Fundamental parameters of the Standard Model

Progress report and outlook for remainder of LQCD project

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Objective

Calculate weak interaction matrix elements between strongly interacting particles to the accuracy needed to determine many of the fundamental parameters of the Standard Model (SM), and to make precise tests of the SM.

- **Calculations are needed to interpret important parts of the experimental work at SLAC (BaBar), FNAL (CDF and D0) and Cornell (CLEO-c)**
- **For more details and plans beyond FY09, see the associated white paper**

Outline

- Aims
- Approach: choices of fermions and ensembles (present and future)
  - MILC improved staggered
  - Domain Wall fermion (DWF)
- Results: past, present and near future
- Examples of what can be achieved in this project
  - Unitarity of first row of CKM matrix
  - Lattice constraints on the unitarity triangle analysis
  - New quantities
- Summary and outlook

Parameters we aim to calculate

- Quark masses: m_u, m_d, m_s, m_c and m_b
  - Input into beyond the standard model (BSM) physics
  - Significant progress made—known to 7-10 % with all errors controlled

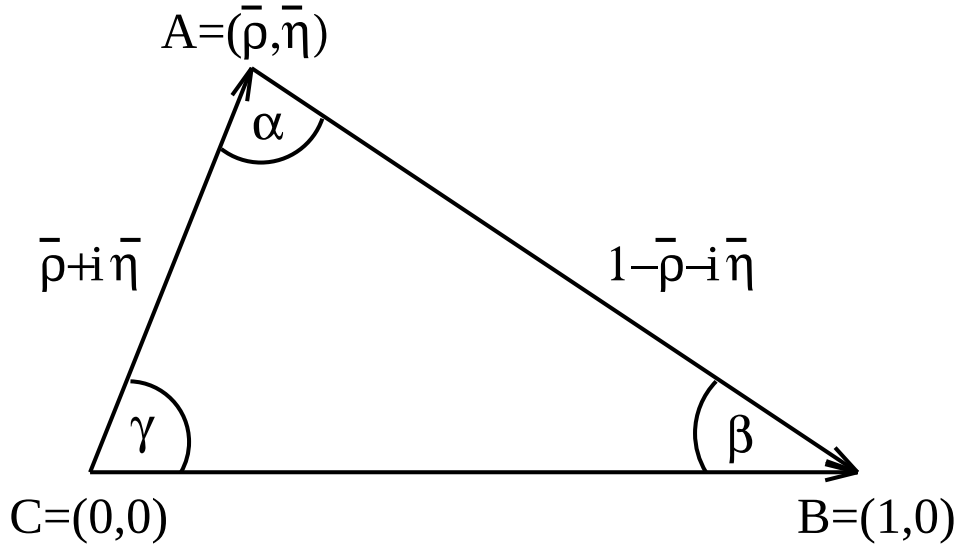
- Strong coupling constant α_S
  - Lattice result is most accurate determination $\alpha_S(m_Z) = 0.1170 \pm 0.0012$
  - Agreement with perturbative QCD determinations (average $= 0.1185 \pm 0.0015$) is highly non-trivial test of QCD as theory of strong interactions on all scales

- Elements of CKM matrix
  - Requires calculation of more complicated matrix elements
  - Test unitarity (V_{us}, V_{cs}, V_{ts})
  - Test SM description of CP violation (V_{td}, ρ, η)

Cabibbo-Kobayashi-Maskawa matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} =$$

$$\begin{pmatrix} 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda+\frac{A^2\lambda^5}{2}[1-2(\rho+i\eta)] & 1-\frac{\lambda^2}{2}+\frac{\lambda^4}{8}(1+4A^2) & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2+\frac{A\lambda^4}{2}(1-2\rho)-i\eta A\lambda^4 & 1-\frac{A^2\lambda^2}{2} \end{pmatrix} + O(\lambda^6)\,,$$



USQCD approach for fundamental parameters

Two lattice fermion actions

- Improved staggered fermions ("asqtad" [MILC,Lepage])
  - \+ Computationally fast
  - – Rooting trick
  - – Complicated fits needed to take chiral-continuum limits
- Domain-wall fermions [Kaplan,Shamir]
  - – 10-20 times slower than staggered
  - \+ Almost exact chiral symmetry
  - \+ Correct number of fermions

Recent advances

- RHMC provides $\gtrsim 6$ speed up [Clark & Kennedy]
- Highly improved staggered quarks (HISQ) [Follana *et al*]
- Tuning of gauge action with DWF to reduce chiral symmetry breaking [RBC/UKQCD]

Ensembles needed to achieve aims

For each fermion discretization:

- Unquenched $2+1$ flavor simulations
- Several lattice spacings for continuum extrapolation
  - Asqtad: $a = 0.15$, 0.12, 0.09 fm and now 0.06 fm
  - DWF: $a = 0.12$ fm with $a = 0.09$ fm starting
- Light quark masses ranging down to $m_\ell/m_s \approx 1/10$
- Several (≥ 2) volumes—so far 2, 3 fm

Heavy quarks (c, b) treated separately:

- Internal loops integrated out perturbatively
- Valence quarks use either NRQCD or "FNAL formalism" [El-Khadra *et al*]

Status:

- Asqtad ensemble in place and being extended
- DWF ensemble underway

MILC ensemble: **past** and present

MILC0: completed 2003-4 (before project). Coarse and Fine lattices

a (fm)	am'_ℓ / am'_s	L(fm)	dims.	# lats.	$m_\pi L$
≈ 0.12	0.03 / 0.05	2.4	$20^3 \times 64$	564	7.6
≈ 0.12	0.02 / 0.05	2.4	$20^3 \times 64$	484	6.2
≈ 0.12	0.01 / 0.05	2.4	$20^3 \times 64$	658	4.5
≈ 0.12	0.01 / 0.05	3.4	$28^3 \times 64$	241	6.3
≈ 0.12	0.007 / 0.05	2.4	$20^3 \times 64$	493	3.8
≈ 0.12	0.005 / 0.05	2.9	$24^3 \times 64$	527	3.8
≈ 0.09	0.0124 / 0.031	2.5	$28^3 \times 96$	531	5.8
≈ 0.09	0.0062 / 0.031	2.5	$28^3 \times 96$	583	4.1

MILC ensemble: past and **present**

MILC1: completed FY07. Extra-fine, Medium-coarse, and extended **Coarse**/**Fine**

a (fm)	am'_ℓ / am'_s	L(fm)	dims.	# lats.	$m_\pi L$
≈ 0.15	0.0290 / 0.0484	2.4	$16^3 \times 48$	600	6.6
≈ 0.15	0.0194 / 0.0484	2.4	$16^3 \times 48$	600	5.5
≈ 0.15	0.0097 / 0.0484	2.4	$16^3 \times 48$	600	3.9
≈ 0.15	0.00484 / 0.0484	2.4	$20^3 \times 48$	600	3.5
≈ 0.12	0.03 / 0.05	2.4	$20^3 \times 64$	564	7.6
≈ 0.12	0.02 / 0.05	2.4	$20^3 \times 64$	484	6.2
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$\approx \mathbf{0.12}$	**0.03 / 0.03**	**2.4**	$\mathbf{20^3 \times 64}$	**350**	**7.6**
$\approx \mathbf{0.12}$	**0.01 / 0.03**	**2.4**	$\mathbf{20^3 \times 64}$	**349**	**4.5**
≈ 0.09	0.0124 / 0.031	2.5	$28^3 \times 96$	531	5.8
≈ 0.09	0.0062 / 0.031	2.5	$28^3 \times 96$	583	4.1
$\approx \mathbf{0.09}$	**0.0031 / 0.031**	**3.6**	$\mathbf{40^3 \times 96}$	**473**	**4.4**
≈ 0.06	0.0072 / 0.018	2.9	$48^3 \times 144$	550	6.3
≈ 0.06	0.0036 / 0.018	2.9	$48^3 \times 144$	350^*	4.5

MILC ensemble: possible **future**

a (fm)	m_ℓ/m_s	Size	L (fm)	MC traj.	TF-Yr R-alg	**TF-Yr RHMD**	Label
0.09	0.10	$40^3 \times 96$	3.6	3000	1.5	0.14	MILC1
0.09	**0.05**	$\mathbf{56^3 \times 96}$	**5.0**	**4200**	**23**	**1.2**	**MILC2**
0.06	0.20	$48^3 \times 144$	2.9	$3750^\dagger$	1.9	0.7	MILC1
0.06	**0.10**	$\mathbf{64^3 \times 144}$	**3.8**	**4500**	**22**	**2.4**	**MILC2**
0.06	0.05	$84^3 \times 144$	5.0	6300	280	19	MILC3
0.06	1/27	$100^3 \times 144$	6.0	7454	–	55	MILC4
0.045	**0.40**	$\mathbf{56^3 \times 192}$	**2.5**	**4000**	**–**	**1.1**	**MILC2**
0.045	**0.20**	$\mathbf{56^3 \times 192}$	**2.5**	**5000**	**10**	**3.0**	**MILC2**
0.045	0.10	$80^3 \times 192$	3.6	6000	135	14	MILC3
0.045	0.05	$112^3 \times 192$	5.0	8400	2100	130	MILC4
0.045	1/27	$124^3 \times 192$	5.6	9940	–	320	MILC5

Note order of magnitude speed-up, mainly due to **RHMC** algorithm.

FY07-09: project produces 36 TF-Yr to be divided between different aims.

$\Rightarrow$ can achieve **MILC2**$^-$ ensemble, including overhead for calculating valence propagators.

DWF ensemble: **present** and possible future

FY06-07: Coarse lattice at two volumes; Fine lattices underway†

a (fm)	m_ℓ/m_s	Size	L_5	L (fm)	MC traj.	TF-Yr	Label
0.12	0.59	$16^3 \times 32$	16	2.0	4050		DWF0
0.12	0.33	$16^3 \times 32$	16	2.0	4000		DWF0
0.12	0.3	$24^3 \times 64$	16	3.0	9000 †	0.7	DWF1
0.12	0.19	$24^3 \times 64$	16	3.0	9000 †	0.8	DWF1
0.09	0.20	$32^3 \times 64$	16	3.0	4500 †	1.3	DWF1

Speed-up due to RHMC algorithm $\gtrsim 6$

DWF ensemble: present and possible **future**

By FY09: **Major milestone**—two lattice spacings with light quark masses

a (fm)	m_ℓ/m_s	Size	L_5	L (fm)	MC traj.	TF-Yr	Label
0.12	0.59	$16^3 \times 32$	16	2.0	4050		DWF0
0.12	0.33	$16^3 \times 32$	16	2.0	4000		DWF0
0.12	0.3	$24^3 \times 64$	16	3.0	9000	0.7	DWF1
0.12	0.19	$24^3 \times 64$	16	3.0	9000	0.8	DWF1
0.09	0.20	$32^3 \times 64$	16	3.0	4500	1.3	DWF1
0.09	**0.136**	$\mathbf{32^3 \times 64}$	**16**	**3.0**	**4500**	**1.4**	**DWF2**
0.09	**0.136**	$\mathbf{48^3 \times 64}$	**16**	**4.4**	**5000**	**7.0**	**DWF2**
0.09	0.065	$48^3 \times 64$	16	4.4	5000	8.6	DWF3
0.09	1/27	$64^3 \times 128$	24	5.9	10000	230	DWF5
0.06	0.144	$48^3 \times 64$	16	3.0	10000	18	DWF3
0.06	0.084	$64^3 \times 128$	16	4.0	10000	130	DWF4
0.06	1/27	$96^3 \times 128$	16	5.9	10000	680	DWF6

Speed-up due to RHMC algorithm $\gtrsim 6$

Results

- What has been achieved during project so far?
- What can be achieved during the remainder of the project?
- What is the significance of these (past and future) results for testing the SM?

Staggered results: prehistory 2004-5

Using MILC0 ensemble, calculate simpler quantities with all errors controlled: [MILC,FNAL,HPQCD] :

- f_π, f_K, $m_{u,d,s}$, α_S, f_D, m_{B_c}, $m_{c,b}$, ...

$\Rightarrow$ Validation and predictions (f_D, m_{B_c})

Using coarse lattice only, calculate (with discretization errors estimated)[FNAL]

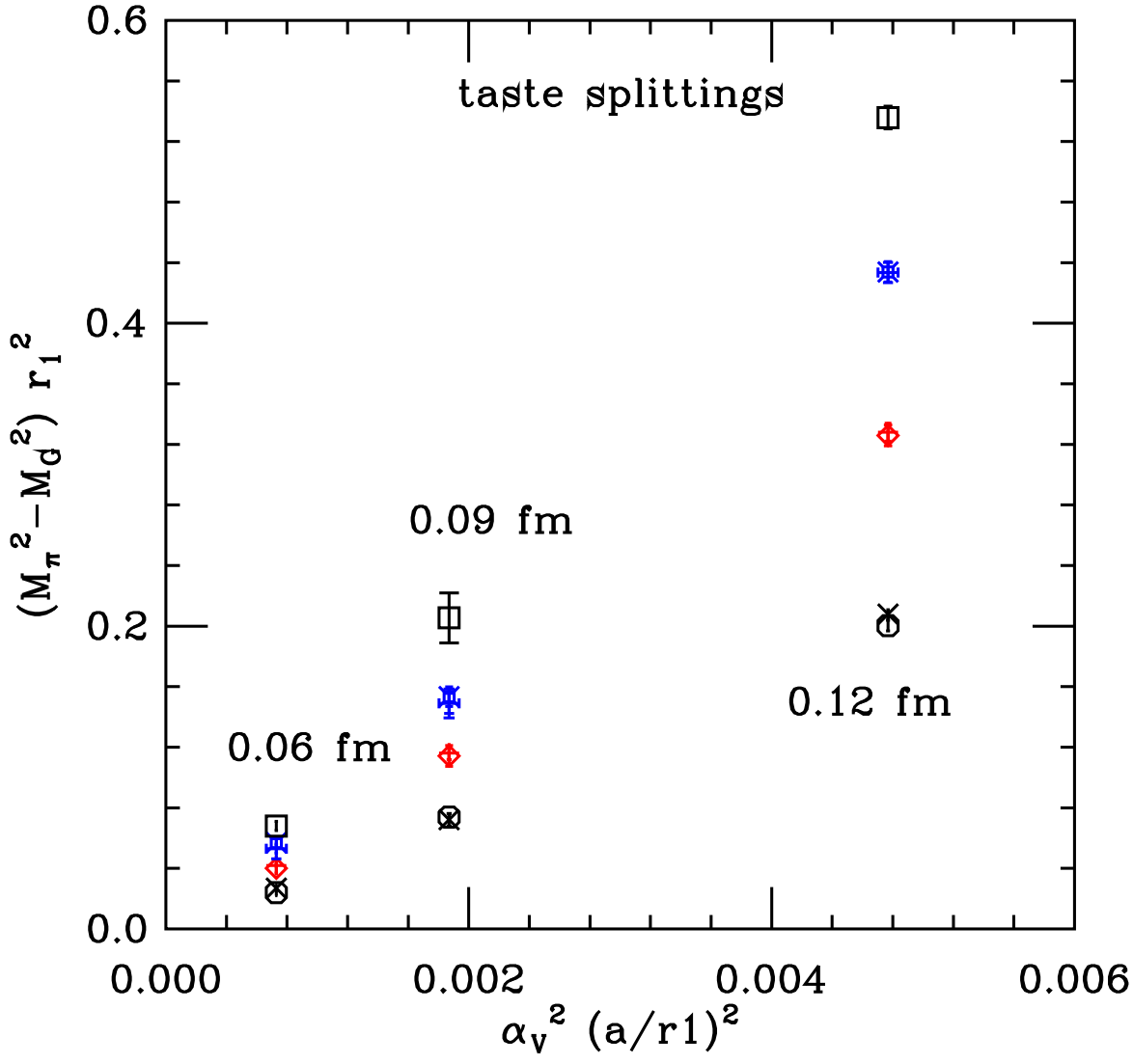
- $B \to D\ell\nu$ and $D \to K/\pi\ell\nu$ form factors
- Prediction ($D \to K$) and first lattice determination of V_{cs}, V_{cd}, V_{cb}

Staggered results: 2006 (project year 1)

- Extended previous results to larger "MILC0$^+$" ensemble (includes lightest quark masses on fine lattices and first ultra-fine lattices)
  - Tests fitting to staggered χPT forms (successfully predict ultra-fine results) [MILC]
  - Incremental improvement in errors in $f_{\pi,K}$, m_q, ...
- Calculated further weak matrix elements of bilinear operators [FNAL/MILC]
  - $B \to \pi \ell \nu \Rightarrow V_{ub}$ smaller than that from (lattice-independent) inclusive B decays
- First four-fermion operator matrix element (coarse lattice only)
  - $f_{B_s}\sqrt{B_{B_s}} \Rightarrow |V_{ts}|$ and test of SM [HPQCD]
- Results from Highly Improved Staggered Fermions (HISQ) [HPQCD]
  - Reduced taste violations, improved precision for c quark

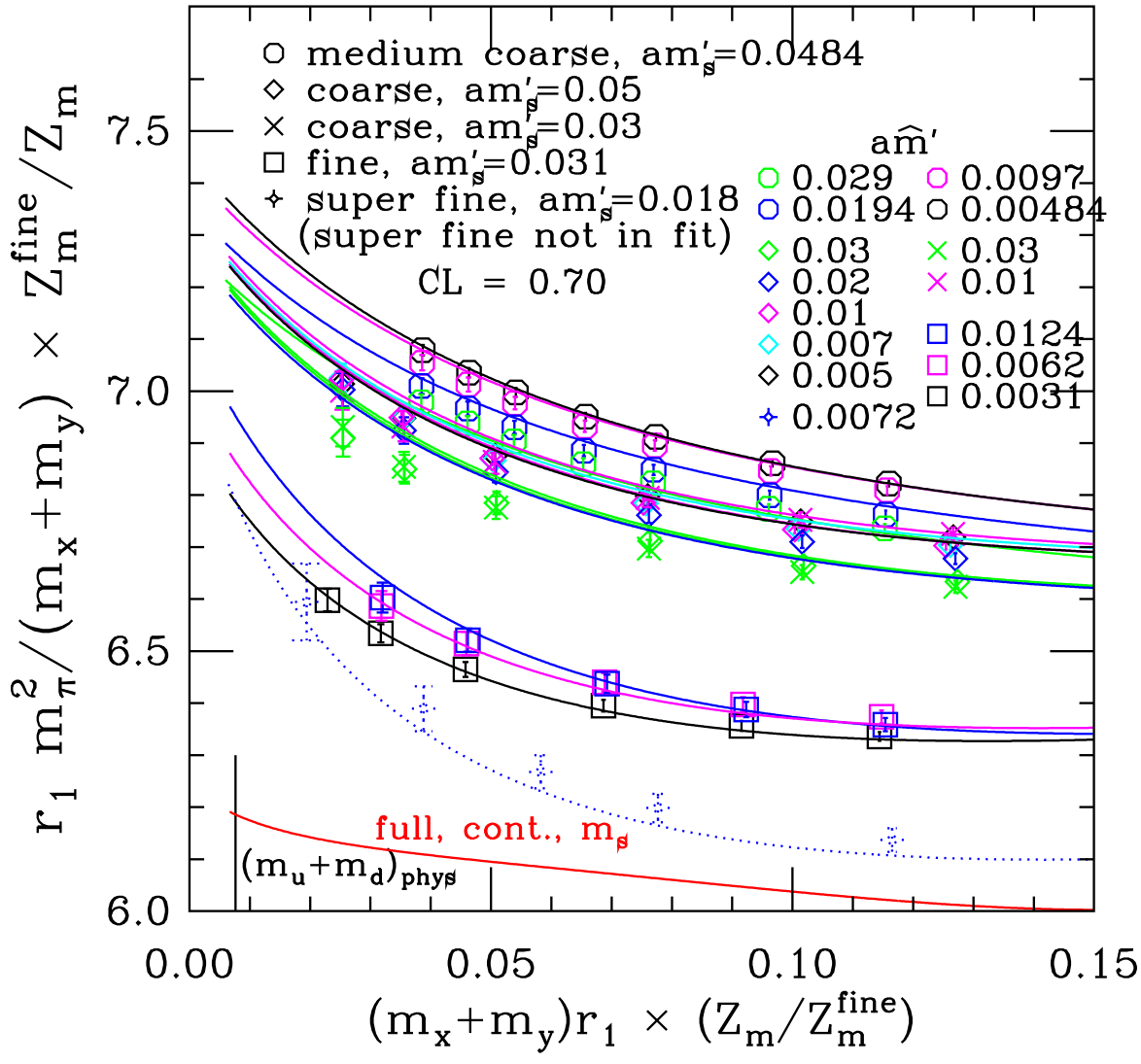
2006 staggered results: taste-splitting

- Fall as $\alpha_S^2 a^2$
- Important consistency check



2006 staggered results: spectrum

- Results for $m_\pi^2/(m_x + m_y)$
- Part of global fit to PGB properties
- Super-fine lattice results agree with predictions
- Partially quenched staggered chiral perturbation theory describes data well



2006 staggered results: quark masses

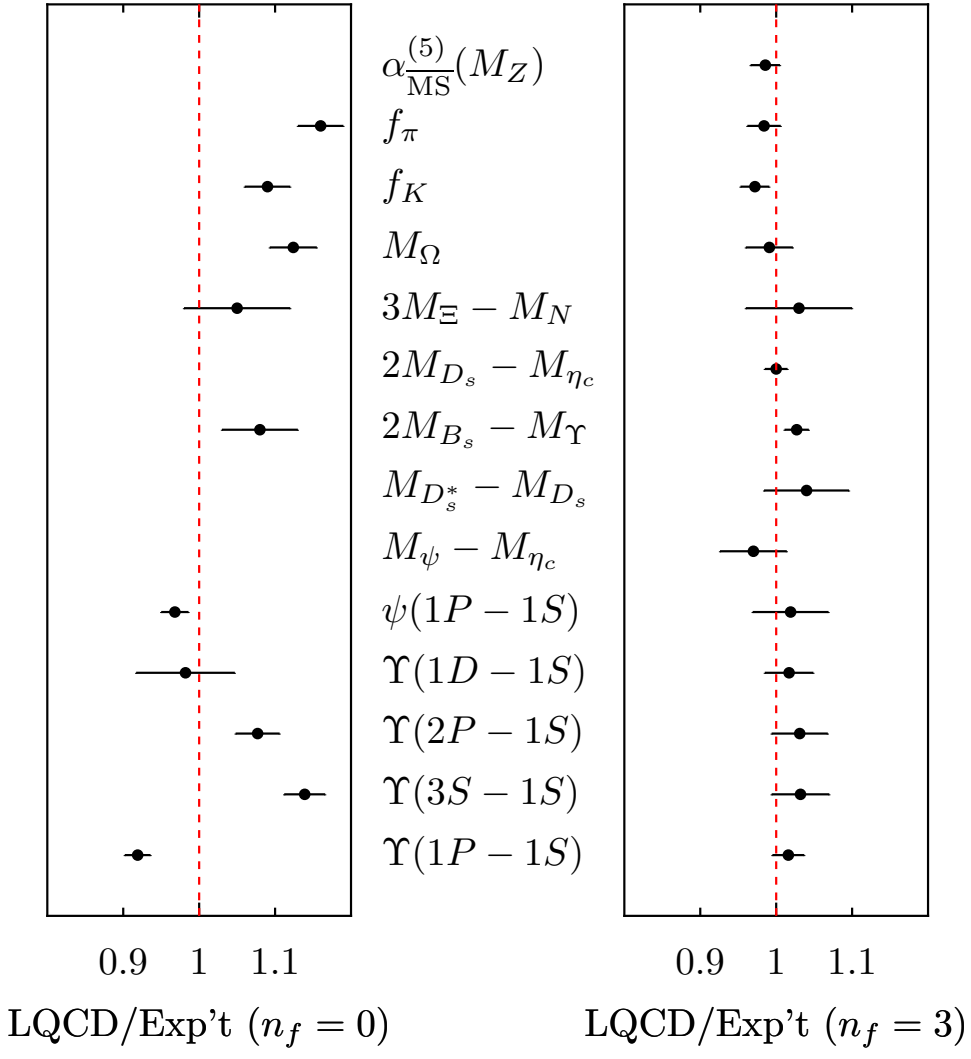
Super-fine (and coarser) lattices confirm earlier results, e.g.

$$m_s^{\overline{\rm MS}}(2\ {\rm GeV}) = 90(0)(5)(4)(0)\ {\rm MeV} \qquad [87(0)(4)(4)(0)\ {\rm MeV}]$$
$$m_s/\hat{m} = 27.2(0)(4)(0)(0) \qquad [27.4(1)(4)(0)(1)]$$
$$m_u/m_d = 0.42(0)(1)(0)(4) \qquad [0.43(0)(1)(0)(8)]$$

- Old results (2005) in red
- Errors are from statistics, simulation, perturbation theory, and EM effects.

2006 staggered results: ratio plot

Comparison with experiment for "gold-plated" quantities updated and extended:



Staggered results: 2007 (expected)

- Extend basic results to larger MILC1 ensemble
  - Errors reduced to $\delta f_\pi \approx 2\%$, $\delta(f_K/f_\pi) \approx 0.7\% \Rightarrow$ unitarity test
  - Valence HISQ quarks give $\delta f_D \approx 1 - 2\%$
- Extend fully controlled bilinear weak matrix element results (MILC0$^+$)
  - $B \to \pi\ell\nu$ and $B \to D\ell\nu \Rightarrow$ improved lattice V_{ub}
  - First $B \to D^*\ell\nu$ results $\Rightarrow$ alternative determination of V_{cb}
- **First fully controlled calculation of heavy-light four-fermion matrix elements**
  - $f_{B_s}\sqrt{B_{B_s}}$ and $f_B\sqrt{B_B}$ (and thus $\xi = (f_{B_s}\sqrt{B_{B_s}}/f_B\sqrt{B_B})$)
  - Constrain $|V_{ts}|$, $\bar\rho$ and $\bar\eta$ and test of SM
  - Major milestone

DWF results

2006: project year 1

Coarse ($a \approx 0.12\,\text{fm}$) lattices, $m_\ell/m_s \gtrsim 0.33$ [RBC/UKQCD]

- Validation of spectrum at 5% level
- Precision calculation of $K \to \pi\ell\nu$ with 0.1% errors from statistics and chiral extrapolation
- Precision calculation of B_K with 2% statistical and 2% matching errors

$\Rightarrow$ DWF extend reach of precision calculations

DWF results

2006: project year 1

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$\Rightarrow$ DWF extend reach of precision calculations

2007: project year 2

Important DWF1 milestone: first fine lattices analyzed

- First DWF results with all errors controlled (B_K ...)
- Precision validation and cross-checks with staggered

2009: end of project

DWF2 ensemble comparable to MILC0$^+$ but without taste-breaking

- Multiple precision results

What can be achieved in course of project?

Example 1: V_{us} and unitarity of first row of CKM matrix

Lattice result for f_K/f_π allows alternate determination of V_{us}

- Present MILC0$^+$ result (error dominated by chiral/continuum extrap.)

$$f_K/f_\pi = 1.208^{+7}_{-14} \quad \Rightarrow \quad |V_{us}| = 0.2223^{+26}_{-14} \quad [\text{cf. } 0.2257(21) \text{ from } K \to \pi\ell\nu]$$

- Implies unitarity sum consistent with unity:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9977(5)(12)(0) \qquad [\text{cf. } 0.9992(5)(9)(0) \text{ PDG}]$$

- Use of MILC2 lattices estimated to halve error [Bernard]
- If so, by end of project could have

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9977(5)(6)(0) = 0.9977(8)$$

- DWF2 calculations of $K \to \pi\ell\nu$ form factor should reduce error in unitarity sum to similar level

What can be achieved?

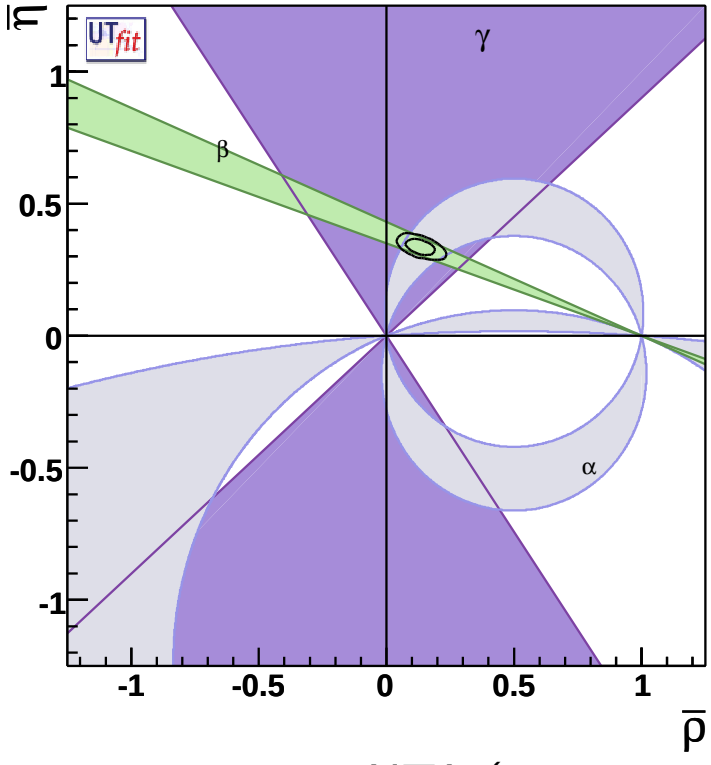
Example 2: Calculations of weak matrix elements using both DW and staggered/NRQCD fermions will allow precision tests of SM

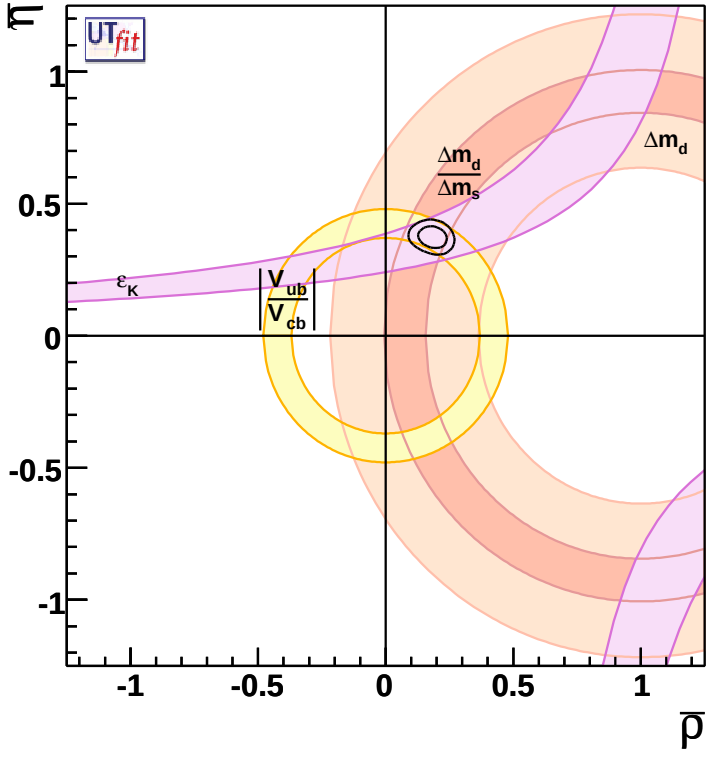
Hadronic Matrix Element	Lattice Estimate Current	UTA Result Current	Lattice Errors 10. TF-Yr **2007**	Lattice Errors 50. TF-Yr **2009**
$\widehat{B}_K$	0.77 ± 0.08	0.75 ± 0.09	± 0.05	± 0.03
$f_{B_s}\sqrt{\widehat{B}_{B_s}}$	282 ± 21 MeV	261 ± 6 MeV	± 16 MeV	± 9 MeV
ξ	1.23 ± 0.06	1.24 ± 0.08	± 0.04	± 0.02

- Comparison of UTA (experiment) and lattice tests SM
- **2007:** all lattice errors controlled
- **2009:** all lattice errors at or below experimental errors

Precision tests of SM (cont.)

Present constraints on $\bar{\rho}, \bar{\eta}$ [UTfit]





UTA (experiment) vs lattice-based results

Impressive consistency–will be solidified and made much more precise by this project

What can be achieved?

Example 3: Extend calculations to many additional quantities that test SM or constrain beyond the SM theories (complementary to LHC direct search)

- Precision results during present project testing SM
  - $(\Delta\Gamma/\Gamma)_{B_s}$, $B \to (K^*, \rho, \omega)\gamma$ and $B \to K\ell^+\ell^-$ amplitudes
  - Moving NRQCD will allow moving B and improve $B \to \pi\ell\nu$
- Precision results focused on constraining BSM theories
  - Non-SM operators giving $B - \bar{B}$, $D - \bar{D}$ and $K - \bar{K}$ mixing
  - Proton decay matrix elements
- First controlled results during project, but require more resources for accurate calculation
  - $K \to \pi\pi$ amplitudes
  - Neutron EDM, hadronic contribution to $(g-2)_\mu$
  - Nucleon matrix elements relavent for dark matter searches, e.g. $\langle N|\bar{s}s|N\rangle$

Calculations will need both DW and staggered/NRQCD fermions

Summary and Outlook

- Entered the era of precision LQCD calculations with all errors controlled
- Staggered ensemble has allowed validation and predictions for simpler quantities
- Approaching a milestone in 2007: first fully controlled results for four-fermion matrix elements constraining CKM matrix
- By end of project:
  - Fully controlled, precision results for growing list of more complicated quantities
  - Continuous improvement in stringency of tests of SM, and predictions for fundamental parameter
  - Complementary constraints on new physics searches at LHC
- Mix of staggered and DW essential for this progress