Kaon physics on the lattice*

Steve Sharpe (UW)

Lattice QCD Meets Experiment Workshop 2010
Fermilab, April 27, 2010

*Plus strange and charm quark masses
*Minus K->ππ, so "gold-plated" only

Outline

Status and future prospects for lattice calculations of:

* Decay constants: f_K, f_K/f_π, (& f_π)
* $K \to \pi\, l\nu$ form factors
* B_K (and related matrix elements)
* m_s and m_c

How reliable are the results, what are the dominant errors, and by how much can they be reduced over the next 1-5 years?

Recent Reviews

P. Boyle, Kaon 09, "Lattice Kaon Physics," arXiv:0911.4317

[LLV] = J. Laiho, E. Lunghi, R. Van de Water, "Lattice QCD inputs to the CKM unitarity triangle analysis," PRD81, 034503 (2010), arXiv:0910.2928

- Contains averages of 2+1 flavor lattice results

V. Lubicz, Lat09 review, "Kaon Physics from lattice QCD," arXiv:1004.3473

- Contains FLAG (Flavianet Lattice Averaging Group) averages (in preparation)

C. Sachrajda, Chiral dynamics 09, "Kaons on the lattice," arXiv:0911.1560

E. Scholz, Lat09 review, "Light Hadron Masses and Decay Constants," arXiv:0911.219

R. Van de Water, Lat09 review, "The CKM matrix and flavor physics from lattice QCD," arXiv:0911.3127

Collaborations & fermions

ALV = Aubin, Laiho & Van de Water: Domain wall valence on staggered (MILC) sea

BMW = Budapest, Marseille, Wuppertal = Durr et al: Improved Wilson fermions

ETMC = European Twisted-mass Collab: Further improved Wilson fermions

HPQCD = High precision lattice QCD = Davies et al: Highly improved staggered valence on staggered (MILC) sea

MILC (= MIMD lattice collaboration) = Bernard et al: Improved/Highly improved staggered fermions

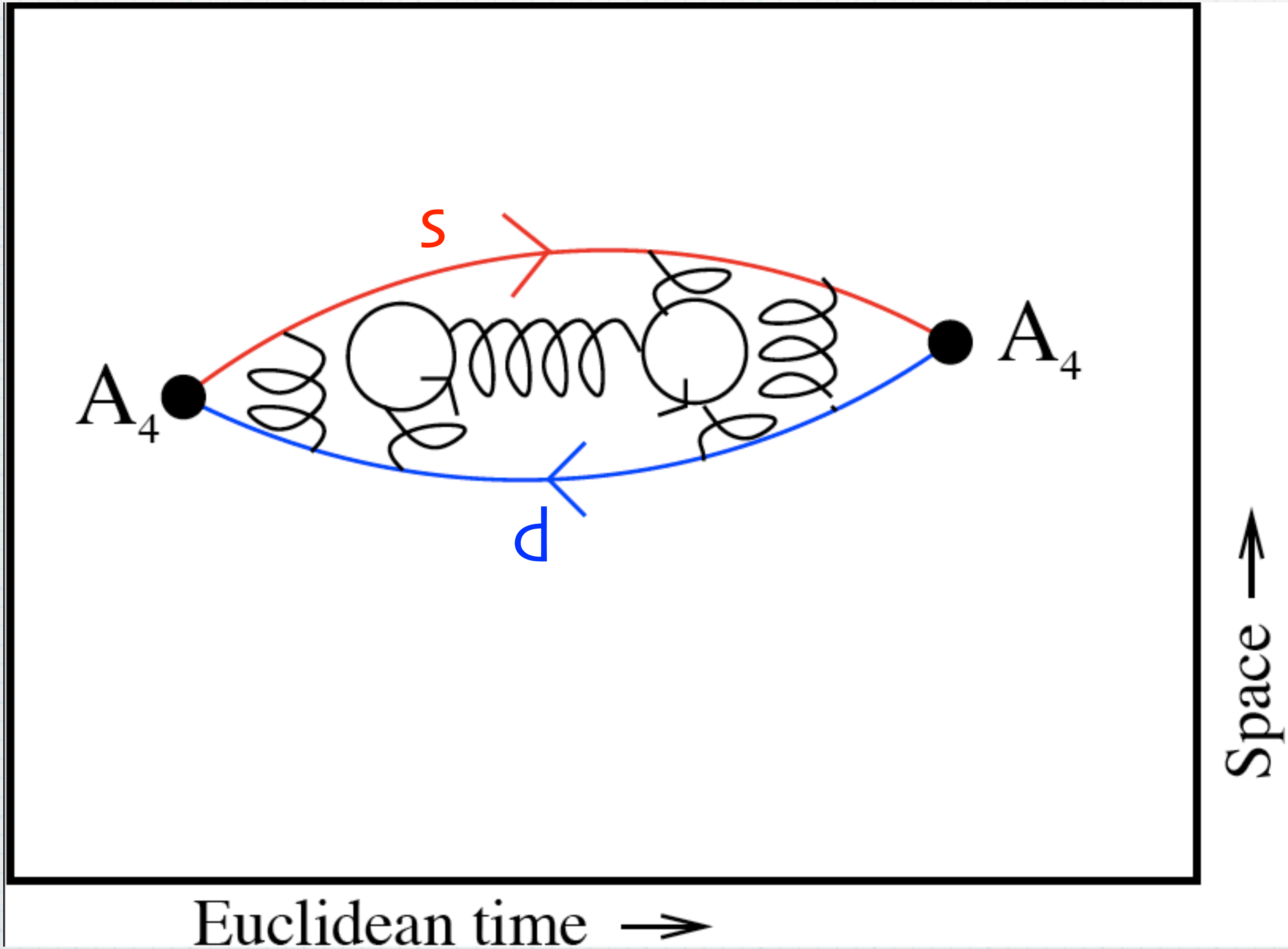
PACS-CS = Tsukuba-centered collab.: Improved Wilson fermions

RBC/UKQCD = Riken, Brookhaven, Columbia / UK lattice QCD: Domain wall fermions (DWF)

Decay constants

f_K & $f_K/f_\pi \Rightarrow V_{us}$ or V_{us}/V_{ud}

$f_\pi \Rightarrow V_{ud}$ or lattice spacing


s
A4
A4
d
Space
Euclidean time

f_K/f_π :FLAG coding scheme

[Lubicz]

Collaboration	Ref.	N_f	publication status	chiral extrapolation	finite volume errors	continuum extrapolation	f_K/f_π
ALVdW 09	[30]	2+1	C	●	●	●	1.192(12)(16)
BMW 09	[31, 32]	2+1	P	★	★	★	1.192(7)(6)
RBC/UKQCD 09	[33]	2+1	C	●	★	●	1.225(12)(14)
MILC 09b	[34]	2+1	A	★	★	★	$1.198(2)(^{+6}_{-8})$
MILC 09a	[35]	2+1	A	★	★	★	$1.197(3)(^{+6}_{-13})$
JLQCD/TWQCD 09	[36]	2+1	C	●	■	■	$1.210(12)_{stat}$
PACS-CS 08	[37]	2+1	A	★	■	■	1.189(20)
HPQCD/UKQCD 07	[38]	2+1	A	★	●	★	1.189(2)(7)
RBC/UKQCD 08	[20]	2+1	A	●	★	■	1.205(18)(62)
NPLQCD 06	[39]	2+1	A	●	■	■	$1.218(2)(^{+11}_{-24})$
MILC 04	[40]	2+1	A	★	●	●	1.210(4)(13)
ETMC 09	[41]	2	A	●	●	★	1.210(6)(15)(9)
ETMC 07	[42]	2	A	●	●	■	1.227(9)(24)
QCDSF/UKQCD 07	[43]	2	C	●	★	●	1.21(3)

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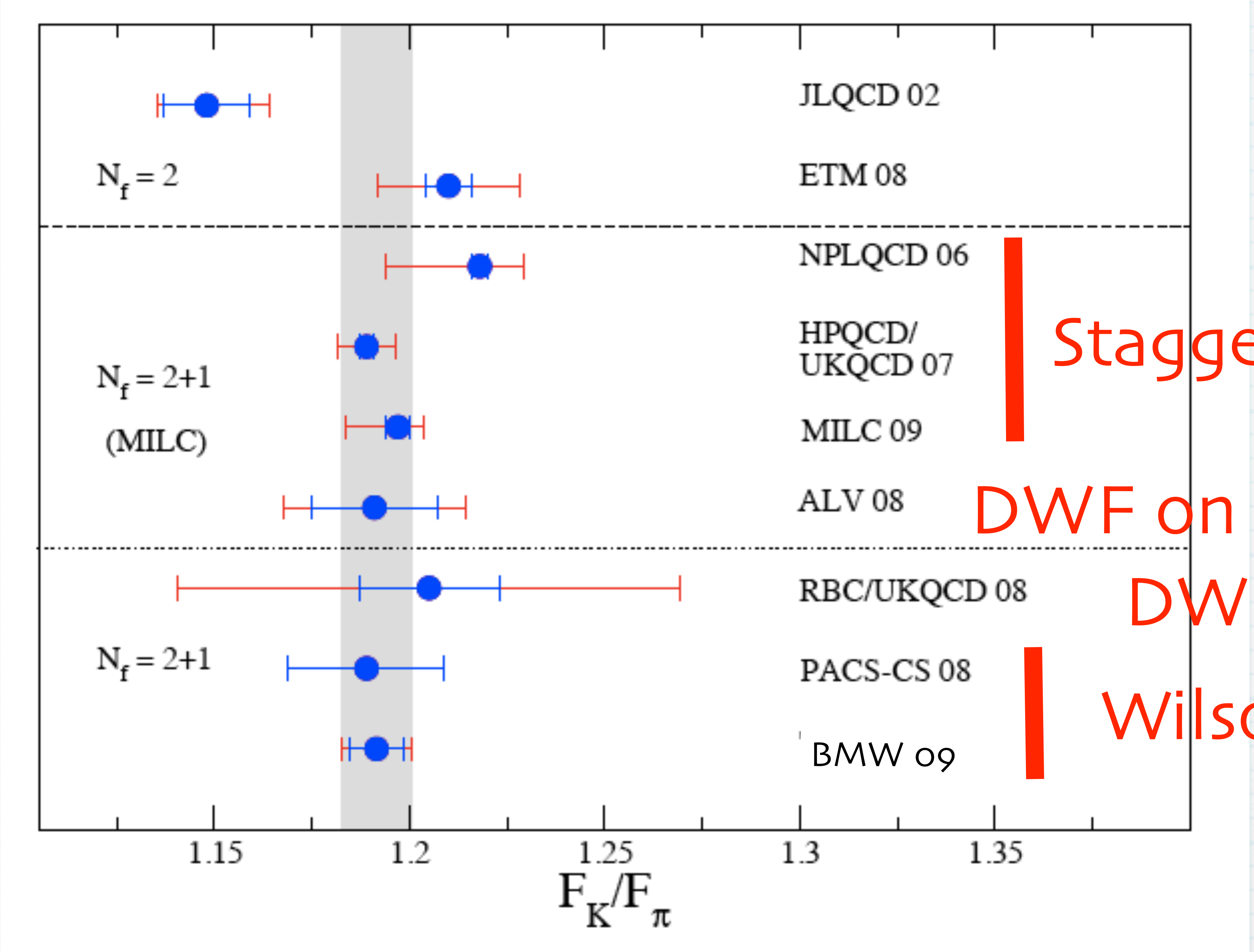
Included in average

Status of f_K/f_π

from BMW 09



Staggered

DWF on stagg.

DWF

Wilson

Good agreement! Reliable calculation!

Lattice average: f_K/f_π=1.196(1)(10) [Lubicz]

Comparison with SM

Lattice average:

f_K/f_π=1.196(1)(10) [Lubicz]

First row unitarity+
K_{l3}+K_{l2}/π_{l2}+ V_{ud}
⇒f_K/f_π=1.1925(56) [FLAG]

Consistent at 1% precision!

Can lattice calculations reduce errors towards few per mil?

- Statistical errors of 2 per mil already attained[MILC,HPQCD]
- Stumbling block is systematic errors

Future prospects

[BMW09]

Source of systematic error	error on F_K/F_π
Chiral Extrapolation:	
- Functional form	3.3×10^{-3}
- Pion mass range	3.0×10^{-3}
Continuum extrapolation	3.3×10^{-3}
Excited states	1.9×10^{-3}
Scale setting	1.0×10^{-3}
Finite volume	6.2×10^{-4}

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Chiral extrapolation (functional form, pion mass range) ↔ $m_\pi \geq 190$ MeV

Continuum extrapolation ↔ $a \geq 0.064$ fm

Future prospects

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[BMW09]

$m_\pi \geq 190$ MeV

$a \geq 0.064$ fm

To reduce dominant systematics:

- $m_\pi \rightarrow$ physical value (error removed)
- $a \rightarrow a/f$ (error reduced by $\sim f^2$, cost $\sim f^6$)

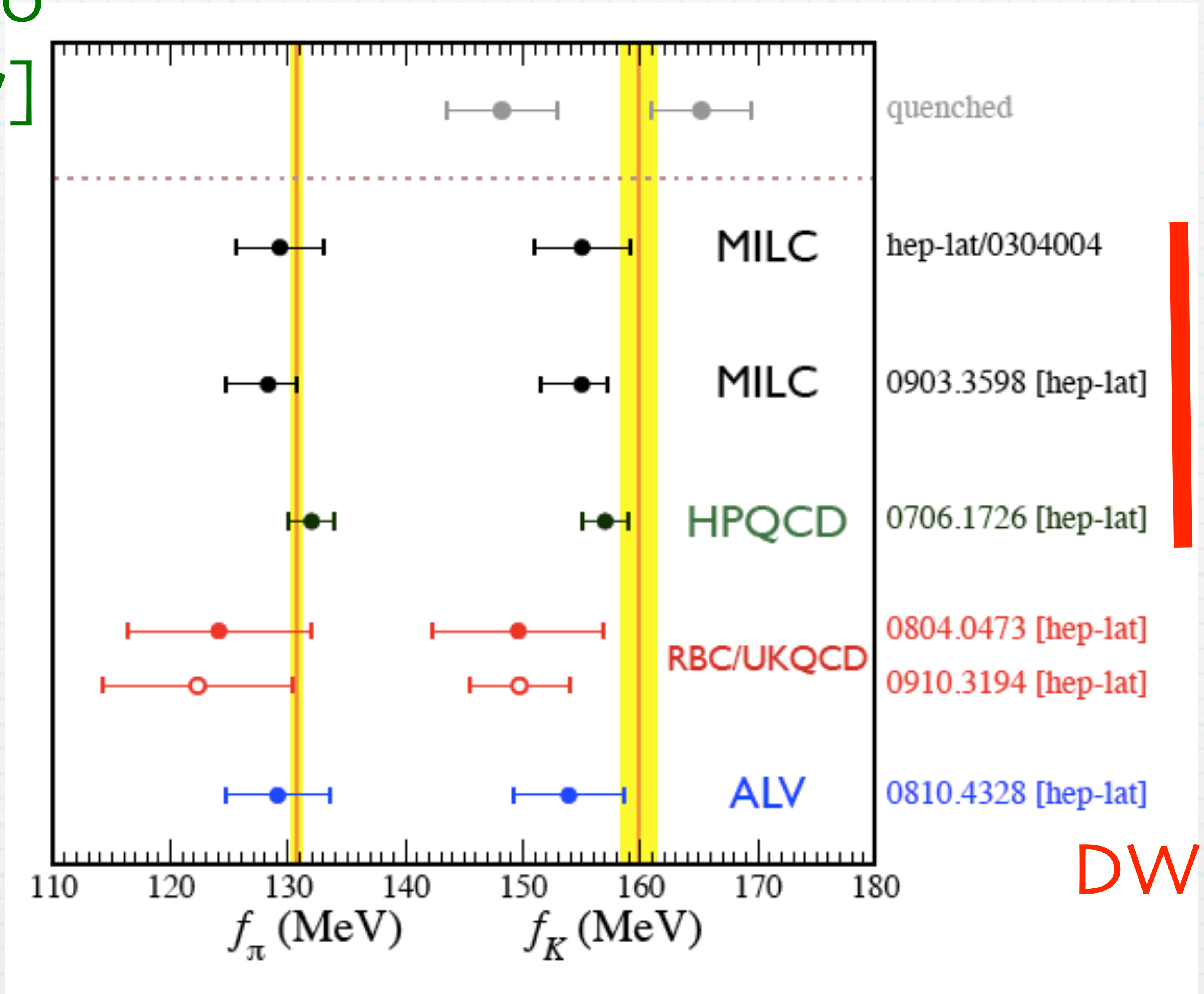
Possible on 2-5 year timescale (need PFlops-yrs)

At some level, will run into other systematics, e.g. EM effects (under study in some quantities), and effects of (omitted) charmed sea

Status of f_K & f_π

[Kronfeld, 2010 USQCD review]

Scale mostly set with Υ splittings



Staggered

DWF

DWF on stagg.

Normalized axial current ⇒ staggered results most accurate

Important to have results with Wilson/DWF

Error budgets & prospects

[HPQCD, arXiv: 0706.1726]

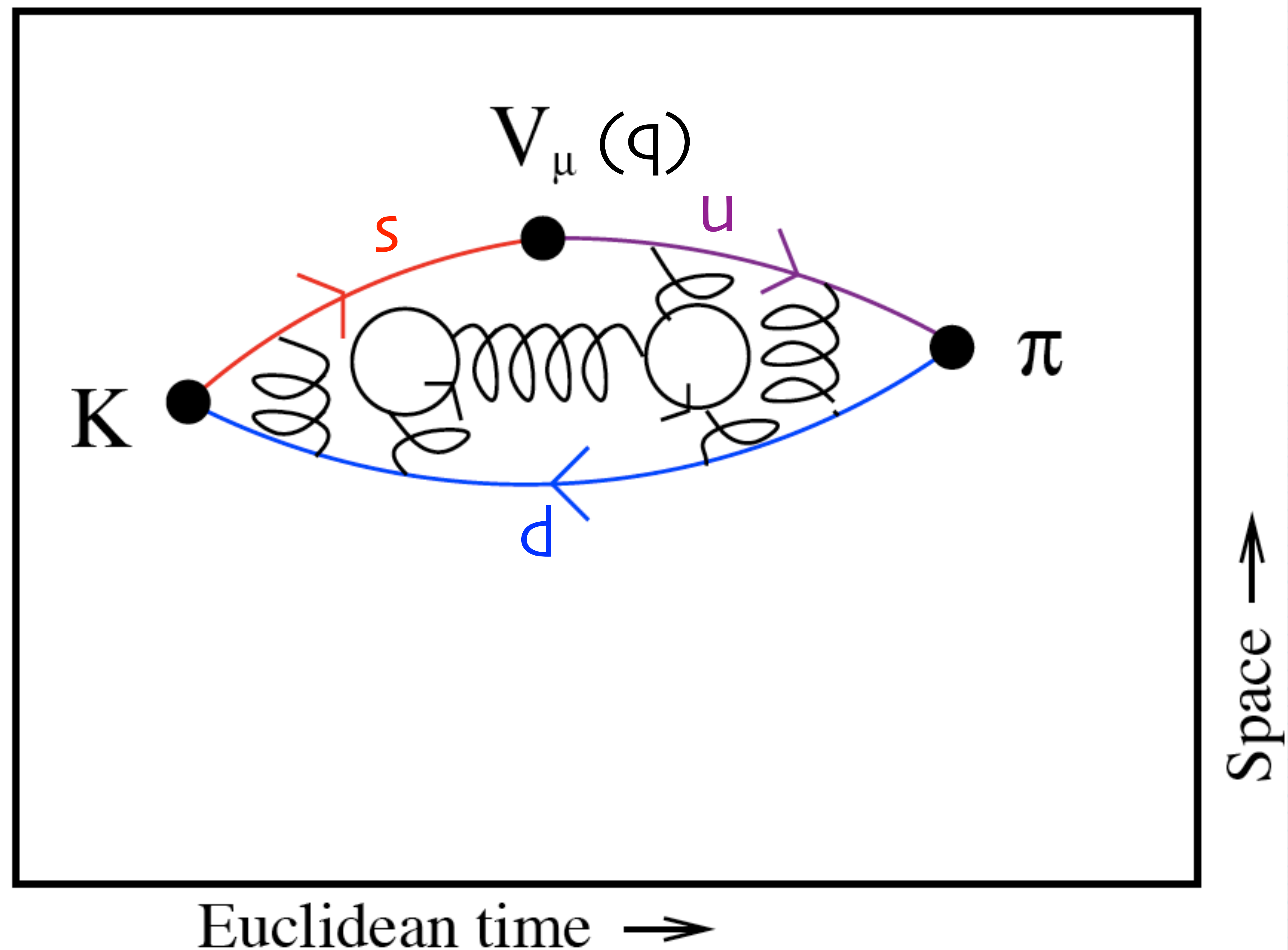
	f_K/f_π	f_K	f_π
r_1 uncerty.	0.3	1.1	1.4
a^2 extrap.	0.2	0.2	0.2
Finite vol.	0.4	0.4	0.8
$m_{u/d}$ extrap.	0.2	0.3	0.4
Stat. errors	0.2	0.4	0.5
m_s evoln.	0.1	0.1	0.1
m_d, QED, etc.	0.0	0.0	0.0
Total %	0.6	1.3	1.7

Dominant error is scale uncertainty

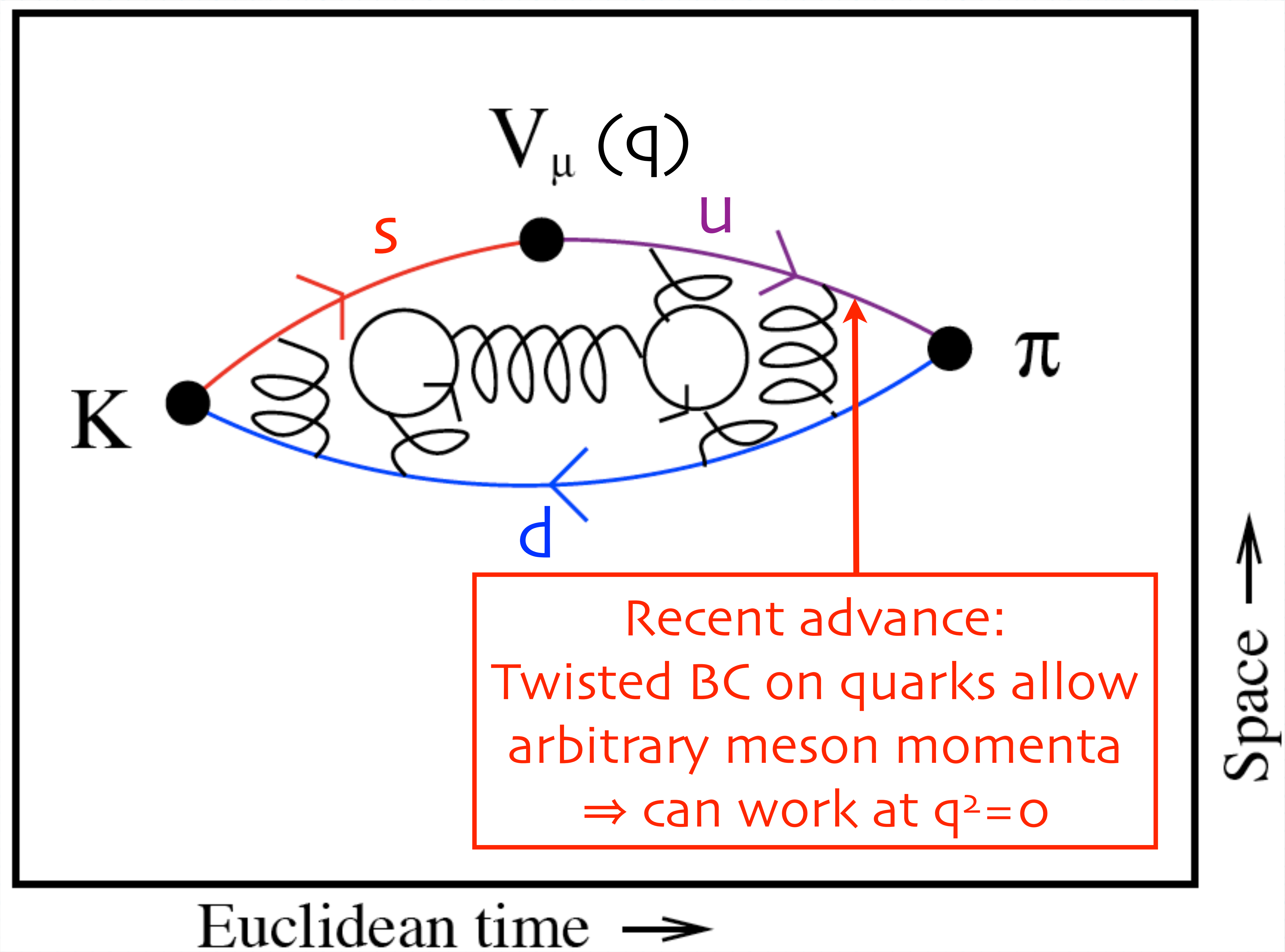
- Expect gradual improvement, with < 1% errors in few years
- f_π may be used to set the scale in future

$K \to \pi$ form factors

$f_+(0) \Rightarrow V_{us}$



V_μ (q)
s
u
K
π
d
Euclidean time →
Space →



Vμ (q)
s
u
K
π
d
Recent advance:
Twisted BC on quarks allow
arbitrary meson momenta
⇒ can work at q²=0
Space →
Euclidean time →

$f_+(0)$:FLAG coding scheme

[Lubicz]

Collaboration	Ref.	N_f	publication status	chiral extrapolation	finite volume errors	continuum extrapolation	$f_+(0)$
RBC/UKQCD 07	[9]	2+1	A	●	★	■	0.9644(33)(34)(14)
ETMC 09	[10]	2	A	●	●	●	0.9560(57)(62)
QCDSF 07	[11]	2	C	■	★	■	$0.9647(15)_{stat}$
RBC 06	[12]	2	A	■	★	■	0.968(9)(6)
JLQCD 05	[13]	2	C	■	★	■	0.967(6)
SPQ$_{CD}$R 04	[8]	0	A	■	★	■	0.960(5)(7)

- No calculation with all errors fully controlled
- Few calculations compared to f_K & f_π

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Included in average

- No calculation with all errors fully controlled
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$f_+(0)$:FLAG coding scheme

[Lubicz]

$$f_+(0) = 0.9599(34)(^{+31}_{-43})(14)\ .$$

[RBC/UKQCD 09]

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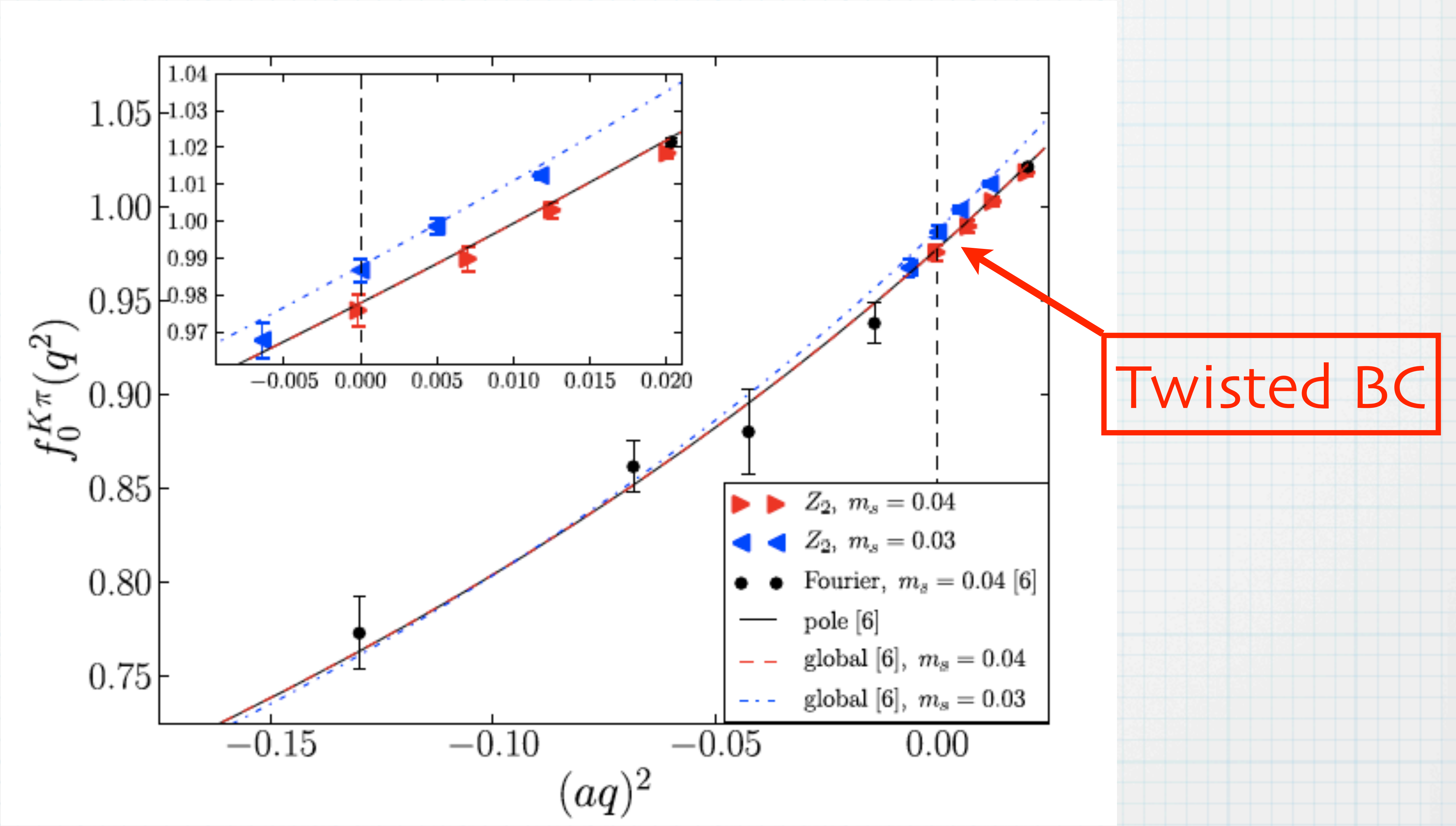
State of the art for $f_+(0)$

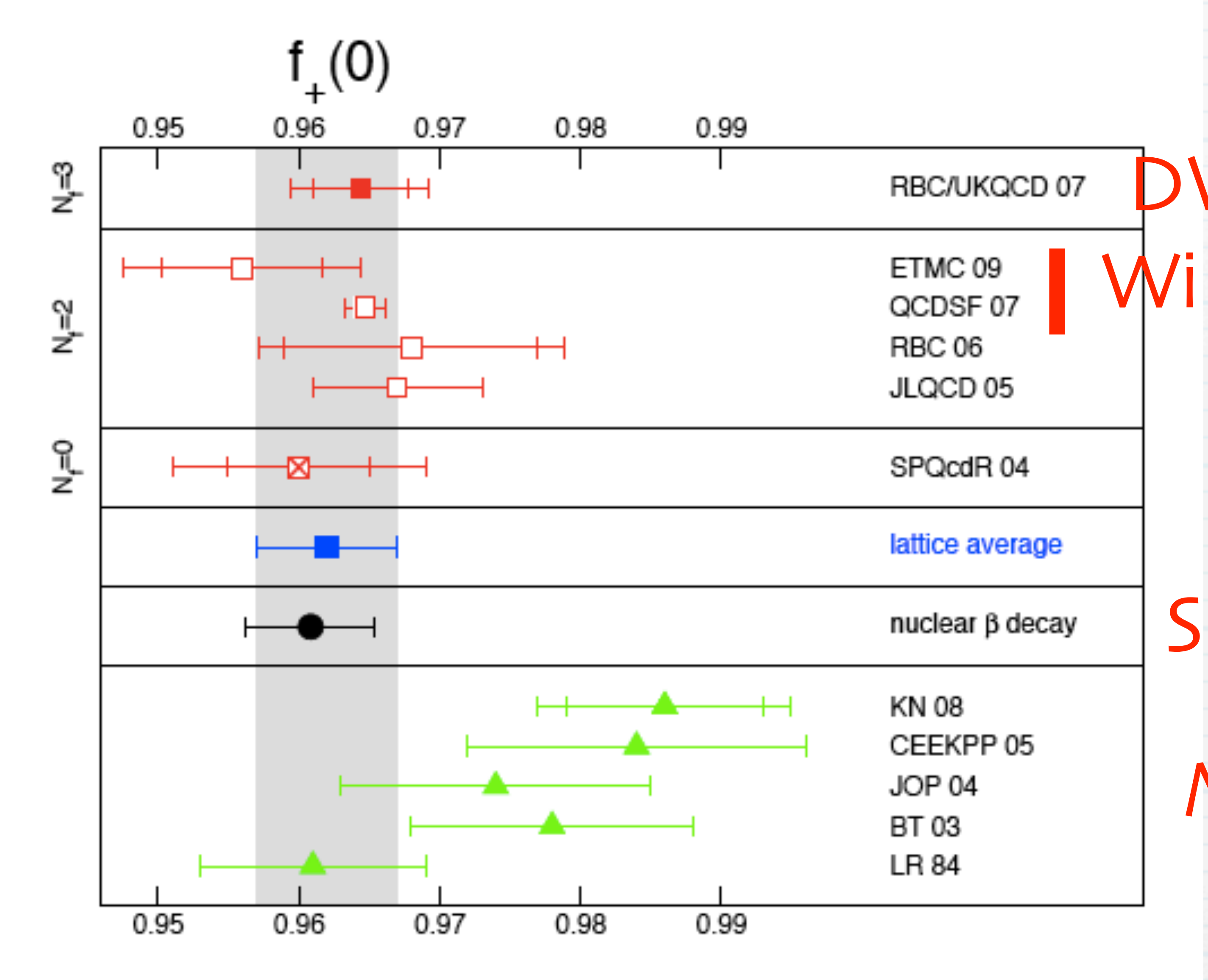


$f_+(0)=0.960(3)(4)(1)$

[RBC/UKQCD09, arXiv:1004.0886]

Status of $f_+(0)$

[Lubicz]



DWF

Wilson

SM + expt.

Models

Lattice average: $f_+(0)=0.962(3)(4)$ [Lubicz]

2+1 flavor result: $f_+(0)=0.960(3)(4)(1)$ [RBC/UKQCD09]

SM+expt+V_{ud}: $f_+(0)=0.9608(46)$ [FLAG]

Future prospects

$f_+(0)=0.960(3)(4)(1)$ [RBC/UKQCD09]

(statistics)(chiral extrap)(continuum extrap)

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$f_+(0)=0.960(3)(4)(1)$ [RBC/UKQCD09]

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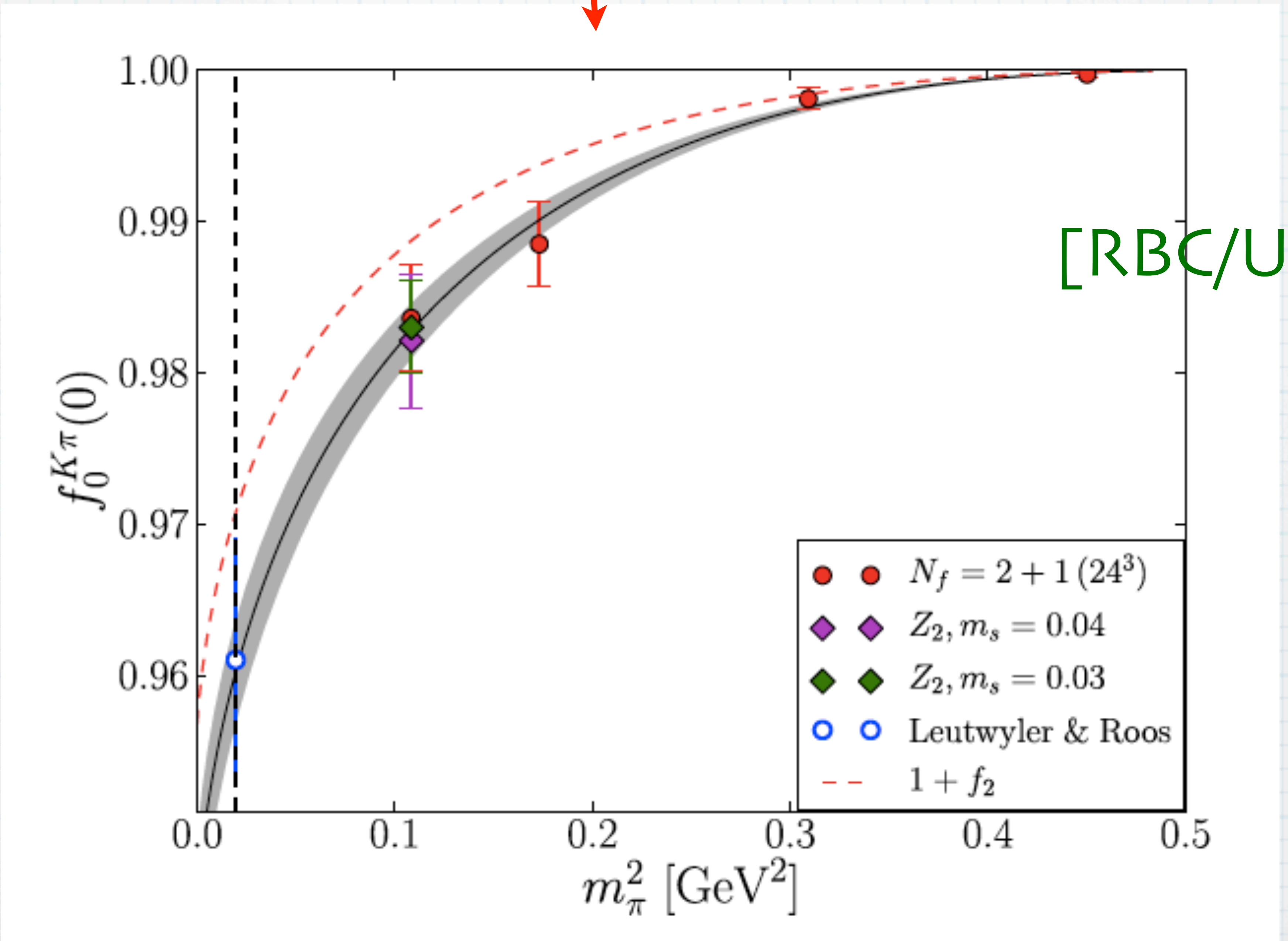


[RBC/UKQCD09]

Future prospects

$f_+(0) = 0.960(3)(4)(1)$ [RBC/UKQCD09]

(statistics)(chiral extrap)(continuum extrap)

To reduce dominant systematics:

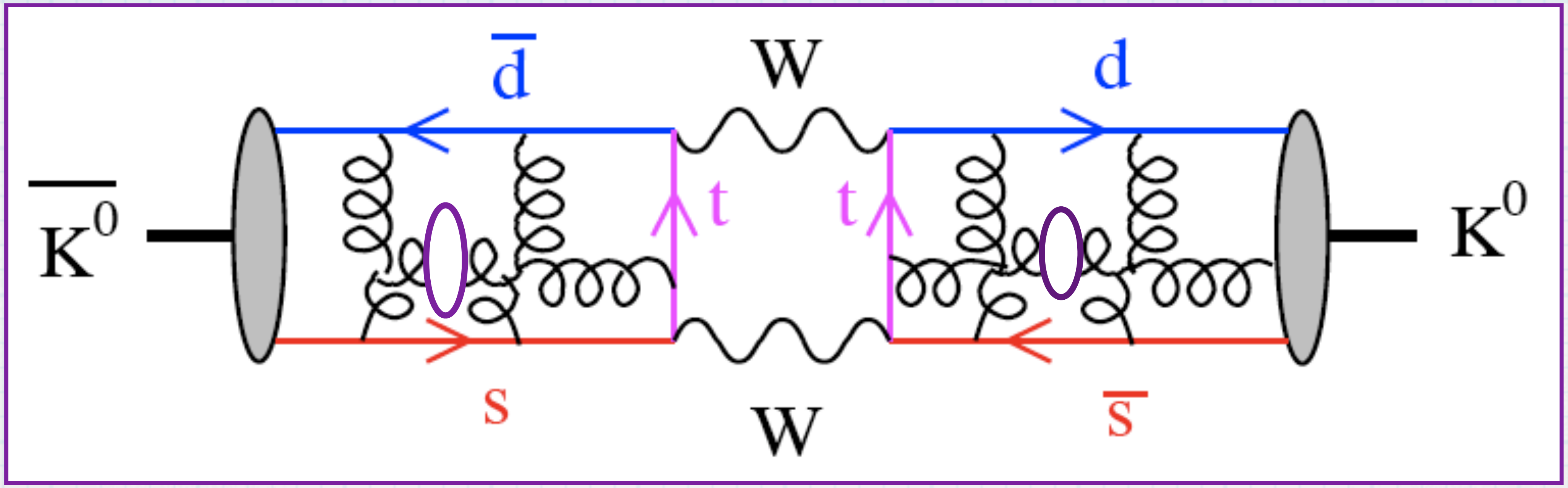
- $m_\pi \rightarrow$ physical value (error removed)
- $a \rightarrow a/f$

Possible on 2-5 year timescale (need PFlops-yrs)

On same timescale, will have results with other fermions (Wilson, staggered?)

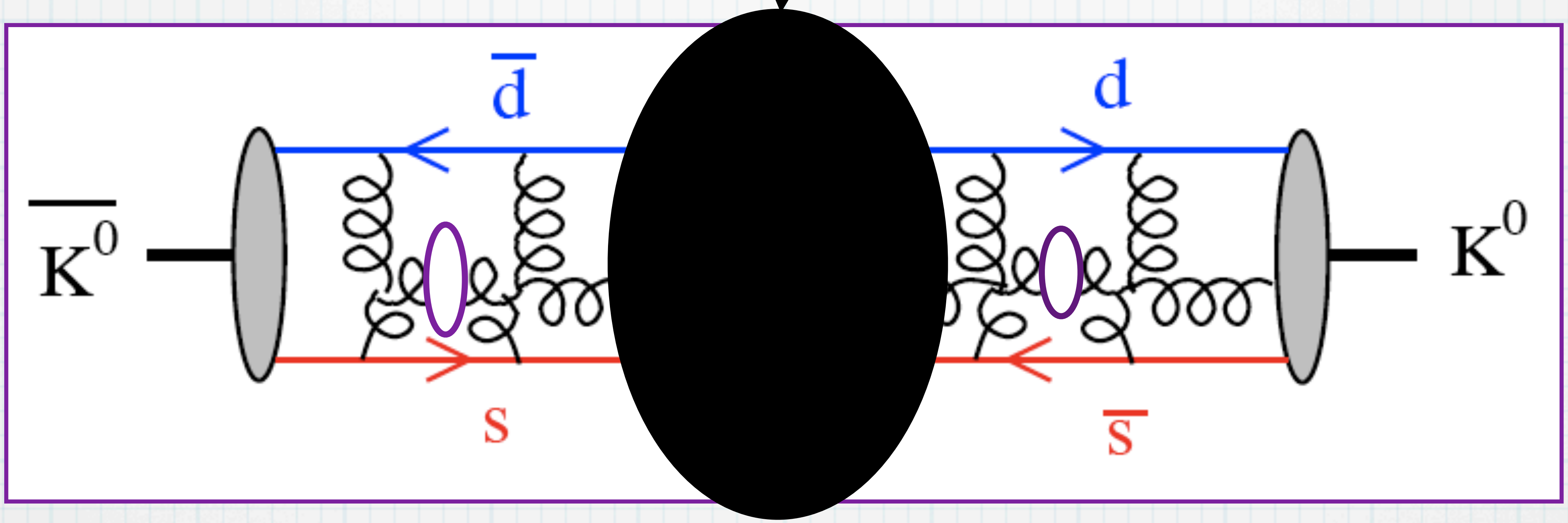
B_K (and related matrix elements)

Calculating B_K



Calculating B_K

Known local four-fermion operator



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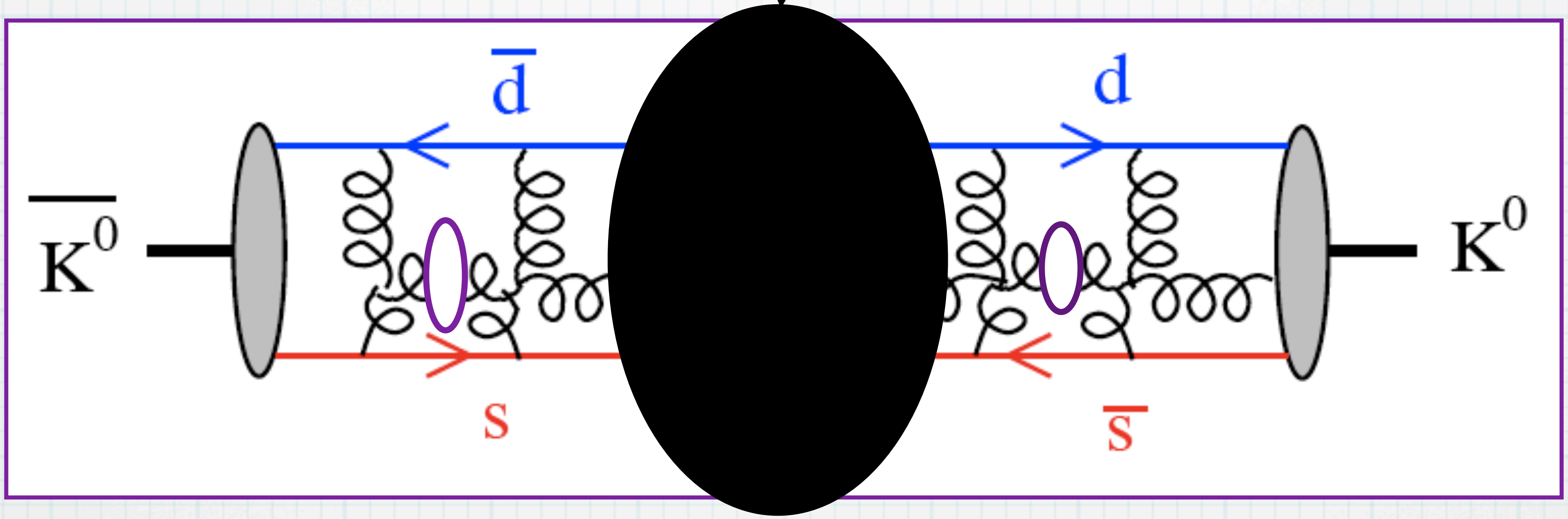

New feature: need to match operator to continuum scheme

B_K :FLAG coding scheme

[Lubicz]

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ALVdW 09	[49]	2+1	A	●	★	●	★	●	0.527(6)(20)	0.724(8)(28)
RBC/UKQCD 09	[50]	2+1	C	●	●	★	★	●	0.537(6)(18)	0.738(8)(25)
SBW 09	[51]-[54]	2+1	C	★	★	■	■	●	0.512(14)(34)	0.701(19)(47)
RBC/UKQCD 07	[55, 20]	2+1	A	■	●	★	★	●	0.524(10)(28)	0.720(13)(37)
HPQCD/UKQCD 06	[56]	2+1	A	■	●	★	■	●	0.618(18)(135)	0.83(18)
ETMC 09	[57]	2	C	★	●	●	★	●	0.518(21)(21)	0.730(30)(30)
JLQCD 08	[58]	2	A	■	●	■	★	●	0.537(4)(40)	0.758(6)(71)
RBC 04	[59]	2	A	■	■	■†	★	●	0.495(18)	0.699(25)
UKQCD 04	[60]	2	A	■	■	■†	■	●	0.49(13)	0.69(18)

- Two calculations with all errors fully controlled
- Several more in near future with different fermions

B_K :FLAG coding scheme

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Included in Lubicz average

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B_K :FLAG coding scheme

[Lubicz]

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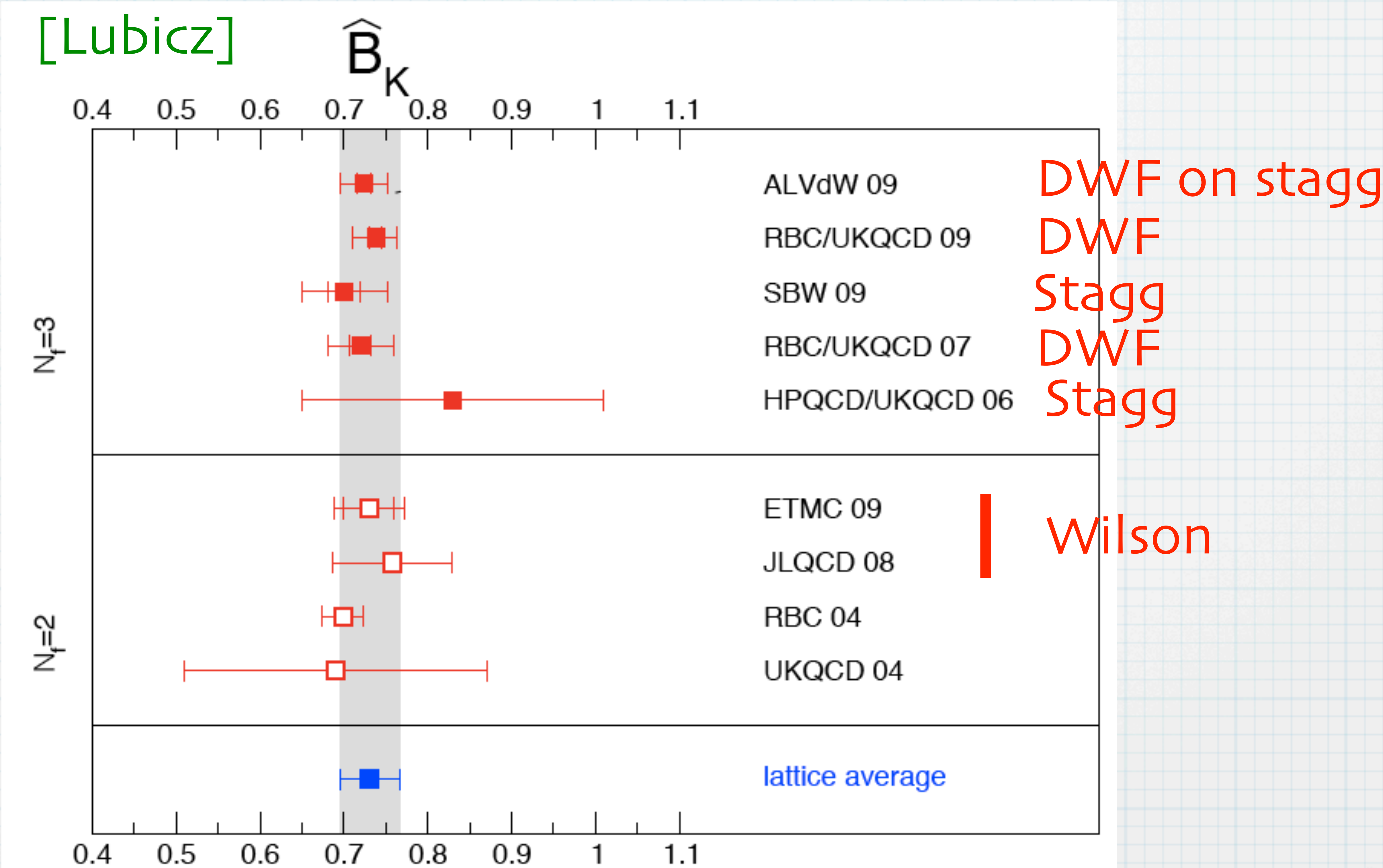
Status of B_K



Good agreement, with 3-5% errors, and several fermion discretizations ⇒ reliable

B_K vs. SM

Lattice averages:

$B_K = 0.725(27)$ [LLV]

$B_K = 0.731(7)(35)$ [Lubicz]

Unitarity triangle fit: [LLV]

$$(\hat{B}_K)_{\text{fit}} = \begin{cases} 1.09 \pm 0.12 & |V_{cb}|_{\text{excl}} \\ 0.903 \pm 0.086 & |V_{cb}|_{\text{incl}} \\ 0.98 \pm 0.10 & |V_{cb}|_{\text{excl+incl}} \end{cases}$$

2-3σ tension

Errors dominated by those in V_{cb}, not those in B_K !

Nevertheless, worth reducing errors to 1% level

Prospects for B_K

[ALV09]

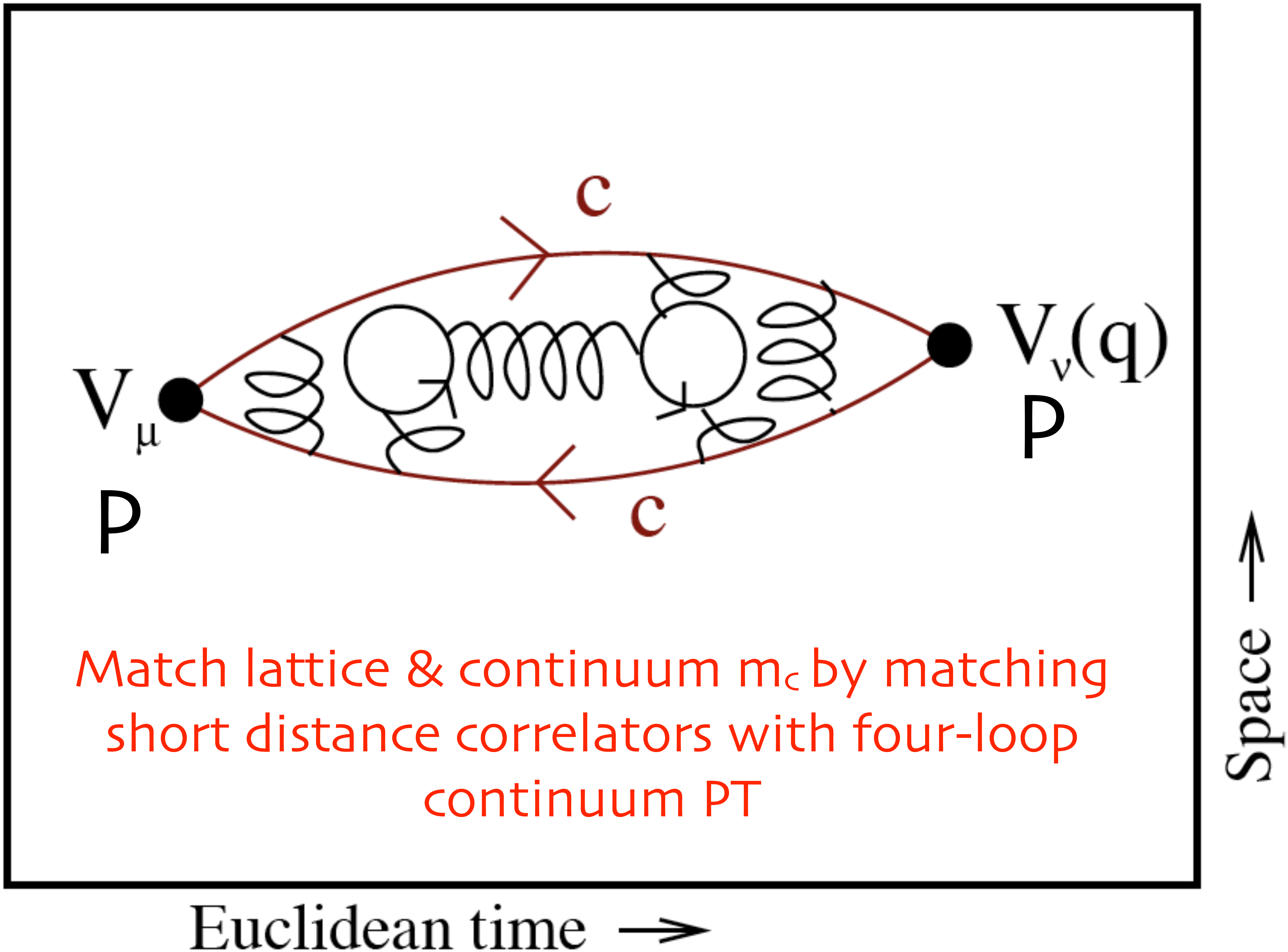
uncertainty	Z_{B_K}
statistics	0.7%
chiral extrapolation fit function	1.2%
strange quark mass dependence	0.3%
chiral symmetry breaking	1.2%
perturbation theory	2.8%
total	3.4%

Dominant error is matching factor

- Expect some improvement by use of finer lattices, higher order continuum PT
- Attaining 1% will be challenging

Calculations will be extended in 1-2 years to four-fermion operators needed to constrain BSM physics, with 5-10% accuracy

Quark masses



Match lattice & continuum m_c by matching short distance correlators with four-loop continuum PT

Results for m_c

Relatively easy to obtain m_c^{lat} using improved fermions
HARDER to match to continuum m_c

Recent advance: matching using short distance correlators:

$m_c(m_c)$=1.268(9) GeV [HPQCD* 08(imp. stagg)]
$m_c(m_c)$=1.273(6) GeV [HPQCD 10(imp. stagg)]

Agrees remarkably well with determination from e^+e^- data:

$m_c(m_c)$=1.268(12) GeV [Kuhn et al, 07]

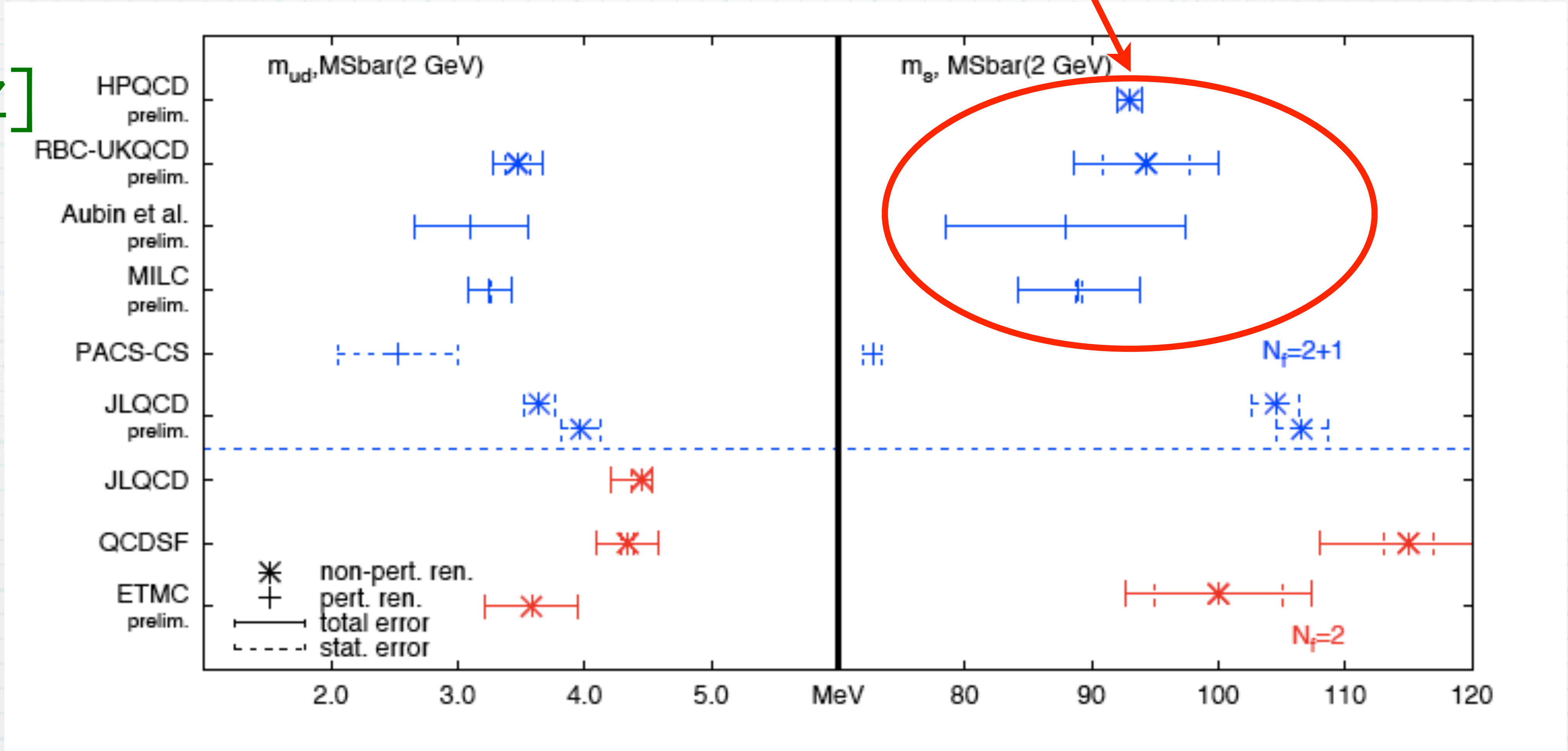
Important to check using other fermion discretizations, and including the charmed sea quark, which will take several years

Results for m_s

Again, matching is dominant source of error

Reasonable agreement if use non-perturbative or 2-loop matching to continuum

[Scholz]



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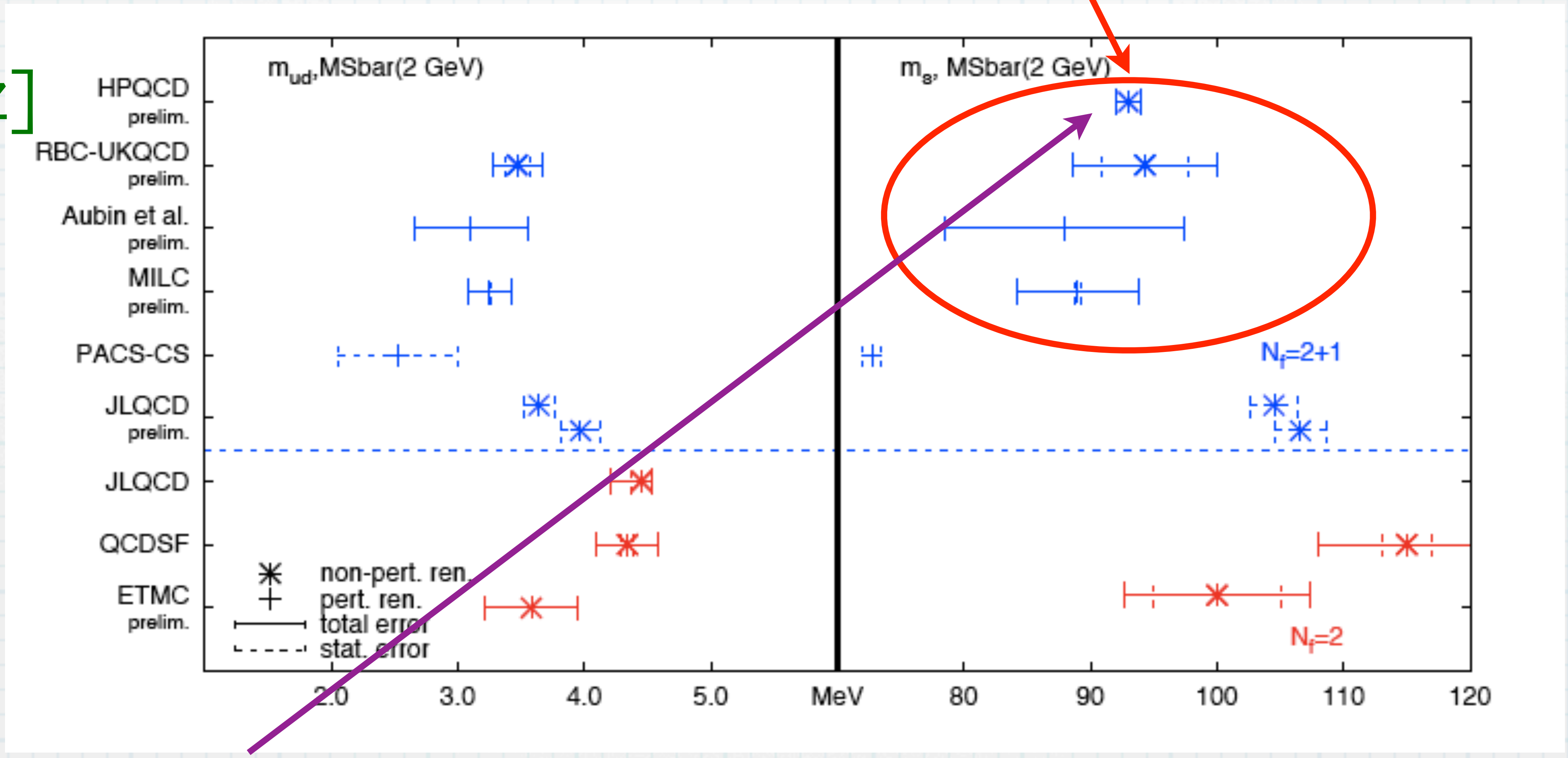

Recent advance: measure m_s/m_c by using same fermions for both, and multiply by accurate m_c

Result for m_s

$$m_s(\overline{\mathrm{MS}}, 2\ \mathrm{GeV}) = 92.4(1.5)\,\mathrm{MeV}$$ [HPQCD 09]

$$m_s(\overline{\mathrm{MS}}, 2\ \mathrm{GeV}) = 92.2(1.3)\,\mathrm{MeV}$$ [HPQCD 10]

Important to check using other fermion discretizations, which will take several years

Very recent result for m_b

$$m_b(m_b) = 4.164(23)\,\mathrm{GeV}$$

[HPQCD 10, arXiv1004.4285]

Using same method as for m_c, but extrapolating to m_b.
Cross checked by independent result for m_c/m_b
In very good agreement with continuum result:

$$m_b(m_b) = 4.163(16)\,\mathrm{GeV}$$

[Chetrykin et al, 09]

Summary

Summary

* Several precise and reliable results!
* Errors will be further reduced by simulations with physical quark masses (including charm)
* Important to have results with multiple discretizations of fermions

References

References

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