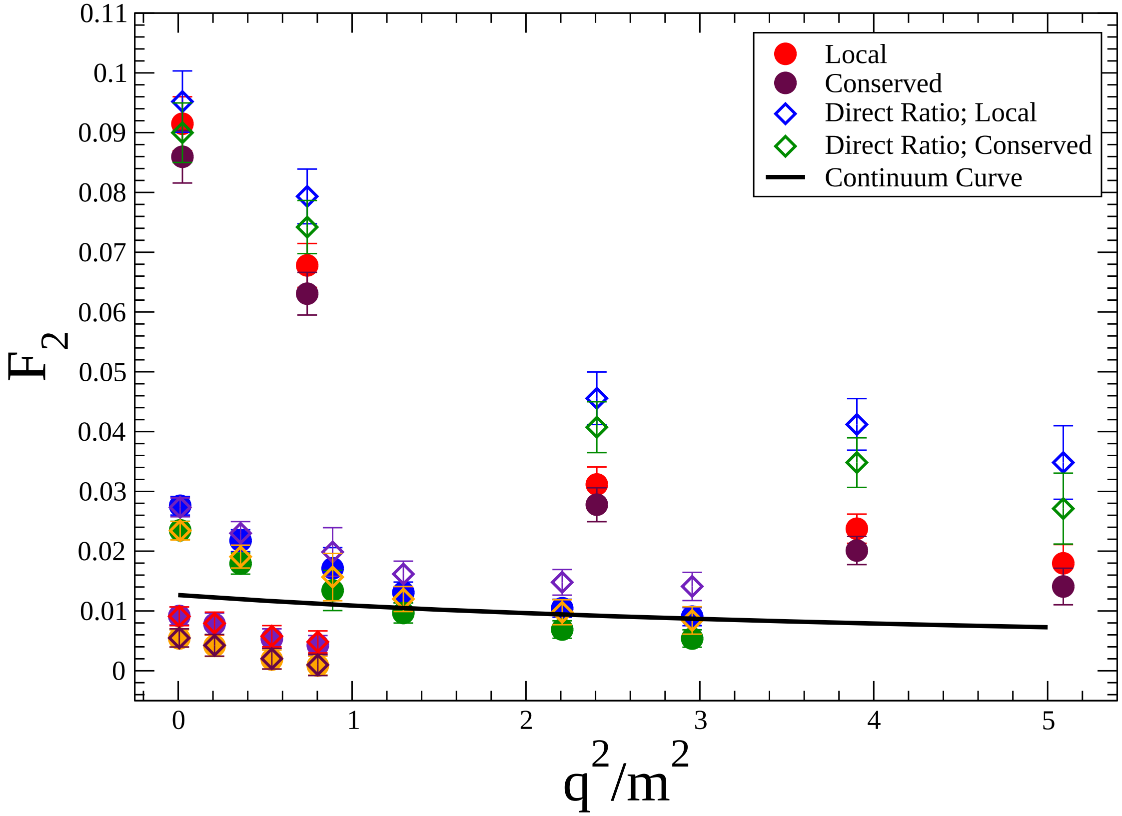
$F_2 = (3.95 \pm 0.40) \times 10^{-4}$ (lowest non-zero momentum)

Continuum PT result: $\approx 10(\alpha/\pi)^3 = 1.63 \times 10^{-4}$ ($e = 1$)

About 2.5 times PT result

Finite volume effects

Large finite volume effect in $O(\alpha)$ Schwinger term ($e = 1$, $m_\mu = 0.2$):



24^3: lbl $F_2 = (0.78 \pm 0.61) \times 10^{-4}$ (lowest non-zero momentum)

Low statistics (consistent with PT), move on to full QCD+QED anyway

F_2 in 2+1 flavor QCD

Include hadronic part in the loop only (same in subtraction)

2+1 flavors of DWF (RBC/UKQCD)

$a = 0.114$ fm, $16^3 \times 32$, $a^{-1} = 1.73$ GeV

$m_q \approx 0.013$ ($m_\pi \approx 400$ MeV)

~ 1000 configurations (one QED conf. for each QCD conf.)

$F_2 = (-1.6 \pm 1.8) \times 10^{-4}$ (lowest non-zero momentum)

Magnitude of error is about 100× model estimates

model calculations (physical mass and charge) about 200 times smaller than QED light-by-light contribution.

Signal has disappeared, but statistical error stayed about the same

Another alternative: $\pi^0 \to \gamma^{(*)}\gamma^{(*)}$

Calculate $\pi^0 \to \gamma^{(*)}\gamma^{(*)}$ vertex function (form factor)

Insert into model calculations of HLbL amplitude (draw)

Can be (partially) measured in experiment (*c.f.*, next talk by Moricciani)

Difficulty: must integrate over **all possible momenta** of *each* photon and the (low) momentum of the *pion* (momentum conservation: only two are independent) (calculate for several momenta and fit?)

First calculations [S. D. Cohen, *et al* (2008) , JLQCD (2009)]

Planning future workshop on HLbL

Working with Dave Hertzog and Lee Roberts

include models, lattice, and experimental input

Invite experts in each topic

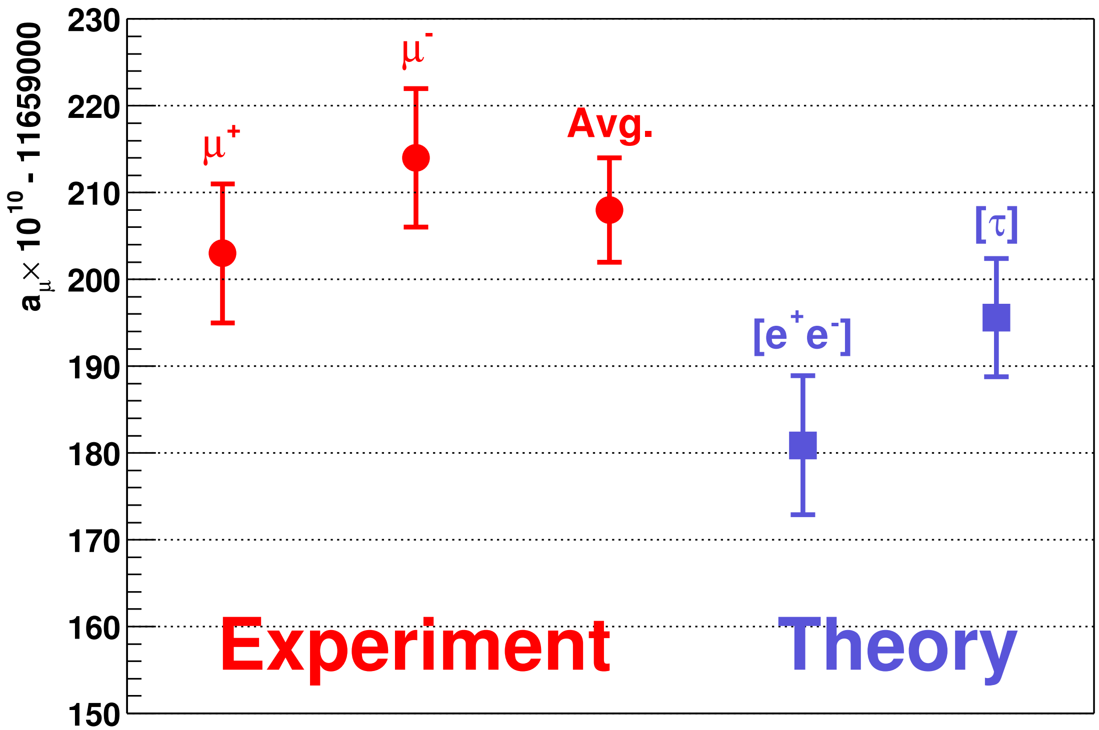
Proposed to Institute of Nuclear Theory (Washington)

Summary/Outlook: light-by-light contribution ($O(\alpha^3)$)

- Pure QED calculation on the lattice roughly reproduces the perturbative result (first ever). Encouraging.
  - Finite volume effects are large, but probably manageable
- Full hadronic contribution is O(10^2) times smaller, so still swamped by the statistical noise (first ever– something to shoot at)
- Small volumes, small statistics. Try
  - Volume (low-mode) averaging for the loop
  - Larger volumes
  - More statistics, i.e. more QED configurations per QCD configuration
- and/or attempt conventional calculation and/or $\pi^0 \to \gamma\gamma$ vertex
- two lepton/quark loops not yet attempted!
- Next generation peta-flop computers needed

Acknowledgments: This research was supported by the US DOE and RIKEN BNL Research Center. Computations done on the QCDOC supercomputers at BNL and Columbia University.

Status of the experimental measurement (Muon $(g-2)$ Collaboration, BNL-E821) of a_μ.



$a_\mu(\text{exp}) = 11\,659\,208(6) \times 10^{-10}$ (accurate to about 0.5 ppm)

Theory calculation

(Summary from D. W. Hertzog [E821 Collaboration], hep-ex/0501053.)

Table 1: Comparison of a_μ(SM) with a_μ(Exp)

	$a_\mu \times 10^{10}$	$\Delta a_\mu \times 10^{10}$
QED	11 658 471.94	0.14
QCD	695.4	7.3
Weak	15.4	0.22
Theory	11 659 182.7	7.4
Experiment	11 659 208	6
a_μ(EXP) $-$ a_μ(SM)	25.3	9.5

Table 2: QCD contribution to the muon $g-2$

	$a_\mu \times 10^{10}$	$\Delta a_\mu \times 10^{10}$
hadronic vacuum polarization ($\mathcal{O}(\alpha_{\text{em}}^2)$)	693.4	6.4
hadronic vacuum polarization ($\mathcal{O}(\alpha_{\text{em}}^3)$)	-10.0	0.6
hadronic light-by-light ($\mathcal{O}(\alpha_{\text{em}}^3)$)	12.0	3.5
Total QCD	695.4	7.3