Lattice QCD for HEP: *Standard-Model parameters & matrix elements*

Ruth Van de Water
for the USQCD Collaboration

DOE Annual Progress Review of LQCD-Ext and LQCD-ARRA
May 9, 2013

Beyond-the-Standard-Model search strategies

- The experimental high-energy physics community is presently searching for new physics with two complimentary approaches

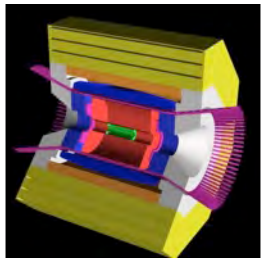
(1) Direct production of new particles at high-energy colliders

- *E.g.*, the LHC has already discovered a ~125 GeV particle that may be the SM Higgs

(2) Precise measurements of Standard-Model parameters and processes

- *E.g.*, the quark-flavor factories dramatically improved determinations of CKM matrix elements & CP-violating phase, and measured decay rates for rare processes
- Upcoming ultrasensitive experiments will improve existing measurements and observe some rare processes for the first time
- Compare measurements to Standard Model predictions and look for inconsistencies

BABAR



Belle II

LHCb

Beyond-the-Standard-Model search strategies

- The experimental high-energy physics community is presently searching for new physics with two complimentary approaches

(1) Direct production of new particles at high-energy colliders

- *E.g.*, the LHC has already discovered a ~125 GeV particle that may be the SM Higgs

(2) Precise measurements of Standard-Model parameters and processes

- *E.g.*, the quark-flavor factories dramatically improved determinations of CKM matrix elements & CP-violating phase, and m

- Upcoming ul existing measu processes for the first time

- Compare measurements to Standard Model predictions and look for inconsistencies

Lattice-QCD calculations are needed to interpret many of their results . . .

Scope of ultrasensitive experiments

- Current and planned experiments cover a broad range of topics in particle and nuclear physics

lepton flavor violation

Mu2e

neutrino physics

NOvA LBNE

Higgs physics

ilc

Muon Accelerator Program

FCC TLEP

B & D physics

Belle II

LHCb

BES-III

muon g-2

kaon physics

KOTO

NA62

Scope of ultrasensitive experiments

- Current and planned experiments cover a broad range of topics in particle and nuclear physics

lepton flavor violation

neutrino physics

Higgs physics

Precise lattice-QCD calculations are crucial to maximize the scientific impact of the current and future experimental high-energy physics program

muon g-2

kaon physics

USQCD scientific objectives

- **USQCD aims to support the US HEP experimental intensity-physics program** by "improv[ing] the accuracy of QCD calculations to the point where they no longer limit what can be learned from high precision experiments that seek to test the Standard Model" — *USQCD HEP SciDAC-3 proposal*

- 2013 White Paper "Lattice QCD at the Intensity Frontier" outlines a 5-year program of calculations matched to experimental priorities developed with input from experimentalists and phenomenologists

  (1) "Improve the calculation of the matrix elements needed for the CKM unitarity fit" (e.g. B-meson decay constants, mixing parameters, & form factors)

  (2) "Calculate … new, more computationally demanding, matrix elements that are needed for the interpretation of planned (and in some cases old) experiments" (e.g. ϵ'_K/ϵ_K, muon g-2, & nucleon axial form factor, …)

- Computational strategy to **support two streams of ensemble generation with different lattice fermion actions that have different advantages**, with several collaborations working on independent physics analyses

USQCD scientific objectives

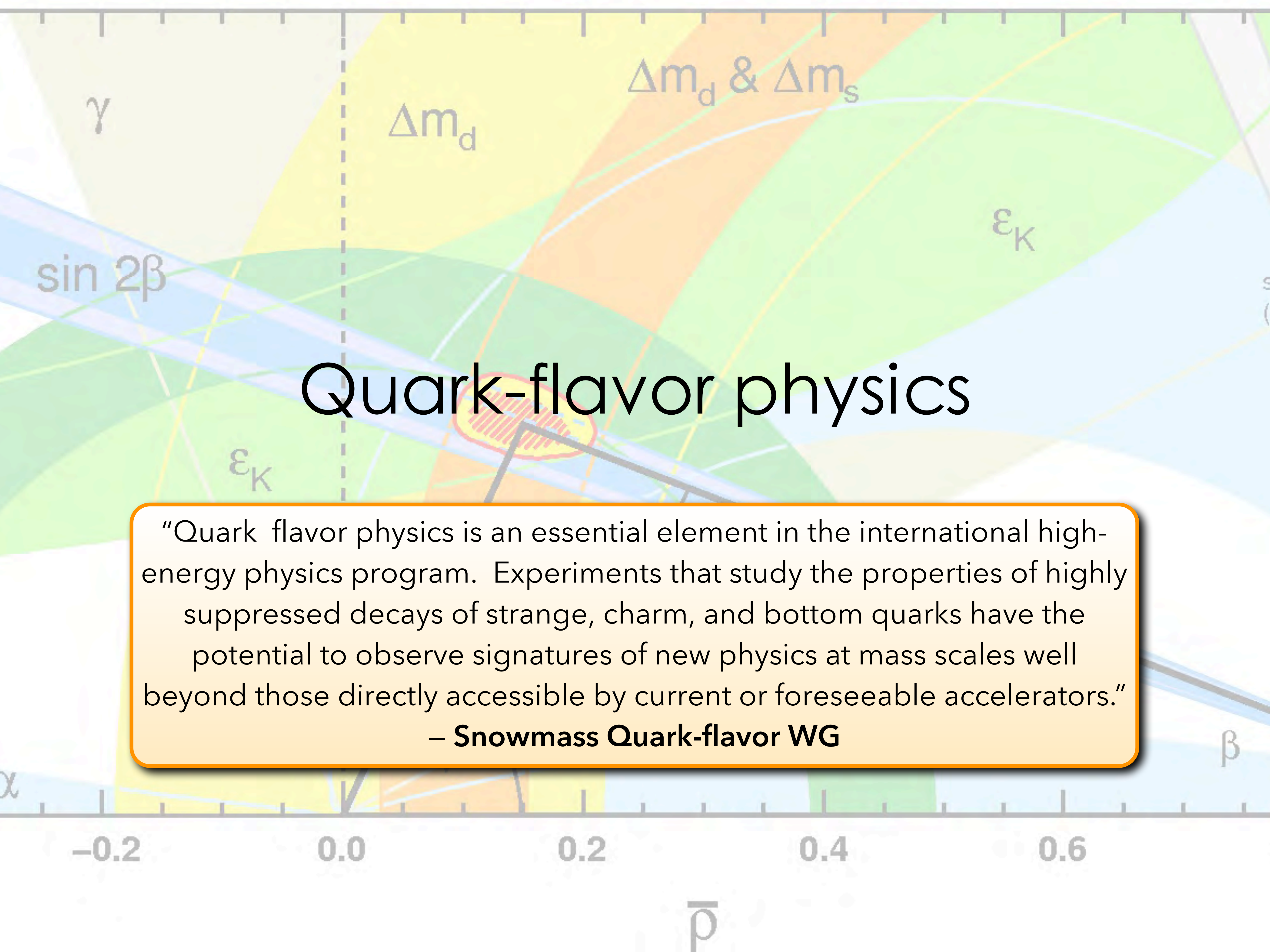
- **USQCD aims to support the US HEP experimental intensity-physics program** by "improv[ing] the accuracy of QCD calculations to the point where they no longer limit what can be learned from high precision experiments that seek to test the Standard Model" — *USQCD HEP SciDAC-3 proposal*

> "The USQCD effort on determining QCD-based quantities needed for measuring ... CKM parameters for precision tests of the Standard Model ... has produced many of the best results available today. The interaction between the lattice community and the experimental community has been crucial ..."
> – 2010 hardware review panel

(e.g. B-meson decay constants, mixing parameters, & form factors)

(2) "Calculate ... new, more computationally demanding, matrix elements that are needed for the interpretation of planned (and in some cases old) experiments" (e.g. ϵ'_K/ϵ_K, muon g-2, & nucleon axial form factor, ...)

- Computational strategy to **support two streams of ensemble generation with different lattice fermion actions that have different advantages**, with several collaborations working on independent physics analyses



Quark-flavor physics

"Quark flavor physics is an essential element in the international high-energy physics program. Experiments that study the properties of highly suppressed decays of strange, charm, and bottom quarks have the potential to observe signatures of new physics at mass scales well beyond those directly accessible by current or foreseeable accelerators."
– **Snowmass Quark-flavor WG**

Lattice-QCD constraints on the CKM matrix

- CKM matrix elements and phase are **fundamental parameters of the Standard Model that enter as parametric inputs to Standard Model predictions for many flavor-changing processes** such as neutral kaon mixing and K → $\pi\nu\nu$ decays

- Simple matrix elements involving single particles allow the determination of almost all CKM matrix elements

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \ell\nu \\ & K \to \pi\ell\nu & B \to \pi\ell\nu \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d|\bar{B}_d\rangle & \langle B_s|\bar{B}_s\rangle & \end{pmatrix}$$

*Neutral kaon mixing (B_K) also gold-plated and can be used to obtain the CKM phase (ρ, η)

- **USQCD leading the world in quark-flavor physics:** single most precise calculation for all of quantities listed by USQCD *(except for B_K, where we are still closely competitive)*

K→πℓν form factor at the physical pion mass

- K→πℓν form factor can be combined with experimentally-measured branching fraction to obtain $|V_{us}|$ in the Standard Model via:

$$\Gamma(K \to \pi \ell \nu) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{\rm EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{K\ell} \left(1 + \delta_{\rm EM}^{K\ell} + \delta_{SU(2)}^{K\pi}\right)^2$$

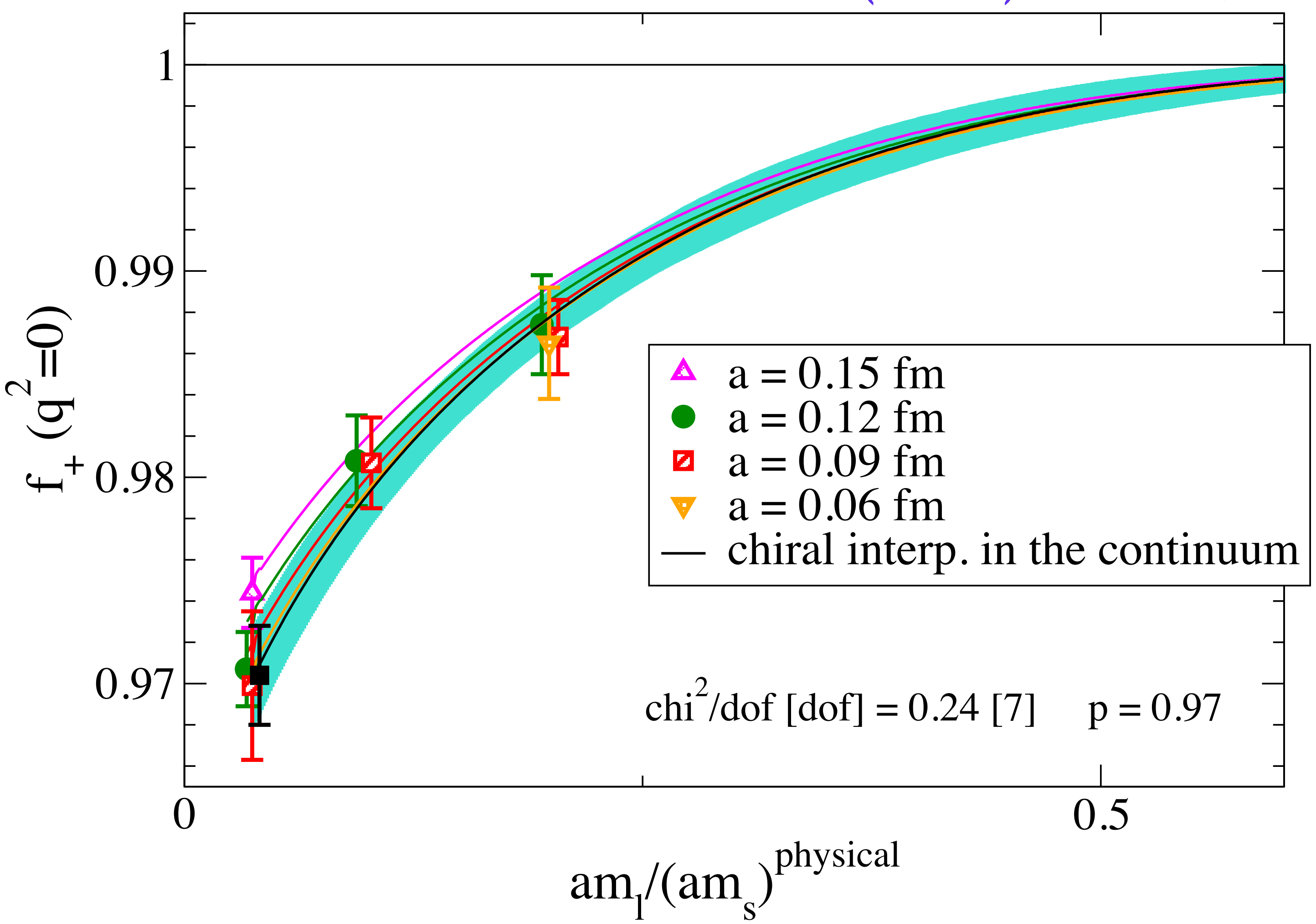
- Fermilab Lattice and MILC recently obtained the first result for the $f_+^{K\pi}(q^2=0)$ at the physical pion mass, removing previously dominant uncertainty from chiral extrapolation

- Single most precise result for $f_+(0)$ enables 0.4% determination of $|V_{us}|$

$f_+^{K\pi}(0) = 0.9704(24)_{\rm stat}(22)_{\rm sys}$

$|V_{us}| = 0.22290(74)_{\rm theo}(52)_{\rm exp}$

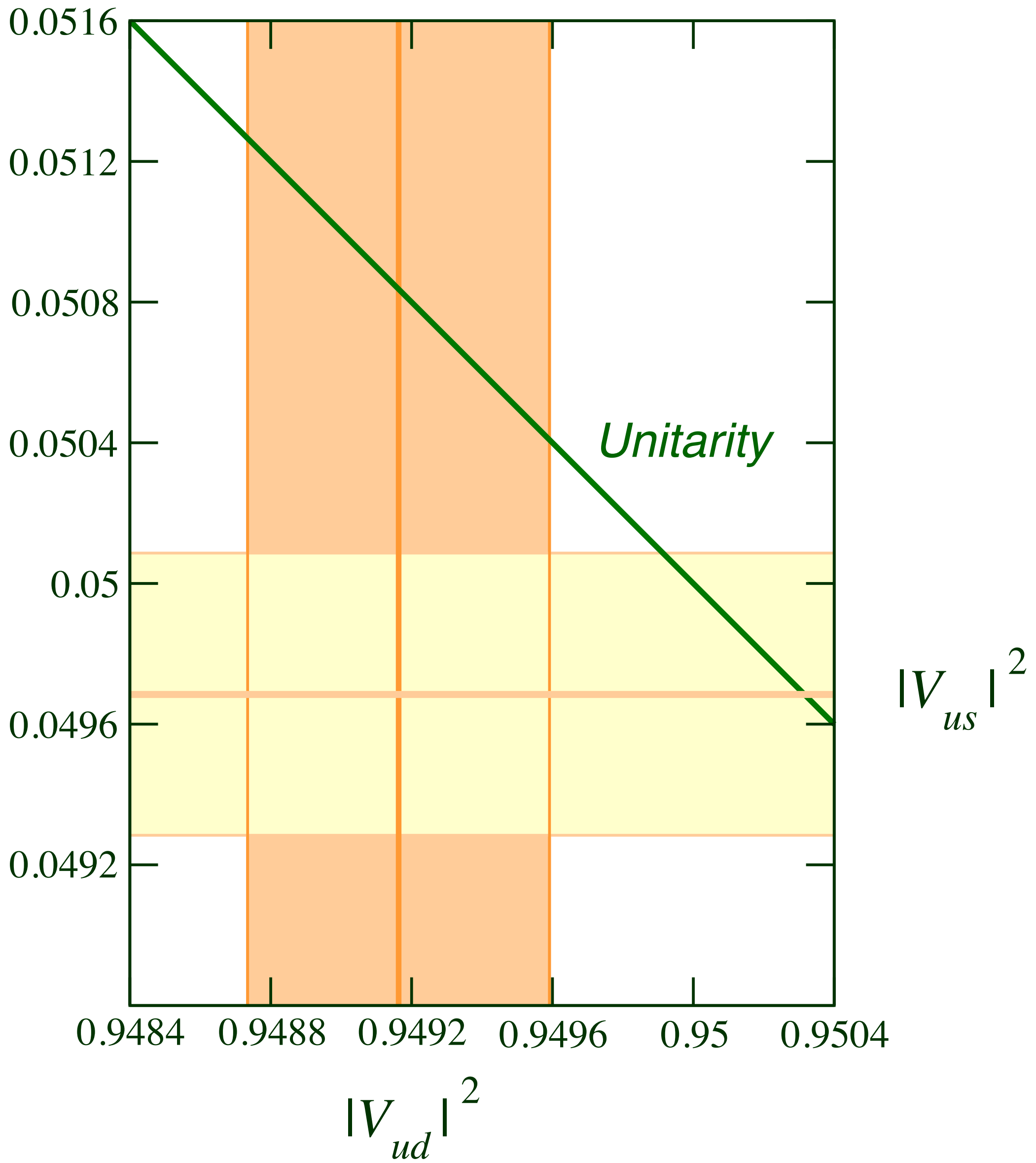
[Bazavov *et al.* PRL 112 (2014) 112001]



2013 highlight:
K→πℓν form factor at the physical pion mass

In test of first-row unitarity, error from $|V_{us}|$ now smaller than that from $|V_{ud}|$:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00115(40)_{V_{us}}(43)_{V_{ud}}$$



➜ Worth revisiting error on $|V_{ud}|$ from nuclear β decays?

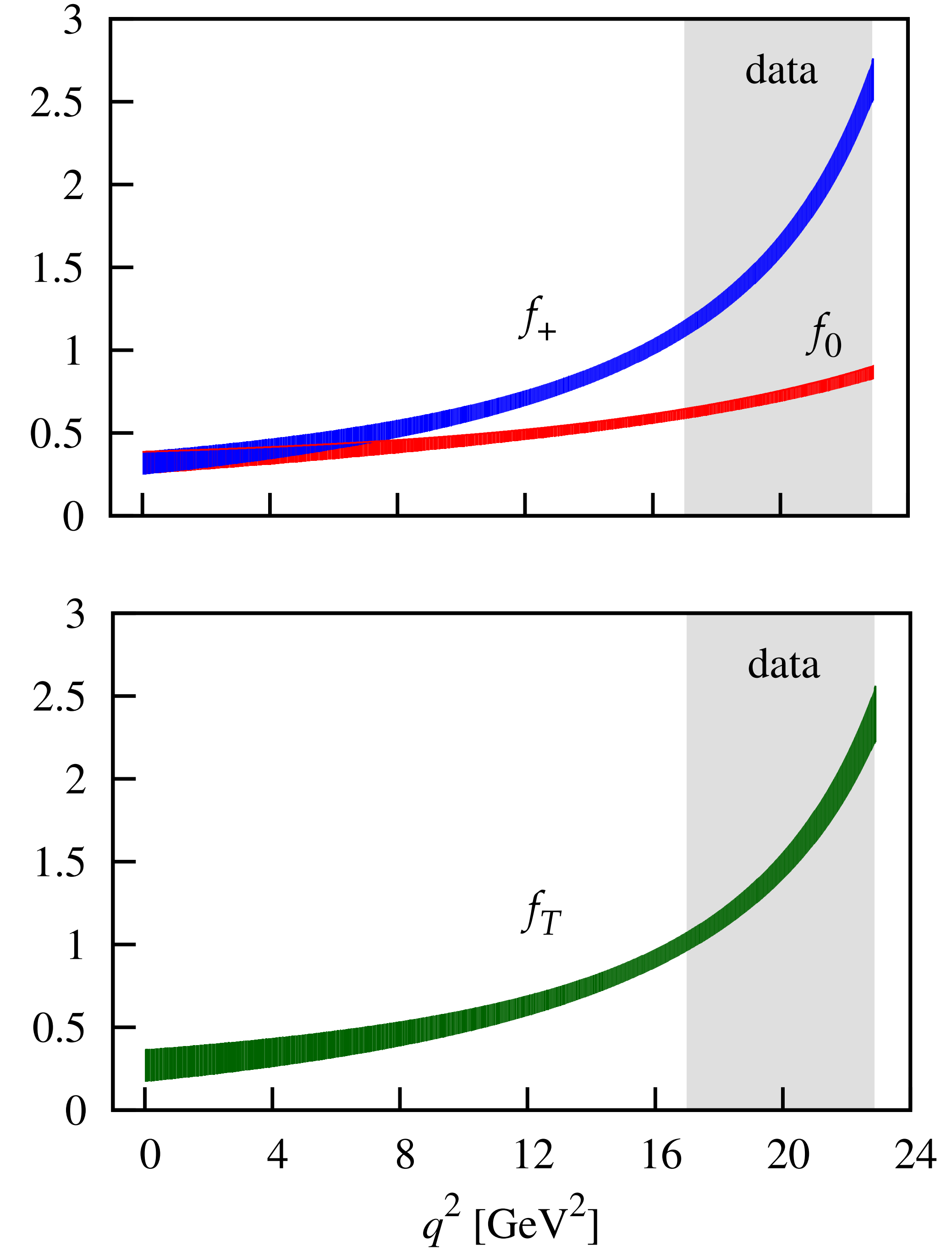
2013 Highlight:
First unquenched B→K$\ell^+\ell^-$ form factors

[Bouchard *et al.* PRL111 (2013) 162002, PRD88 (2013) 054509]

- Rare decay B→K$\ell^+\ell^-$ can proceed only through loop diagrams in the Standard Model, making it a particularly sensitive probe for new physics

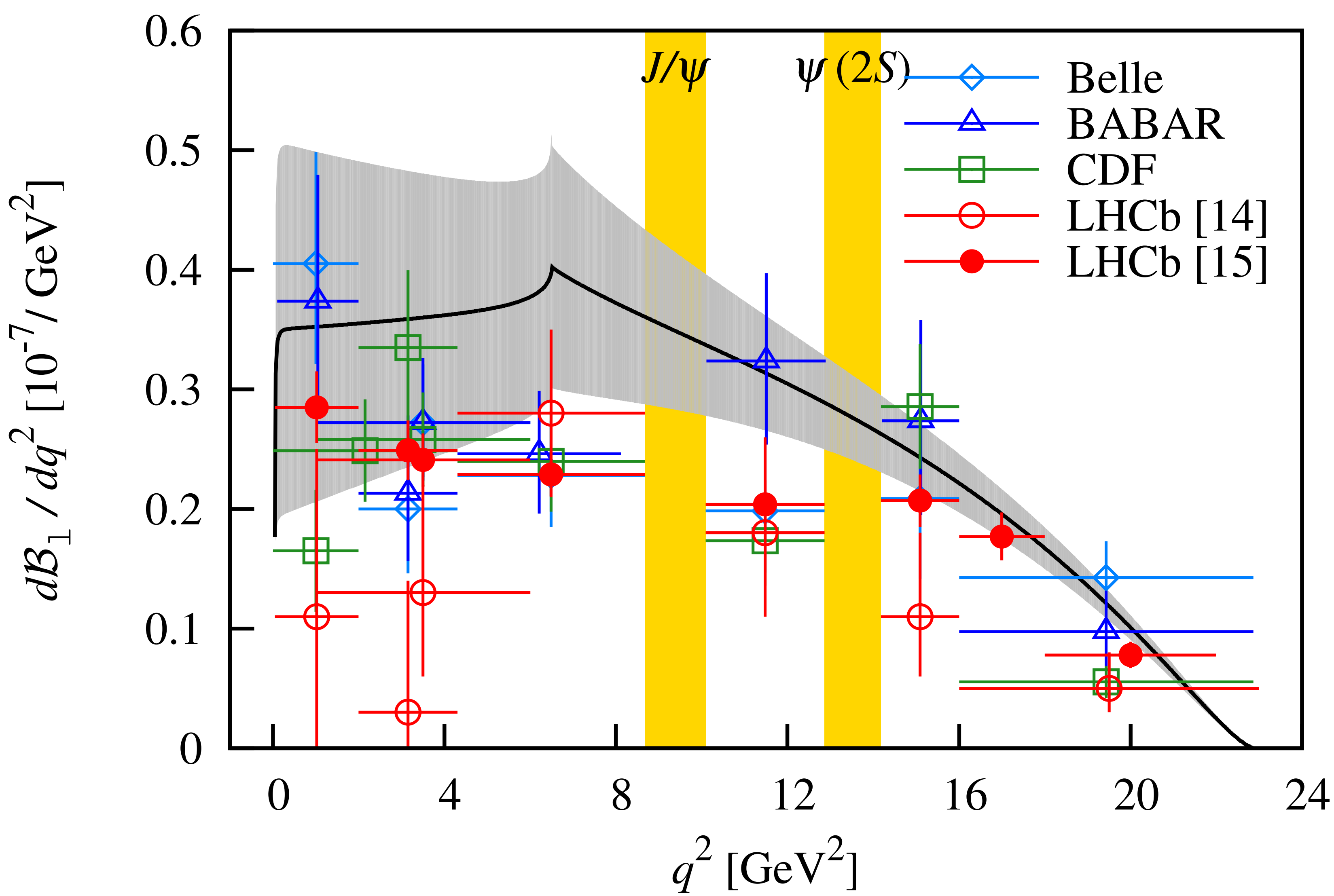
- Accurate Standard-Model predictions are important and timely as experimental measurements becoming more precise, and require parameterization of hadronic form factors over full q^2 range

- HPQCD Collaboration recently obtained the **first (2+1)-flavor result for the three form factors $f_+(q^2)$, $f_0(q^2)$, and $f_T(q^2)$, which are sufficient to parameterize B→Kl$^+$l$^-$ both in the Standard Model and in all possible beyond-the-SM theories**



2013 Highlight:
First unquenched $B \to K \ell^+ \ell^-$ form factors

For $q^2 > 10$ GeV^2, results more precise than previous Standard-Model predictions, and for all q^2, results consistent with previous calculations and experiment.



(But both experimental and theory uncertainties will continue to improve…)

Coming soon:
B→Dℓν form factor at nonzero recoil

- B → Dℓν semileptonic form factor allows determination of $|V_{cb}|$ via:

$$\frac{d\Gamma(B \to D l \nu)}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B + m_D)^2 (w^2 - 1)^{3/2} |\mathbf{V_{cb}}|^2 |\mathcal{G}_{\mathbf{B\to D}}(\mathbf{w})|^2$$

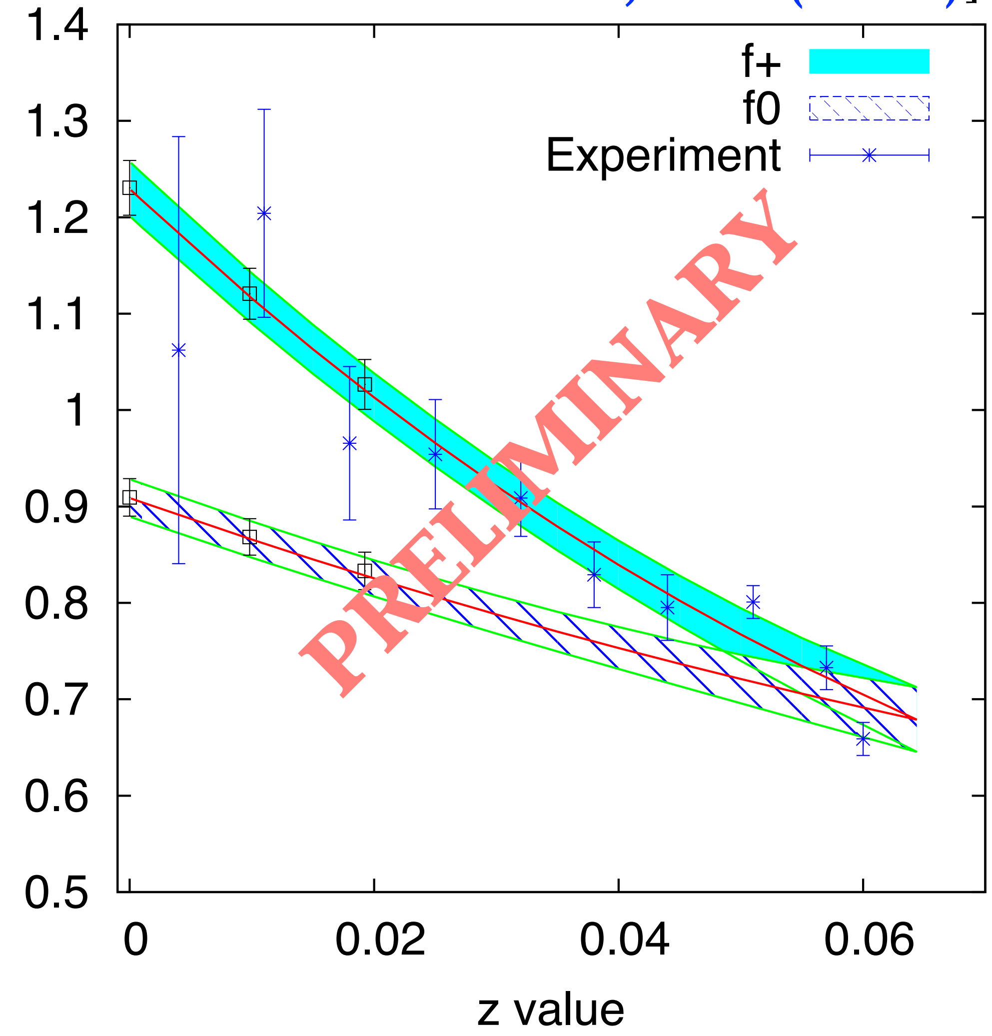
$\}$ $w \equiv v_B \cdot v_D$

- Common practice comparing theory and experiment at zero recoil (w=1) leads to large experimental errors in $|V_{cb}|$ because decay rate kinematically suppressed at low recoil momentum

- Fermilab/MILC presented **first unquenched results for G(w) over full kinematic range at Lattice 2013**, and analysis is now almost finalized

- Following method now standard for B→π exclusive decays, **obtain $|V_{cb}|$ with reduced uncertainties from combined fit of lattice and experimental data to model independent "z-parameterization"** based on analyticity and unitarity [Boyd, Grinstein, Lebed, PRL74 (1995) 4603-4606]

[Qiu *et al.*, PoS LATTICE 2013, 385 (2013)]



In progress: $K \to \pi\pi$ decays

- Direct CP-violation in $K \to \pi\,\pi$ decays (ϵ'_K/ϵ_K) measured experimentally to <10% precision more than a decade ago [NA48, KTeV], but utility for testing Standard Model handicapped by large uncertainty in corresponding weak matrix elements

- RBC/UKQCD presently attacking $K \to \pi\pi$ amplitudes via "direct" Lellouch-Lüscher approach

  - Computed $\Delta I = 3/2$ matrix elements with physical pion and kaon masses, obtaining $\mathrm{Re}(A_2)$ & $\mathrm{Im}(A_2)$ with ~20% errors [PRL108 (2012) 141601], and now analyzing data at a second lattice spacing for a continuum limit [arXiv:1311.3844]

  - Performed successful pilot study of $\Delta I = 1/2$ matrix elements with ~330 MeV pions [PRD84 (2011) 114503; Q. Liu Ph.D. thesis (2012)], and now beginning large-scale calculation with physical pions and kaons

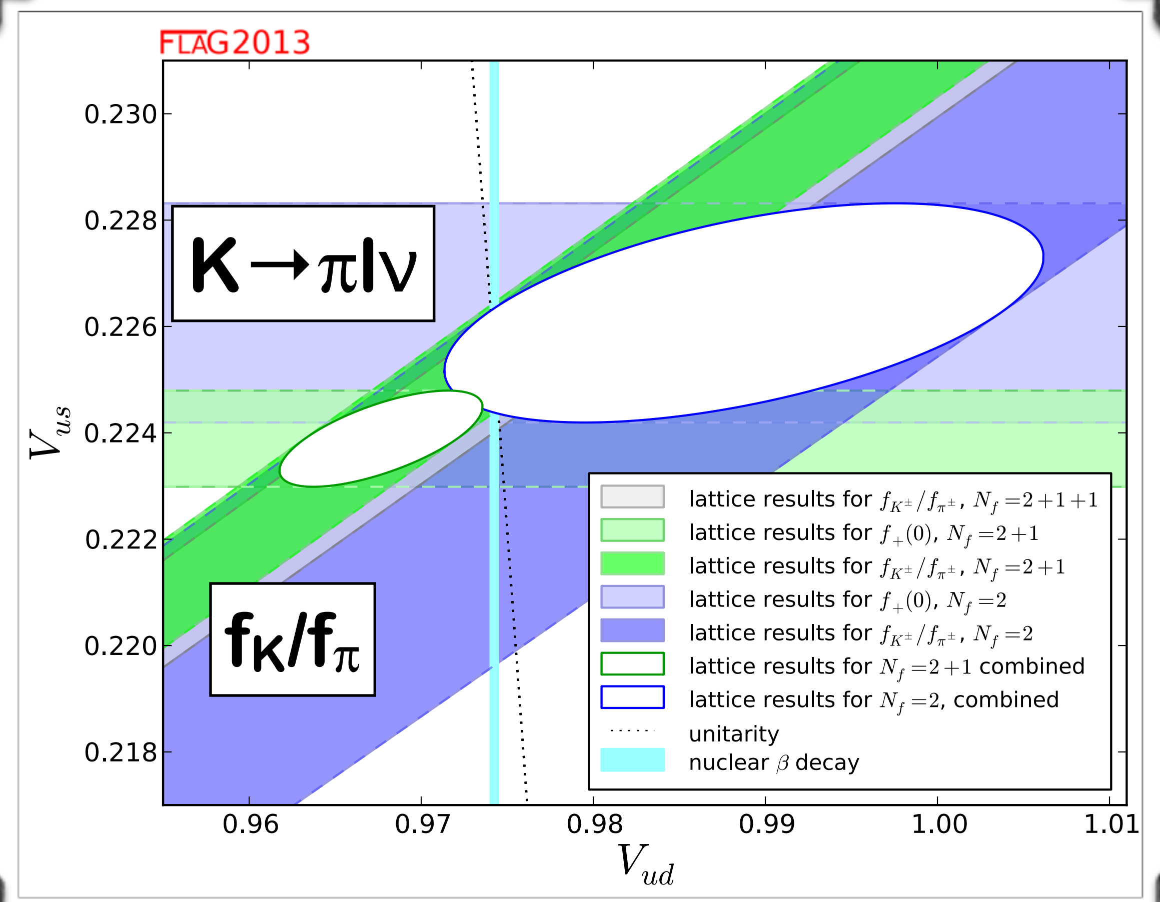
  - **Should yield first ab initio QCD calculation of ΔI=1/2 rule and calculation of $\varepsilon'_K/\varepsilon_K$ with ~20-30% precision in the next one or two years**

- Methods for long-distance matrix elements needed for rare kaon decays also being studied, with approach worked out for simplest quantities $\Delta(M_K)$ and ϵ_K in [Christ, PoS (Lattice2010) 300, LATTICE2011 277], but too soon to predict time needed to obtain controlled results

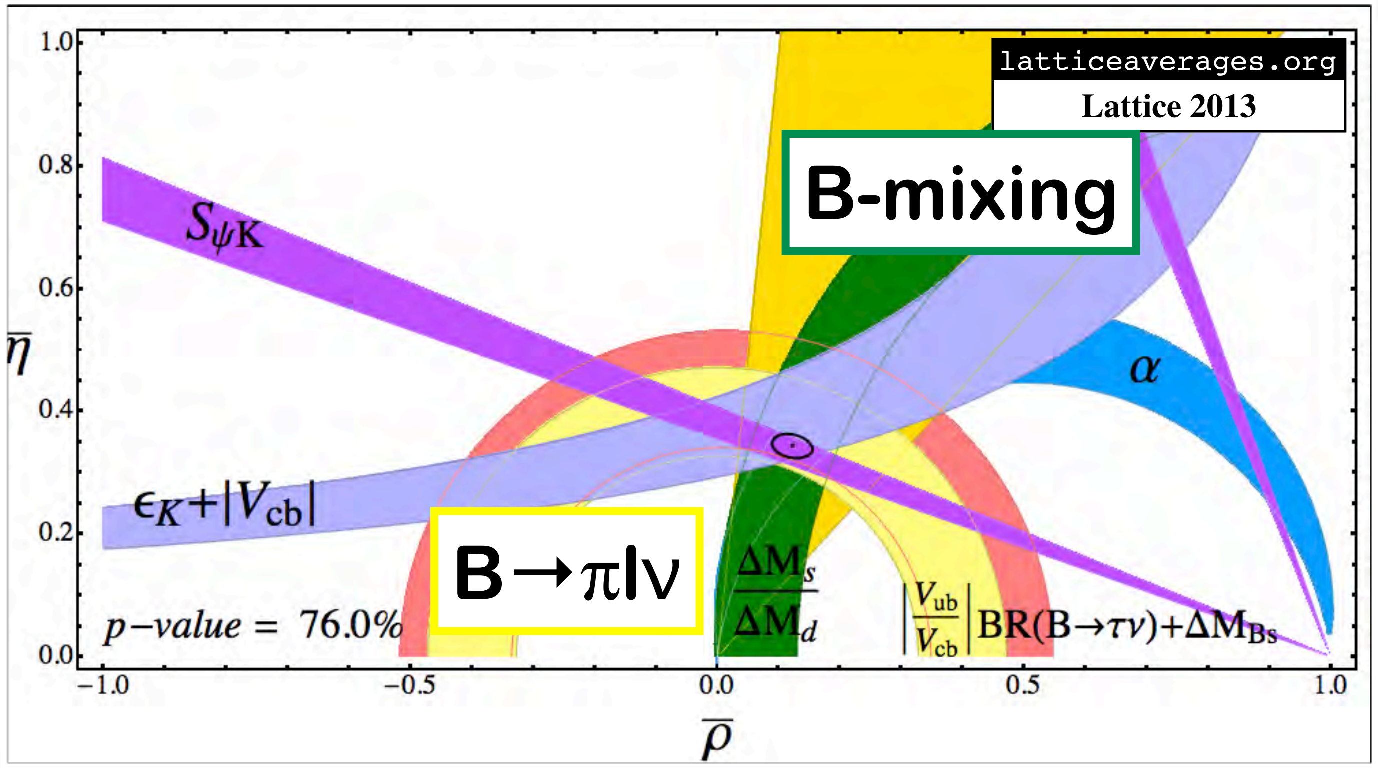
Room for improvement

- For many quark-flavor changing processes, lattice errors still larger than those from experiment (in particular semileptonic form factors and neutral meson mixing parameters)
- Must continue to improve precision on "standard" lattice matrix elements to squeeze the vise on the Standard-Model CKM framework with existing quark-flavor data

First-row unitarity

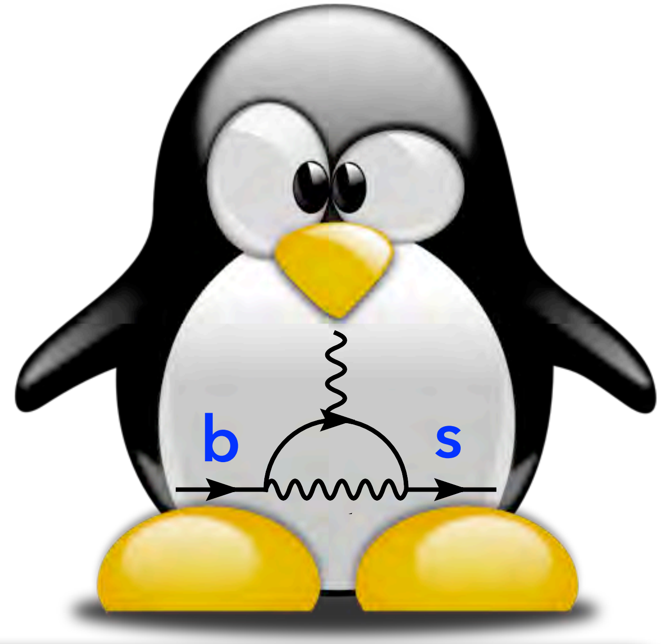


Global CKM UT Fit



Open challenges

- **$B \to K^*$ and related form factors**
  - $B \to K^* ll$, $B \to K^* \gamma$, and $B_s \to \varphi\gamma$ have been observed experimentally and rate measurements will continue to improve; comparisons with SM predictions require form factors over full kinematic range
  - Lattice calculations "very challenging" because final-state K^* and φ are unstable in QCD; moreover, their widths increase as the light-quark masses approach the physical point
  - Initial step recently taken by Prelovsek *et al.*, who completed first lattice study of the $K^*(872)$ decay width [Phys.Rev. D88 (2013) 054508]



FCNCs mediated by b→s penguins potentially sensitive to new physics

- **D-meson matrix elements**
  - Important in light of recent experimental evidence for CP-violation in $D \to \pi\pi(KK)$ decays and mixing ➜ *now in the same situation as we've been in for decades with ε'!*
  - Particularly difficult aspect is dealing with intermediate 4π, 6π, etc., states in finite box
  - Progress with generalization of Lüscher formalism to 3π case [Polejaeva & Rusetsky, Briceno & Davoudi, Hansen & Sharpe], but more ideas and hard work are needed

New opportunities

P5 identified scientific drivers for HEP

- Use the Higgs as a new tool for discovery
- Explore the physics associated with neutrino mass ...
- Search for new particles and interactions; new physical principles

"Each has the potential to be transformative. Expect surprises."

– P5 preliminary comments, March HEPAP meeting

Likely experimental horizon

(Not comprehensive;
later experiments more "off shell")

NOW

ATLAS/CMS
Δm_s, $B_s \to \mu^+\mu^-$, …

E14 "KOTO" @ J-PARC
$K^0 \to \pi^0 \nu\bar{\nu}$

NA62 @ CERN_SPS
$K^+ \to \pi^+ \nu\bar{\nu}$

LHCb
rare $b \to s\gamma$ & $b \to sll$ decays,
$B_s \to \mu^+\mu^-$, D-mixing…

2016

Belle II
$\sin(2\beta)$, $B \to \tau(\mu)\nu$,
$B \to \pi(\rho) l\nu$, $B \to D^{(*)} l\nu$,
rare $b \to s\gamma$ & $b \to sll$ decays, …

Muon g-2

2019

Mu2e

2020s

LBNE
neutrino mixing &
mass hierarchy,
proton decay, …

2030s

next high-energy collider
Higgs decay modes

Likely experimental horizon

(Not comprehensive; later experiments more "off shell")

ATLAS/CMS
Δm_s, $B_s \to \mu^+\mu^-$, …

E14 "KOTO" @ J-PARC
$K^0 \to \pi^0 \nu\bar{\nu}$

Belle II
$\sin(2\beta)$, $B \to \tau(\mu)\nu$,
$B \to \pi(\rho) l\nu$, $B \to D^{(*)} l\nu$,
rare $b \to s\gamma$ & $b \to sll$ decays, …

LBNE
neutrino mixing & mass hierarchy, …n decay, …

NOW — 2020s — 2030s

Lattice-QCD calculations needed for ALL of these experiments -- here focus on new applications to charged leptons, neutrinos, and precision Higgs physics

NA62 @ CERN SPS
$K^+ \to \pi^+ \nu\bar{\nu}$

LHCb
rare $b \to s\gamma$ & $b \to sll$ decays,
$B_s \to \mu^+\mu^-$, D-mixing…

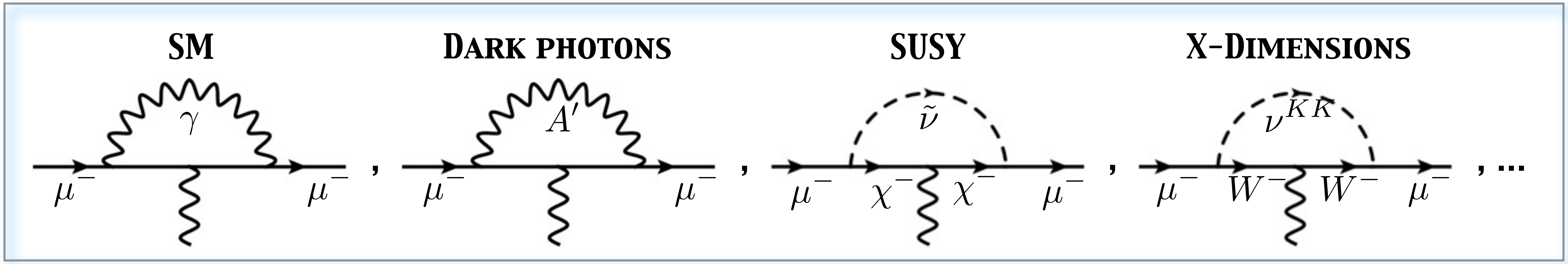
Muon g-2

Mu2e

next high-energy collider
Higgs decay modes

Muon anomalous magnetic moment

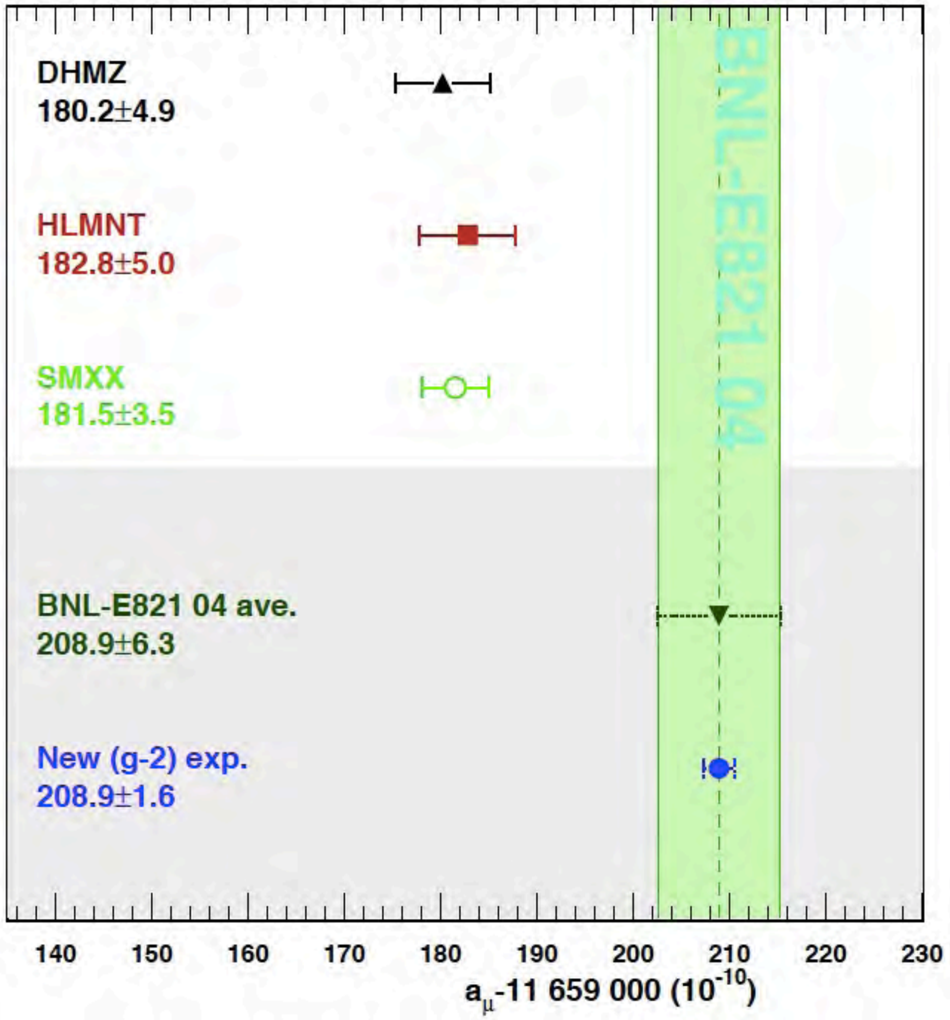
- **Muon g-2 provides precise test of SM and constraints on its extensions**



- BNL measurement disagrees with SM by >3σ, and Fermilab Muon g-2 Experiment aims to reduce experimental error by factor of four

- To leverage improved experimental precision, theoretical uncertainty in SM prediction must be shored-up and brought to a comparable precision

- Lattice QCD can provide hadronic contributions to muon g-2 from first principles with controlled uncertainties that are systematically improvable

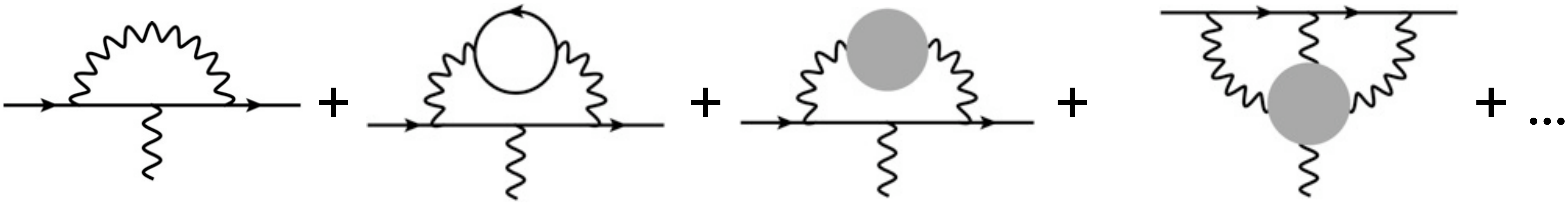


[Blum et al., arXiv:1311.2198]

Standard-Model contributions to g-2

Hadronic vacuum polarization (HVP):

Hadronic light-by-light (HLbL):



QED (4 loops) & EW (2 loops)

from experimental result for $e^+e^- \rightarrow$ hadrons plus dispersion relation

estimated from models such as large N_c, χPT, vector meson dominance, etc...

Contribution	Result ($\times 10^{11}$)		Error
QED (leptons)	116 584 718 $\pm$ 0.14	$\pm$ 0.04_α	0.00 ppm
HVP(lo) [1]	6 923 $\pm$ 42		0.36 ppm
HVP(ho)	-98 $\pm$ 0.9_{exp}	$\pm$ 0.3_{rad}	0.01 ppm
HLbL [2]	105 $\pm$ 26		0.22 ppm
EW	154 $\pm$ 2	$\pm$ 1	0.02 ppm
Total SM	116 591 802 $\pm$ 49		0.42 ppm

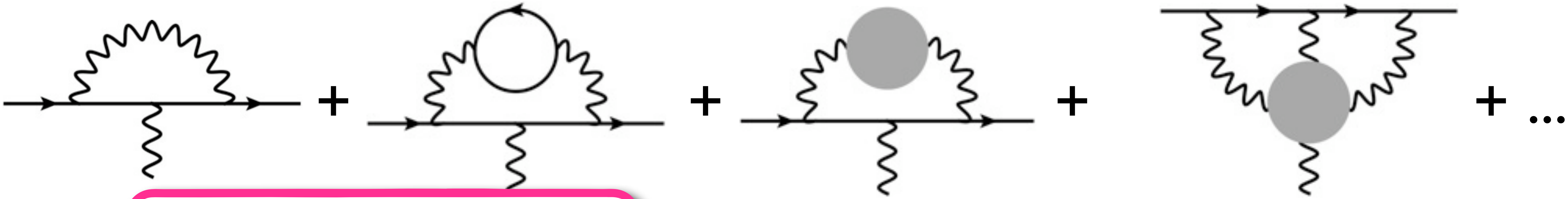
[1] Davier, Hoecker, Malaescu, Zhang, Eur.Phys.J. C71 (2011) 1515

[2] Prades, de Rafael, Vainshtein, arXiv:0901.030

Standard-Model contributions to g-2

Hadronic vacuum polarization (HVP):

Hadronic light-by-light (HLbL):



QE

HLbL error more subjective than for other contributions, and somewhat controversial

from experimental result for $e^+e^- \to$ hadrons plus dispersion relation

estimated from models such as large N_c, χPT, vector meson dominance, etc...

Contribution	Result ($\times 10^{11}$)		Error
QED (leptons)	116 584 718 ± 0.14	± 0.04_α	0.00 ppm
HVP(lo) [1]	6 923 ± 42		0.36 ppm
HVP(ho)	-98 ± 0.9_{exp}	± 0.3_{rad}	0.01 ppm
HLbL [2]	105 ± 26		0.22 ppm
EW	154 ± 2	± 1	0.02 ppm
Total SM	116 591 802 ± 49		0.42 ppm

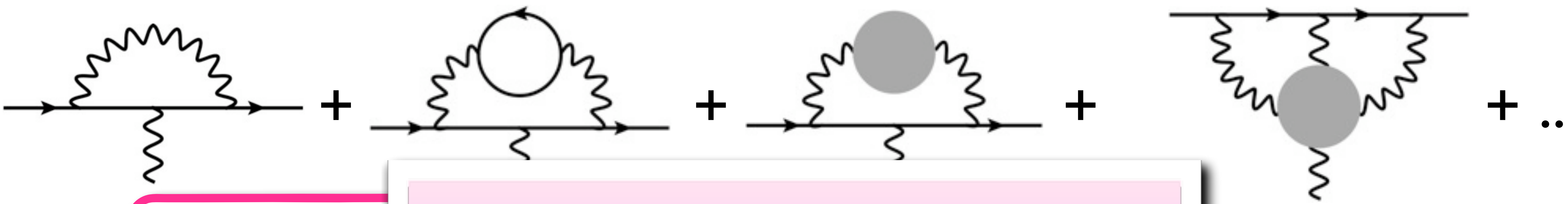
[1] Davier, Hoecker, Malaescu, Zhang, Eur.Phys.J. C71 (2011) 1515

[2] Prades, de Rafael, Vainshtein, arXiv:0901.030

Standard-Model contributions to g-2

Hadronic vacuum polarization (HVP):

Hadronic light-by-light (HLbL):



QE

HLbL error more
for other cont
somewhat c

Precision goals for hadronic contributions set by Muon g-2 Experiment are:

$\delta(a_\mu^{HVP}) \sim 0.2\%$, $\delta(a_\mu^{HLbL}) \sim 15\%$

nated from models
n as large N_c, χPT,
vector meson
minance, etc…

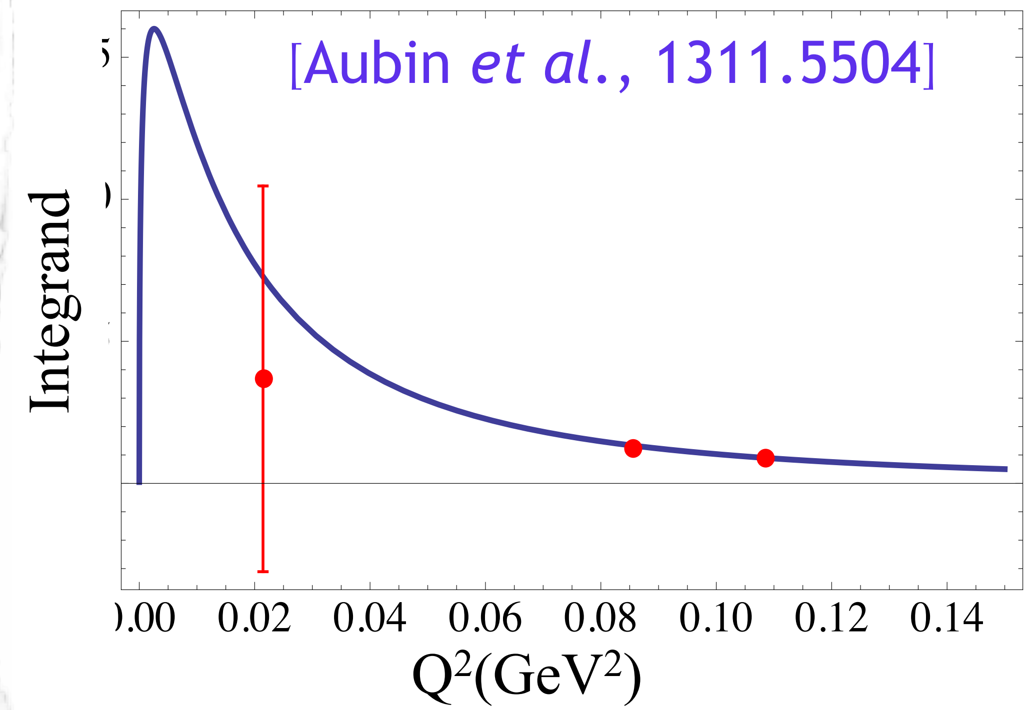
Contribution	Result ($\times 10^{11}$)		Error
QED (leptons)	116 584 718 $\pm$ 0.14	$\pm$ 0.04_α	0.00 ppm
HVP(lo) [1]	6 923 $\pm$ 42		0.36 ppm
HVP(ho)	-98 $\pm$ 0.9_{exp}	$\pm$ 0.3_{rad}	0.01 ppm
HLbL [2]	105 $\pm$ 26		0.22 ppm
EW	154 $\pm$ 2	$\pm$ 1	0.02 ppm
Total SM	116 591 802 $\pm$ 49		0.42 ppm

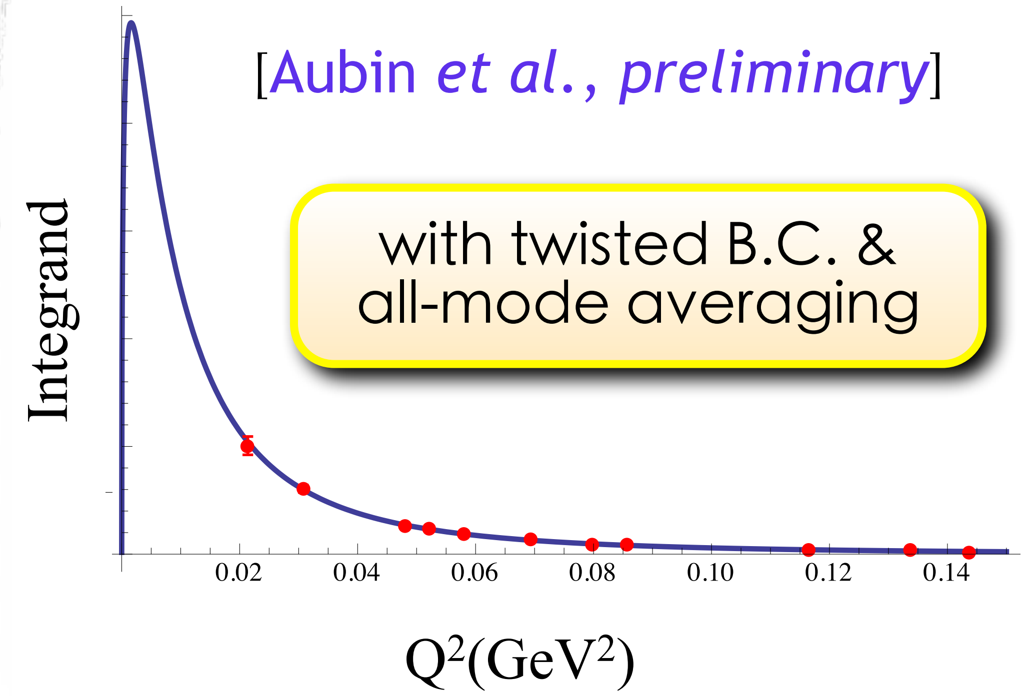
[1] Davier, Hoecker, Malaescu, Zhang, Eur.Phys.J. C71 (2011) 1515

[2] Prades, de Rafael, Vainshtein, arXiv:0901.030

Recent progress on a_μ^{HVP}

- **Three independent USQCD efforts**: each developing new theoretical/numerical methods critical to reaching target uncertainty on a_μ^{HVP}
- Twisted boundary conditions for fermion fields to access momentum values below the minimum discrete lattice momentum ($2\pi/L$) [spatial lattice volume=L^3] [Aubin *et al.*, PRD88 (2013) 7, 074505]
- All-mode averaging to reduce statistical errors [RBC/UKQCD, PRD 88 (2013) 094503]
- Model-independent fitting approach based on analytic structure of $\Pi(Q^2)$ to extrapolate lattice data to low-Q^2 region without hidden systematics [Aubin *et al.*, Phys.Rev. D86 (2012) 054509]
- Calculations with these improvements at the physical pion mass and including dynamical charm quarks are underway





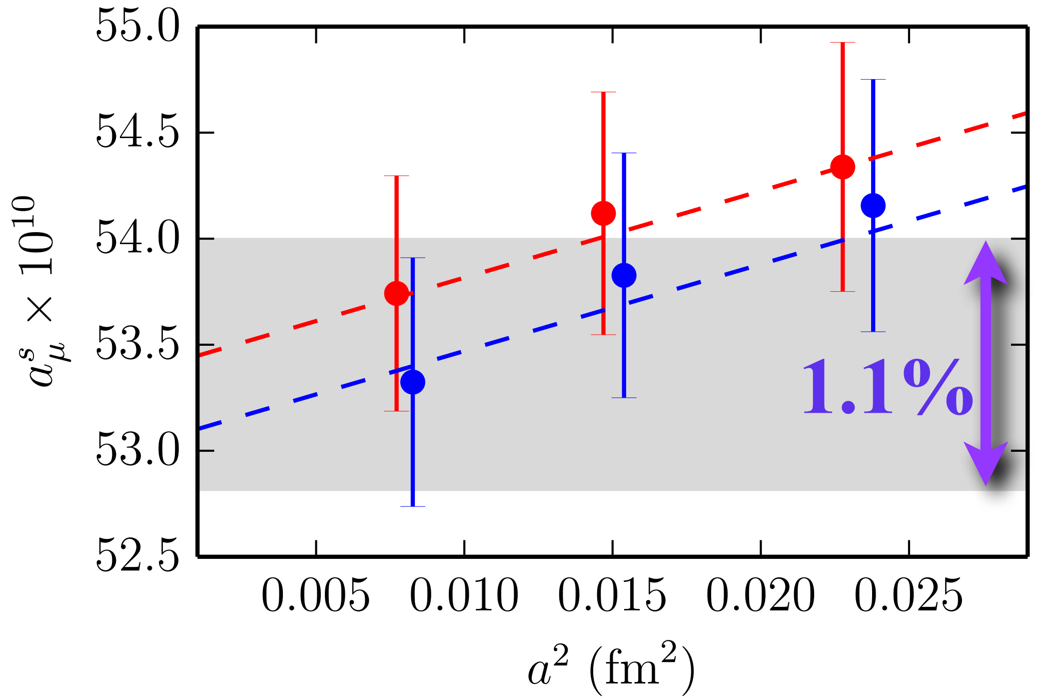
New method: a_μ^{HVP} from current correlators

[Chakraborty et al. (HPQCD), 1403.1778]

- Sidestep $q^2 \to 0$ extrapolation by expressing a_μ^{HVP} in terms of derivatives of vacuum polarization function $\Pi(q^2)$ at $q^2=0$
- Derivatives easily computed on lattice to high statistical precision from time-moments of the electromagnetic current-current correlator at $q^2=0$

$$G_{2n} \equiv a^4 \sum_t \sum_{\mathbf{x}} t^{2n} Z_V^2 \langle j^i(\mathbf{x},t) j^i(\mathbf{0},0)\rangle$$
$$= (-1)^n \frac{\partial^{2n}}{\partial q^{2n}} q^2 [\Pi(q^2) - \Pi(0)]\Big|_{q^2=0}$$

- Illustrate method with strange and charm-quark contributions and obtain a_μ^s to ~1%
- Correlator noisier for light quarks, but **estimate that similar precision can be obtained for $a_\mu^{u,d}$ with 10× larger gauge-field ensembles**
- Beyond ~1%, likely need to directly include EM and isospin-breaking in simulations



	a_μ^s	a_μ^c
lattice spacing (w_0, r_1):	1.0%	0.6%
Uncertainty in Z_V:	0.4%	2.5%
Monte Carlo statistics:	0.1%	0.1%
$a^2 \to 0$ extrapolation:	0.1%	0.4%
QED corrections:	0.1%	0.3%
Quark mass tuning:	0.0%	0.4%
Finite lattice volume:	$<0.1\%$	0.0%
Padé approximants:	$<0.1\%$	0.0%
Total:	1.1%	2.7%

New method: a_μ^{HVP} from current correlators

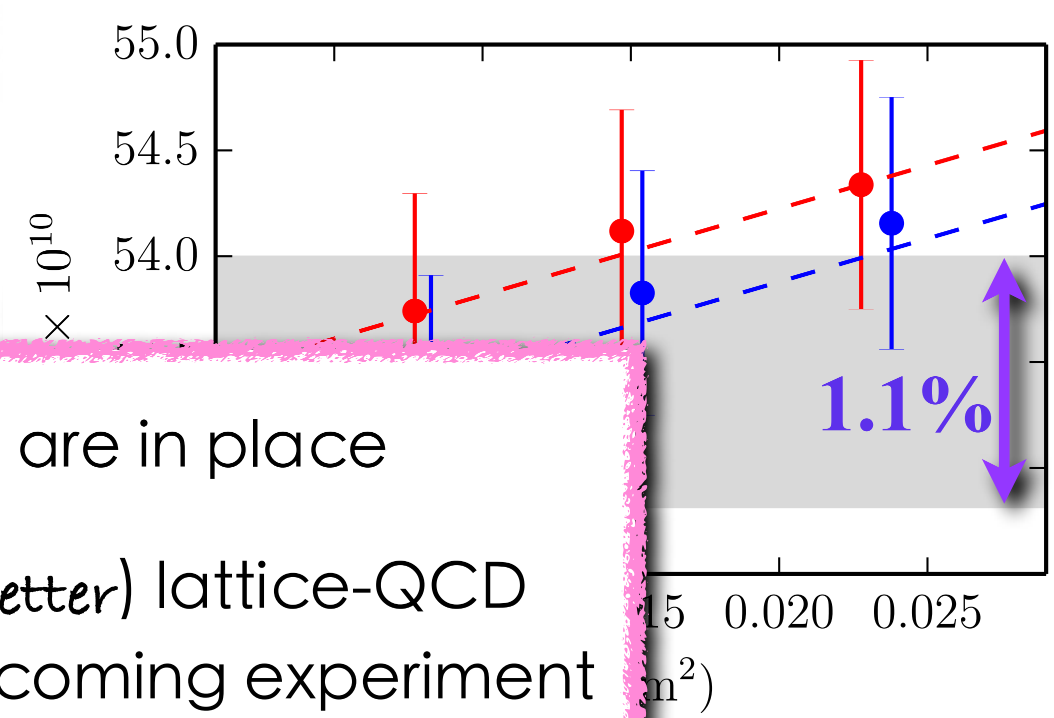
[Chakraborty et al. (HPQCD), 1403.1778]

- Sidestep $q^2 \to 0$ extrapolation by expressing a_μ^{HVP} in terms of derivatives of vacuum polarization function $\Pi(q^2)$ at $q^2=0$

- Derivatives easily computed on lattice to high statistical precision from time-moments of the electromagnetic current-current correlator at $q^2=0$

$G_{2n} \equiv$

$=$



Theoretical methods for a_μ^{HVP} are in place
➜
should enable a few percent (or better) lattice-QCD calculation on the timescale of the coming experiment

	a_μ^s	a_μ^c
lattice spacing (w_0, r_1):	1.0%	0.6%
Uncertainty in Z_V:	0.4%	2.5%
Monte Carlo statistics:	0.1%	0.1%
$a^2 \to 0$ extrapolation:	0.1%	0.4%
QED corrections:	0.1%	0.3%
Quark mass tuning:	0.0%	0.4%
Finite lattice volume:	< 0.1%	0.0%
Padé approximants:	< 0.1%	0.0%
Total:	1.1%	2.7%

- Illustrate method with strange and charm-quark contributions and obtain a_μ^s to ~1%

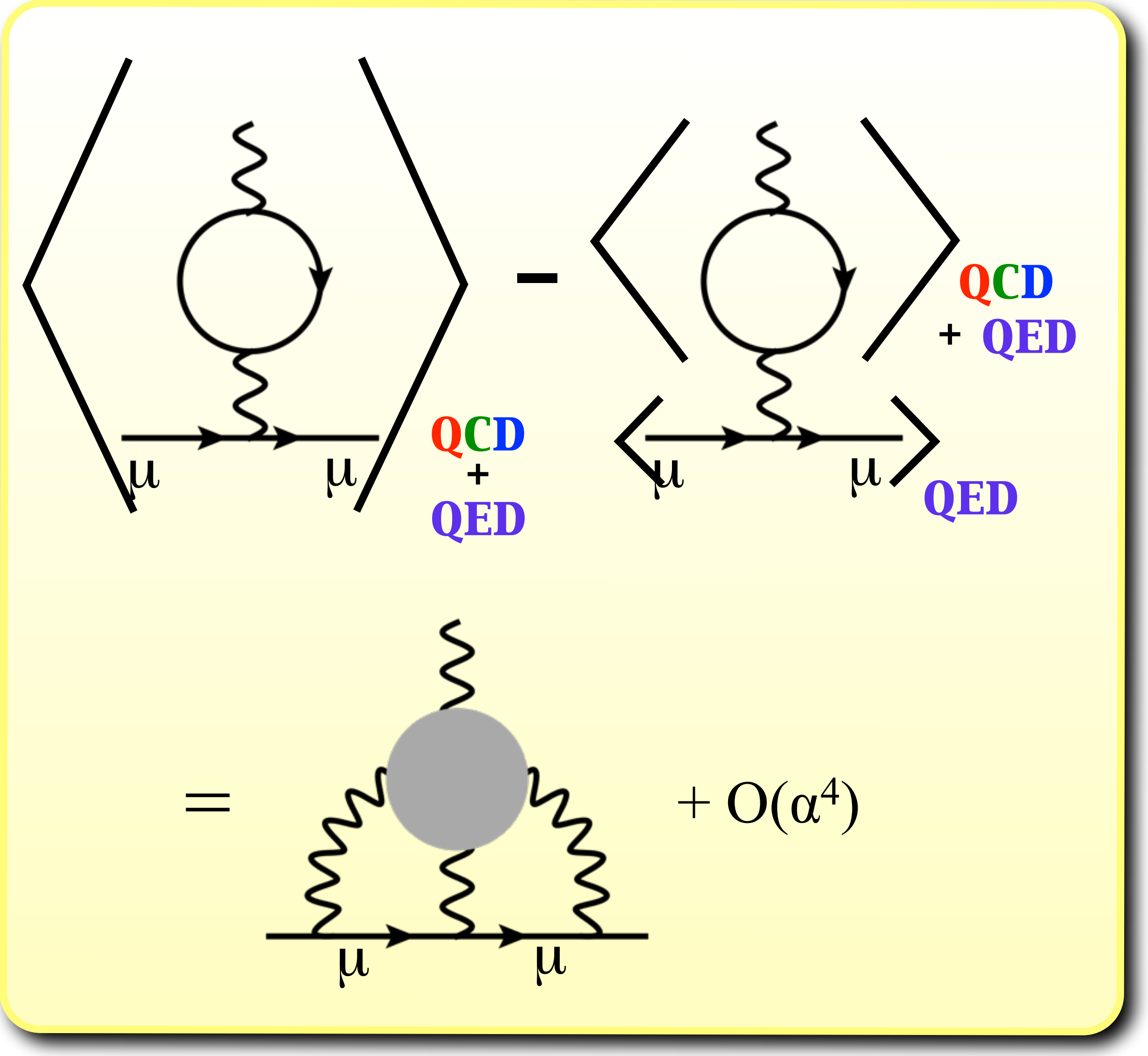
- Correlator noisier for light quarks, but **estimate that similar precision can be obtained for $a_\mu^{u,d}$ with 10× larger gauge-field ensembles**

- Beyond ~1%, likely need to directly include EM and isospin-breaking in simulations

RBC/UKQCD calculation of a_μ^{HLbL}

[Hayakawa et al., PoS LAT2005 (2006) 353]

- Analogous approach to HVP calculation inserting correlation function of 4 EM currents into 2-loop QED integral (prohibitively?) complicated and costly
- **Promising approach to compute full hadronic amplitude nonperturbatively**
  - Include photon field along with gluon field in gauge link, so simulation & analysis follow conventional lattice-QCD calculation
  - Using all-mode-averaging, Blum *et al.* obtain statistically-significant signal emerging in the ballpark of model estimates [PoS LATTICE2012 (2012) 022]
  - Still much to do for realistic result with controlled uncertainty: larger spatial volumes, extrapolation to physical pion mass and continuum, momentum extrapolation $Q^2 \to 0$, quark-disconnected contributions, ...



RBC/UKQCD calculation of a_μ^{HLbL}

[Hayakawa et al., PoS LAT2005 (2006) 353]

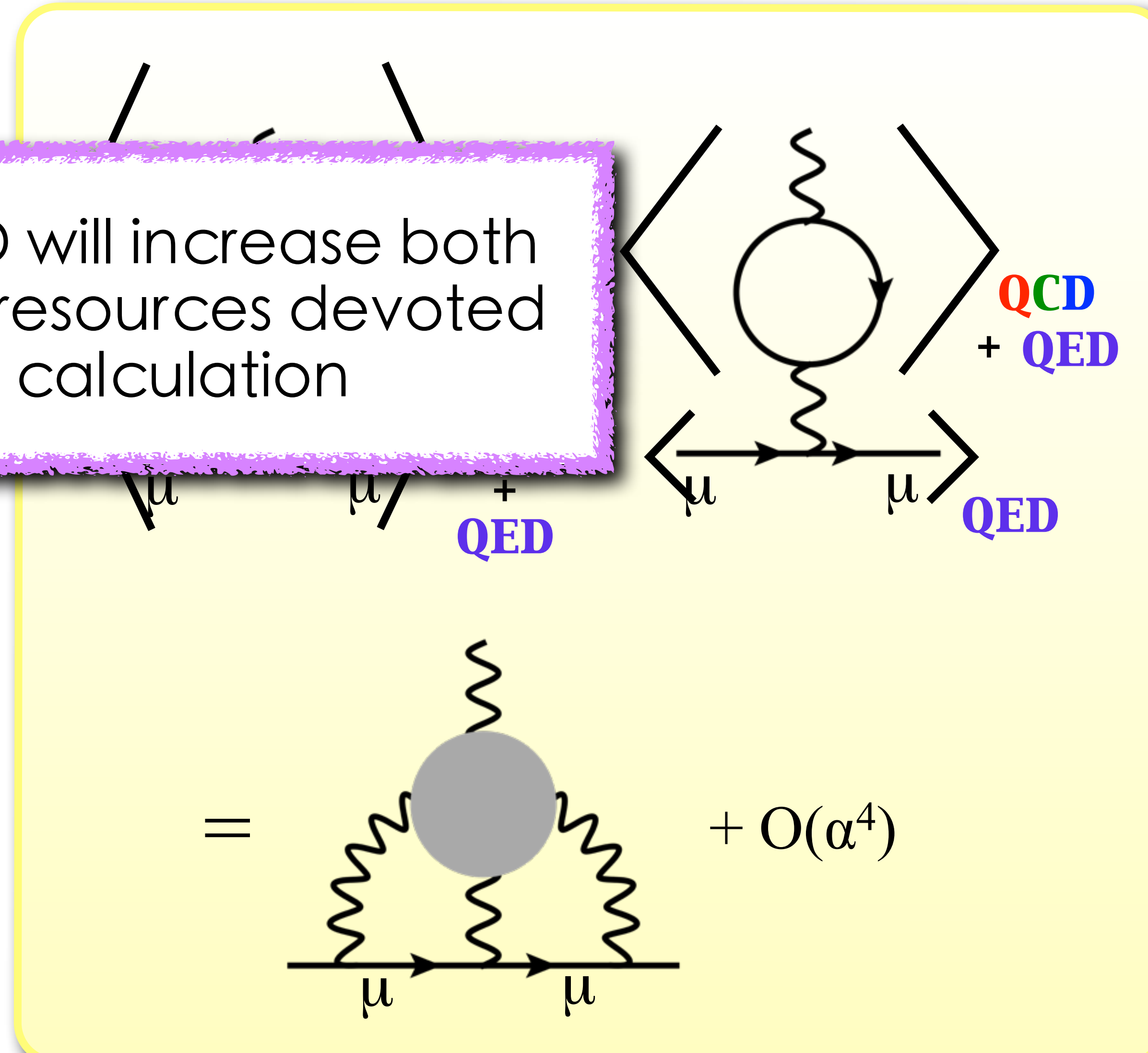
- Analogous approach to HVP calculation inserting correlation function of 4 EM currents into 2-loop QED integral (prohibitively?) complicated and costly

- **Promising approach to compute full hadronic amplitude nonperturbatively**

  - Include photon field along with gluon field in gauge link, so simulation & analysis follow calculation

  - Using all-mod obtain statistic emerging in the ballpark of model estimates [PoS LATTICE2012 (2012) 022]

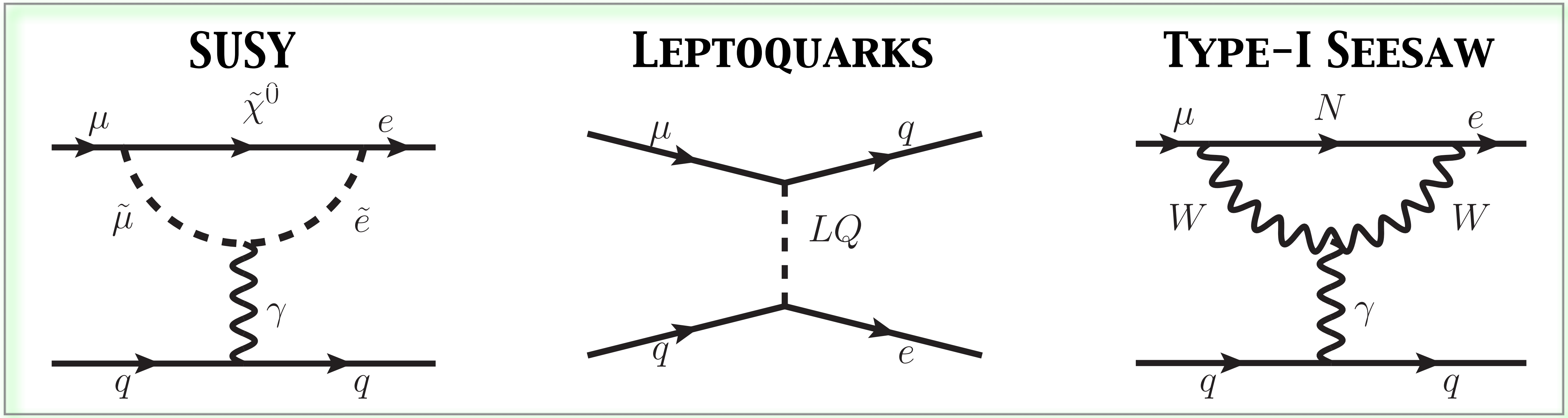
  - Still much to do for realistic result with controlled uncertainty: larger spatial volumes, extrapolation to physical pion mass and continuum, momentum extrapolation $Q^2 \to 0$, quark-disconnected contributions, ...



In coming years, USQCD will increase both human and computing resources devoted to this high-priority calculation

Muon-to-electron conversion

- Charged-lepton flavor violation highly suppressed in the Standard Model
  - ➡ Observation of CLFV would be unambiguous evidence of new physics
- Many new-physics models allow for CLFV and predict rates close to current limits

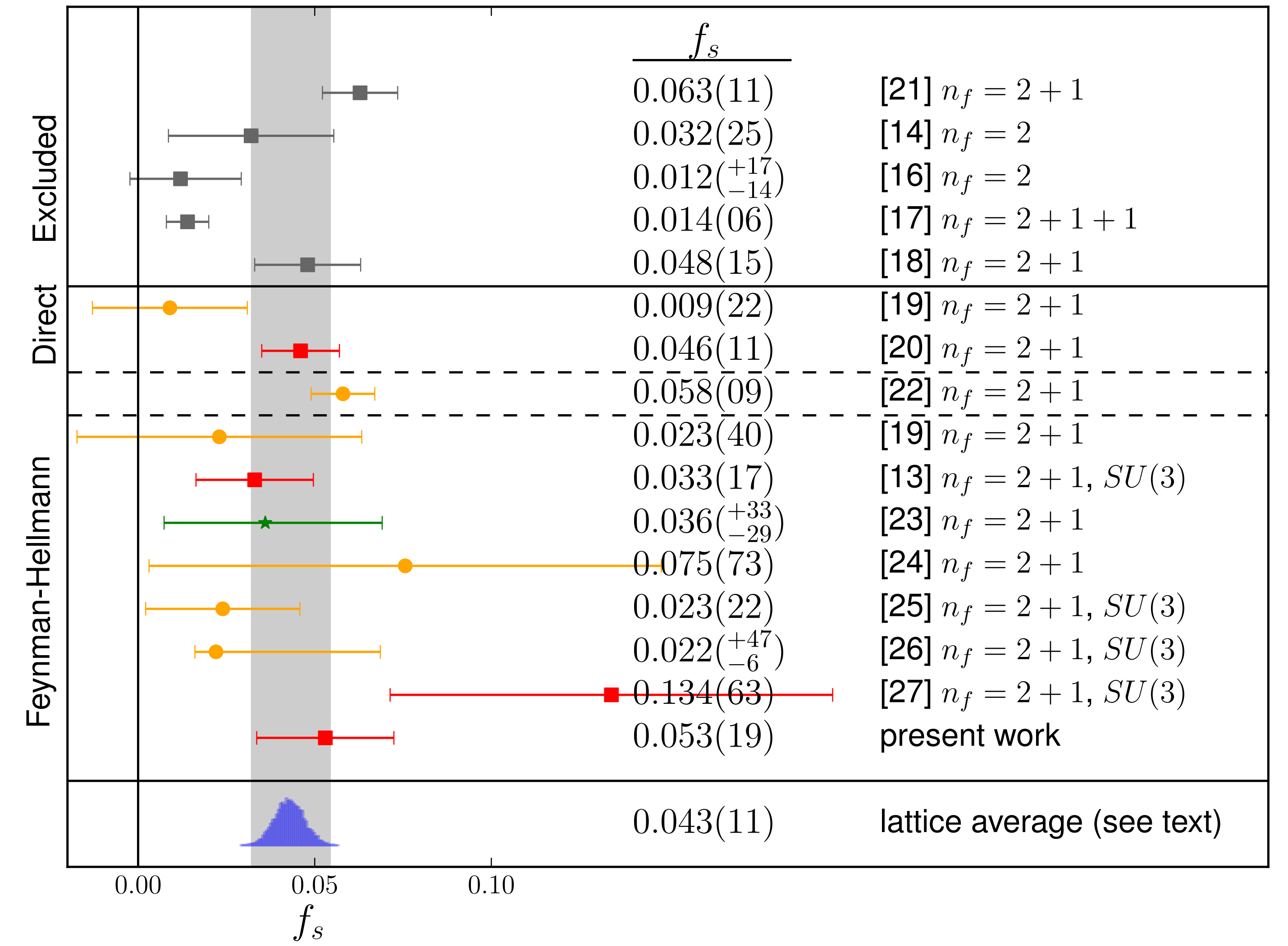


- **Mu2e Experiment @ Fermilab** aims to search for $\mu N \to eN$ with a sensitivity four orders of magnitude below the current best limit
- **MEG@PSI** searching for $\mu \to e\gamma$, while **Mu3e** proposes improved search for $\mu \to eee$
- Combining measured rates of $\mu \to e\gamma$ and $\mu \to e$ conversion on different target nuclei can distinguish between models and reveal information on underlying theory

Model discrimination in CLFV

- Model predictions for μ → e conversion rate off target nucleus depend upon the light- and strange-quark contents of the nucleon
- Lattice-QCD can nonperturbatively determine the pion-nucleon sigma term and strangeness content of the nucleon with controlled uncertainties
- Calculations of $f_s=m_s\langle N|ss|N\rangle/m_N$ improved significantly in recent years, and already rule out much larger values of favored by early non-lattice estimates
- Pinning down values with ~10-20% errors in the next five years is both realistic and sufficient

[Junnarkar & Walker-Loud, PRD87 (2013) 11, 114510]



Model discrimination in CLFV

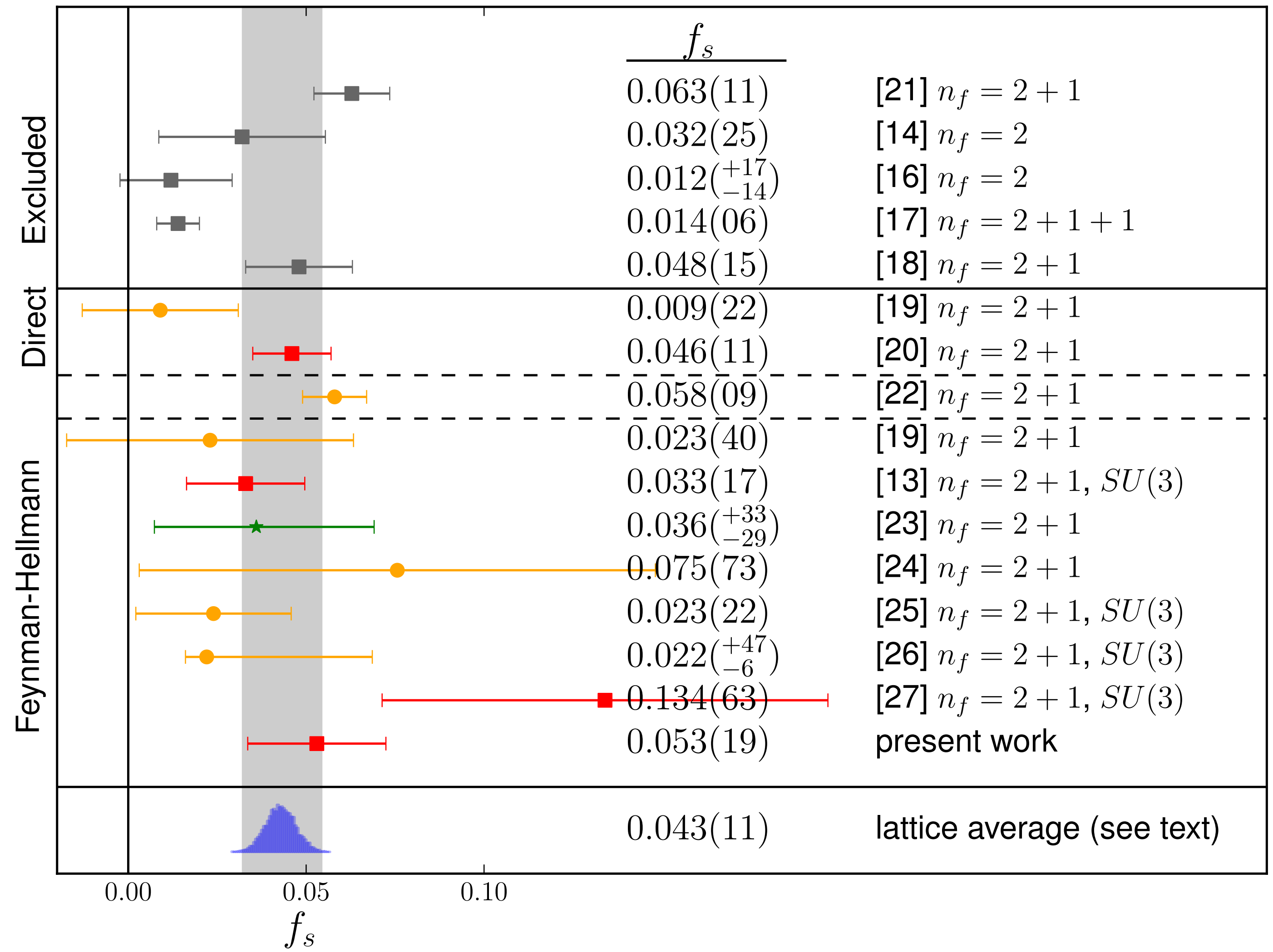
- Model predictions for μ → e conversion rate off target nucleus depend upon the light- and strange-quark contents of the nucleon

- Lattice-QCD can nonperturbatively determine the pion-nucleon sigma term and strangeness content of the nucleon with controlled uncertainties

- Calculations of $f_s = m_s\langle N|ss|N\rangle/m_N$ improved significantly in recent years, and already rule out much larger values of favored by early non-lattice estimates

- Pinning down values with ~10-20% errors in the next five years is both realistic and sufficient

[Junnarkar & Walker-Loud, PRD87 (2013) 11, 114510]

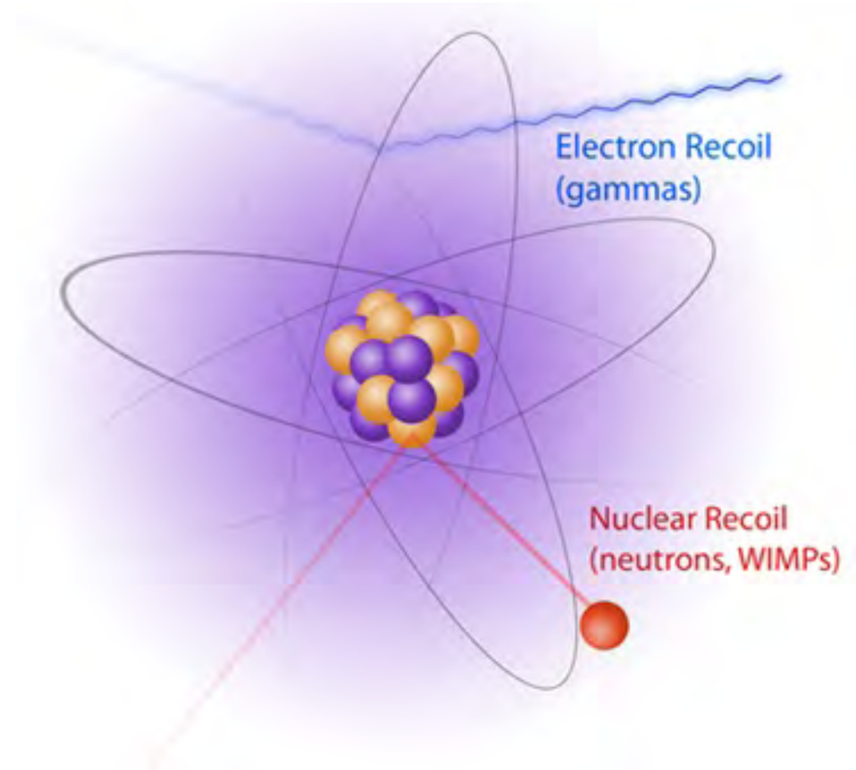


USQCD efforts on these and other nucleon matrix elements will be covered in science talk by Savage

Proton decay & other new interations

- Nucleon matrix elements needed to interpret many other experimental measurements as constraints on TeV-scale or GUT-scale new physics

  - **PROTON DECAY:** large underground detectors for neutrino physics also sensitive to proton decay; GUT model predictions for proton lifetime depend upon expectation values $\langle\pi, K, \eta, \ldots | \mathcal{O}_{NP} | p\rangle$ of new-physics operators



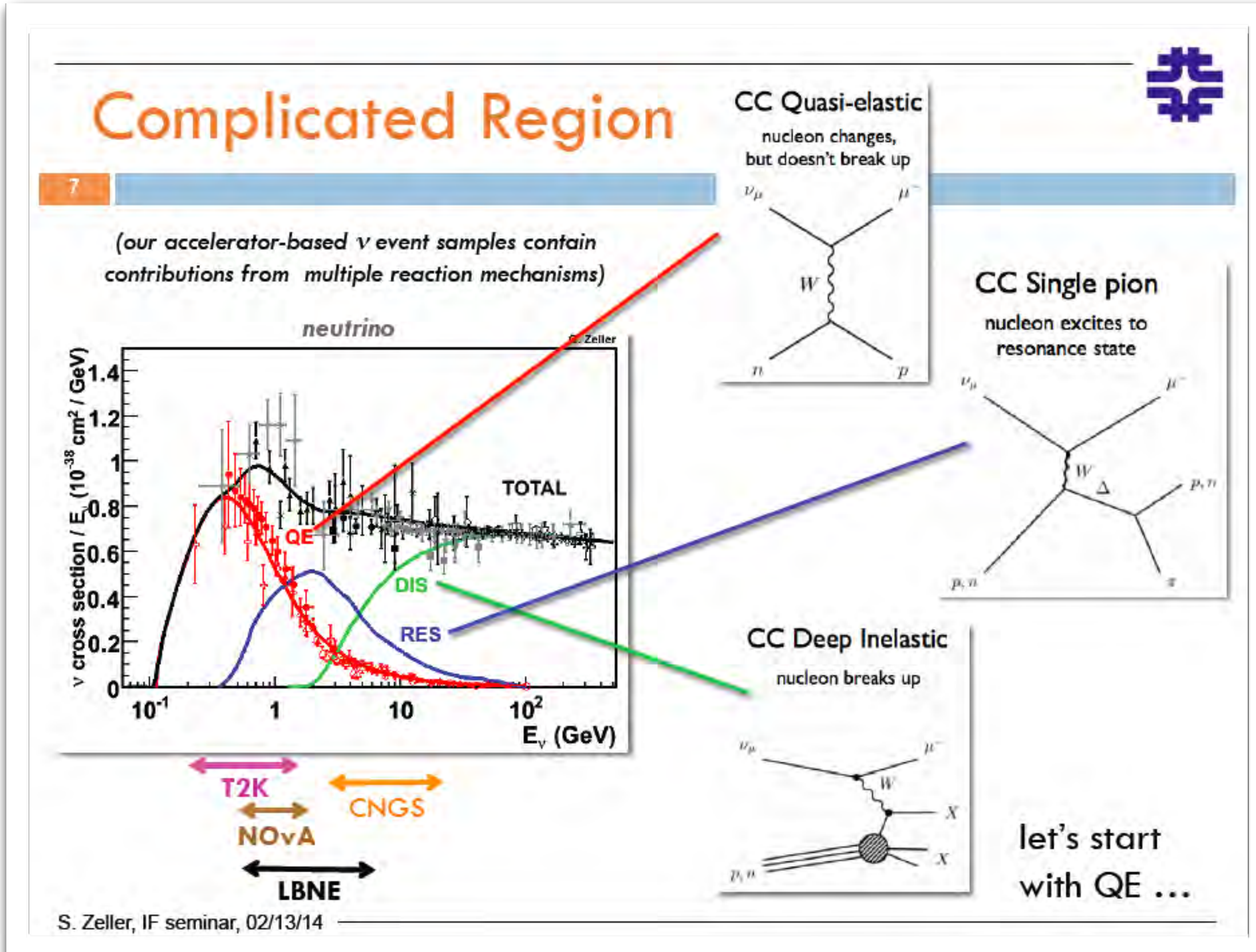
  - **DARK-MATTER DETECTION:** for spin-independent dark matter (*e.g.* mediated by Higgs exchange), cross-section for DM-nucleon scattering depends upon the light- and strange-quark contents of the nucleon (same matrix elements as for $\mu \rightarrow e$)

  - **NEUTRON BETA DECAY:** constraints on new TeV-scale interactions depend on the neutron scalar and tensor charges g_S and g_T

- For these matrix elements, **lattice calculations with 10–20% precision are sufficient for the time being** and can be achieved in the next five years (see Savage talk…)

Neutrino physics

- Accelerator-based ν experiments in low-energy regime complicated by nuclear environment
- Largest signal contribution in most oscillation experiments from charged-current quasielastic (CCQE) scattering on bound neutron
- Measurement of ν oscillation parameters and possible discovery of new ν states limited by understanding of CCQE cross section

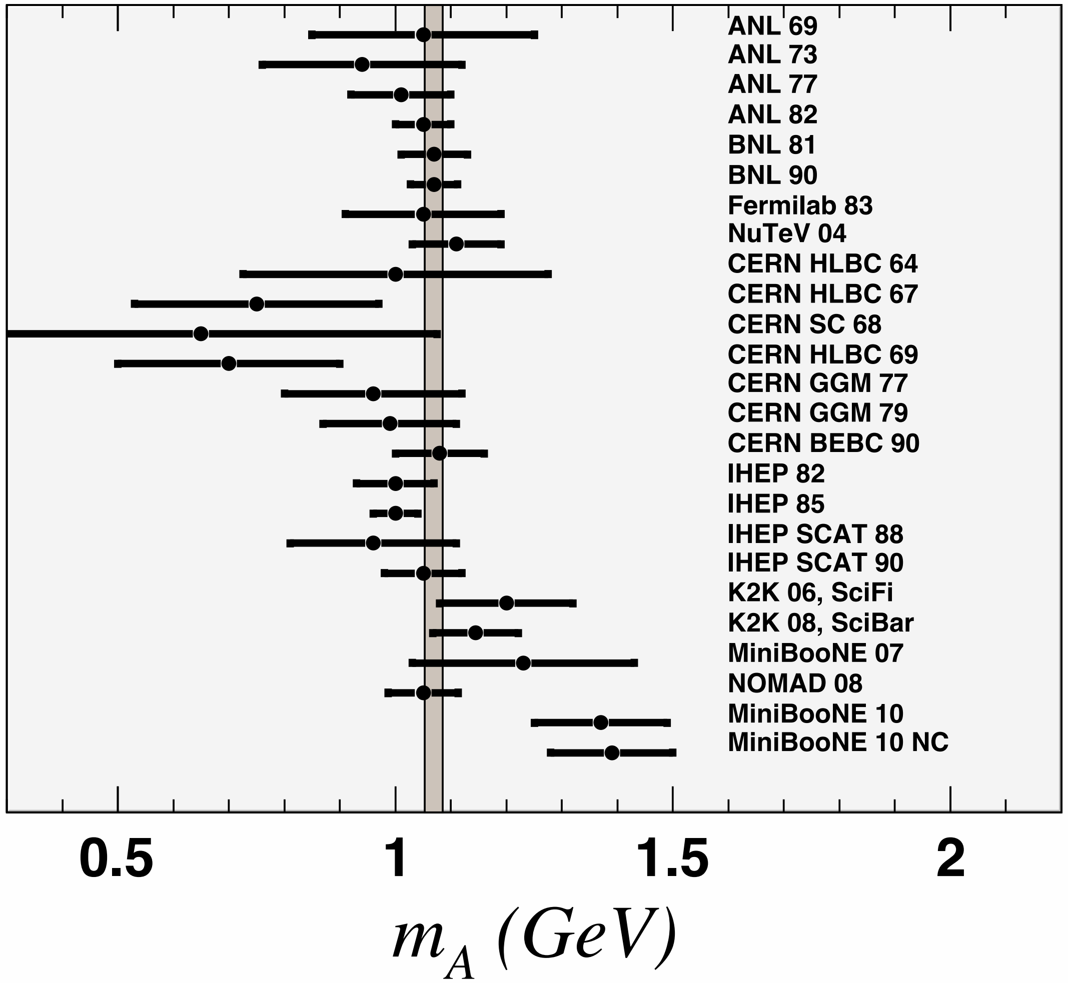


CCQE and the axial form factor

- **CCQE described by nucleon axial-vector form-factor $F_A(q^2)$**
- Typically q^2 dependence modeled by dipole form with g_A taken from neutron decay, but fits over different q^2 ranges and by different experiments lead to inconsistent determinations of m_A
  - Difference may stem from nuclear effects, inadequate model parameterization, or both

$$F_A(q^2) = \frac{g_A}{(1 + q^2/m_A^2)}$$

[Hill, "Lattice Meets Experiment" 2014]



- Shape of $F_A(q^2)$ can be calculated from first principles by merging analyticity constraints [Bhattacharya *et al.*, PRD84 (2011) 073006] with lattice QCD
- In addition to USQCD efforts on g_A and $F_A(q^2)$ (Savage talk), A. Kronfeld and collaborators working with MINERvA experimenters to implement z-expansion and external QCD input into GENIE Monte Carlo

Precision Higgs measurements

- Now that the Higgs mass is known, can predict all Higgs-boson couplings and properties within the Standard Model and look for deviations
- **Future high-energy/luminosity colliders will measure Higgs partial widths to sub-percent precision**, but commensurate theoretical uncertainties on Standard-Model predictions needed to fully exploit measurements
- Parametric errors from m_c, m_b, and α_s are largest sources of uncertainty in SM width predictions for the dominant Higgs decay mode H→bb, many other Higgs decay channels, and the Higgs total width [LHC Higgs X-Section WG, EPJ C71 (2011) 1753]

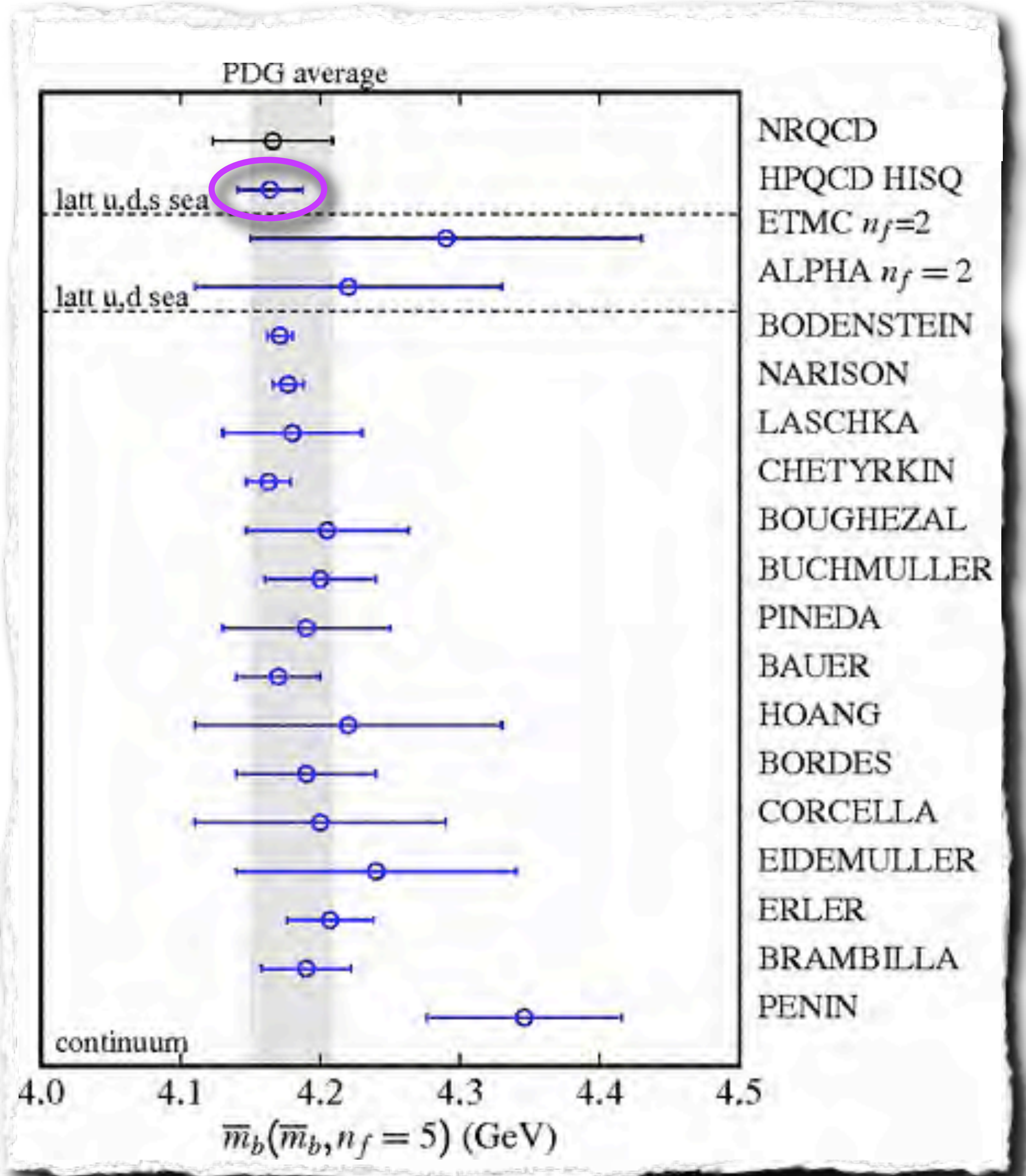
Channel	$\Delta\alpha_s$	Δm_b	Δm_c	Theory Uncertainty	Total Uncertainty
$H \to \gamma\gamma$	0%	0%	0%	±1%	±1%
$H \to b\bar{b}$	$\mp$ **2.3%**	$^{+3.3\%}_{-3.2\%}$	0%	±2%	±6%
$H \to c\bar{c}$	$^{-7.1\%}_{+7.0\%}$	∓0.1%	$^{+6.2\%}_{-6.1\%}$	±2%	±11%
$H \to gg$	$^{+4.2\%}_{-4.1\%}$	∓0.1%	0%	±3%	±7%
$H \to \tau^+\tau^-$	0%	0%	0%	±2%	±2%
$H \to WW^*$	0%	0%	0%	±0.5%	±0.5%
$H \to ZZ^*$	0%	0%	0%	±0.5%	±0.5%

[Snowmass Higgs WG Report, 1310.8361]

Heavy-quark masses from lattice QCD

[McNeile et al. (HPQCD), PRD82 (2010) 034512]

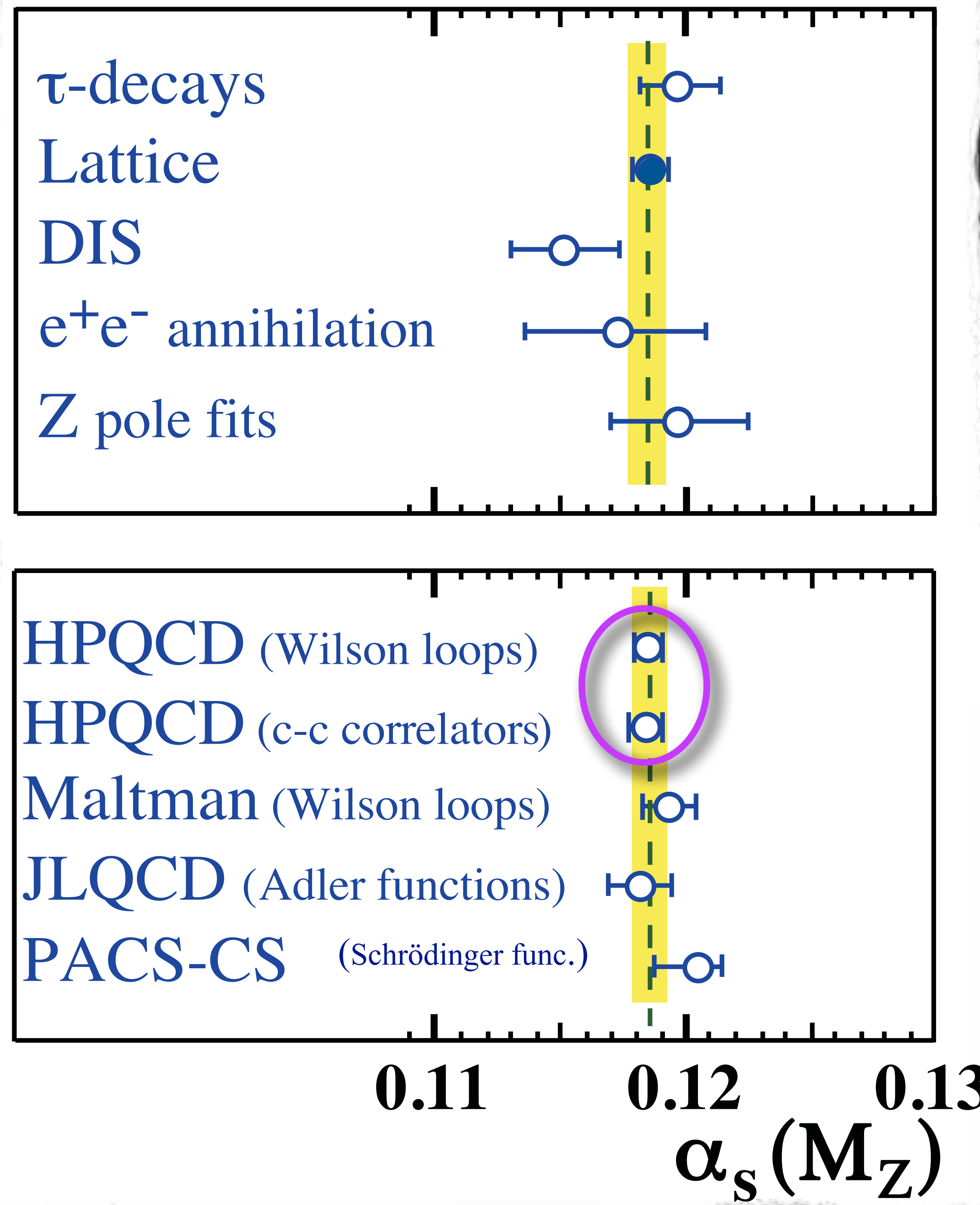
- Only (2+1)-flavor calculations of m_b & m_c from USQCD
- Approach to fit moments of correlation functions of the quarks' electromagnetic current to $O(\alpha_S^3)$ perturbative expressions
  - Lattice moments have *negligible statistical uncertainties,* so cleaner than e^+e^- data
  - Can vary lattice quark-mass between m_c and m_b to control and estimate errors
- Using HISQ c & b quarks, **HPQCD obtains m_c & m_b to ~0.5% precision** and finds good agreement with non-lattice determinations
  - m_c will only improve modestly without higher-order PT calculation, but **m_b will improve significantly with HISQ b quarks on the planned 0.03fm MILC ensembles**



Strong coupling from lattice QCD

- Several good lattice methods available to obtain α_s, all of which yield consistent results with smaller errors than non-lattice determinations
- Most precise calculations by HPQCD using the MILC asqtad ensembles [McNeile et al. (HPQCD), PRD82 (2010) 034512]
  - Obtain ~0.5% precision from NNNLO QCD fits to short-distance lattice quantities built from Wilson loops, and ~0.6% from moments of current current correlators
  - **Errors in both will improve with analysis of planned finer 0.03fm MILC HISQ ensembles**
- Further, in the coming years, anticipate increased corroboration from new calculations

[PDG (2013)]



Strong coupling from lattice QCD

✦ Several good lattice methods available to obtain α_s, all of which yield consistent results with smaller errors than non-lattice determinations

✦ Most preci
the MILC c
(HPQCD), F

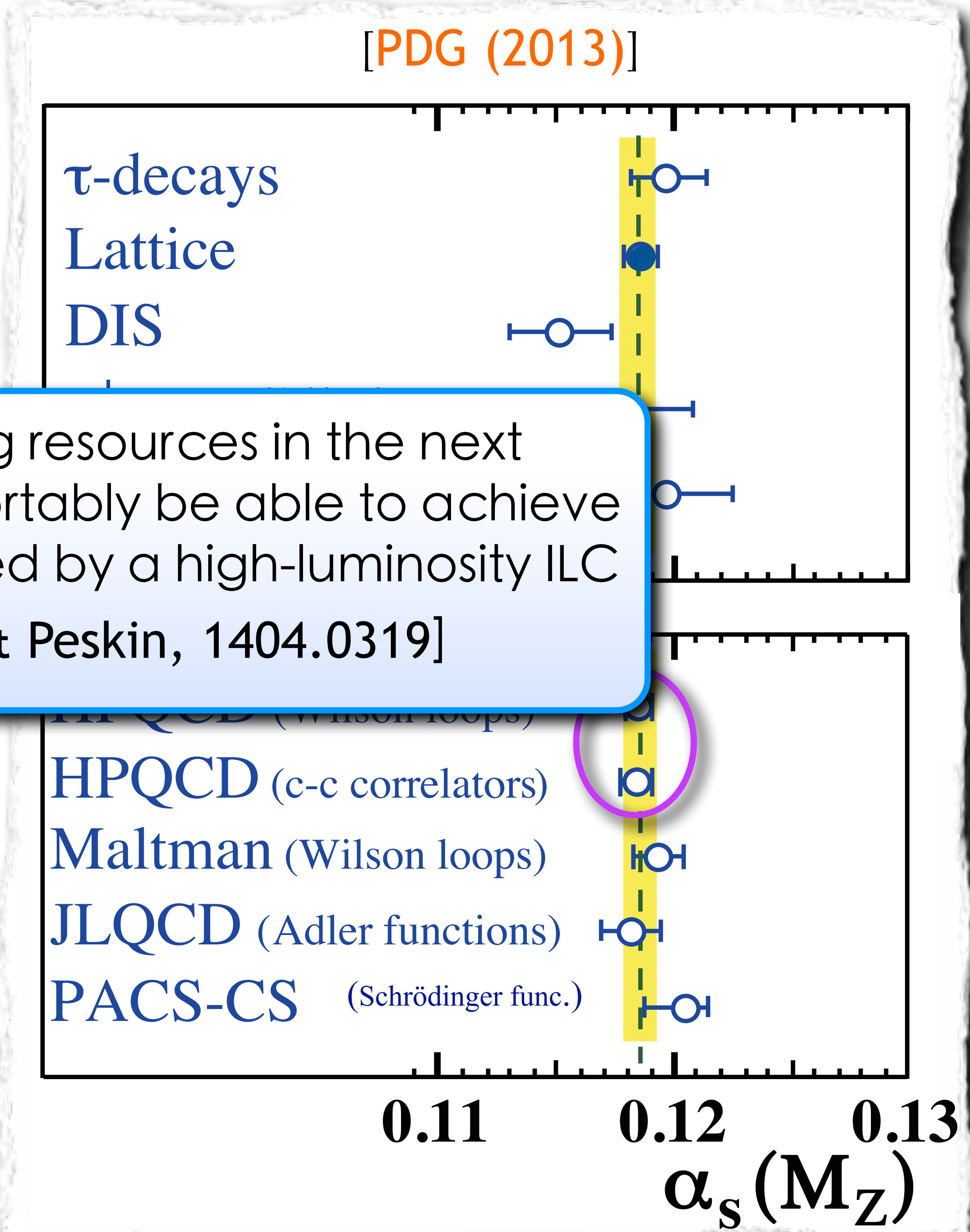
❖ Obtain
QCD fit
built from Wilson loops, and ~0.6% from moments of current current correlators

❖ **Errors in both will improve with analysis of planned finer 0.03fm MILC HISQ ensembles**

✦ Further, in the coming years, anticipate increased corroboration from new calculations

With anticipated computing resources in the next decade, lattice QCD will comfortably be able to achieve m_c, m_b, & α_s to precisions needed by a high-luminosity ILC

[see Lepage, Mackenzie, & Peskin, 1404.0319]




Computing plans

Physics requirements

- Achieving scientific goals will require ensembles with the following properties:

(1) Physical-mass pions

- To improve kaon, D-, and B-meson decay constants, form factors, and mixing; also nucleon matrix elements and HVP contribution to muon g-2 (already enabled calculations of f_K/f_π and Kl3 form factor with unprecedented precision)

(2) Very fine lattice spacings

- Will enable simulations with HISQ charm, and eventually HISQ bottom: anticipate dramatic improvement in precision for b-quark mass, D- & B-meson decays

(3) Dynamical charm quarks

- For calculations such as D decays & K_L-K_S mass difference (GIM cancellation)

(4) Systematic inclusion of isospin-breaking and electromagnetism

- Dynamical QED will help m_u/m_d and HLbL contribution to g-2

(5) Large physical volumes

- For nucleon matrix elements and muon g-2

Physics requirements

✦ Achieving scientific goals will require ensembles with the following properties:

(1) Physical-mass pions

❖ To improve kaon, D-, and B-meson decay constants, form factors, and mixing; also nucleon matrix elements and HVP contribution to muon g-2 (already enabled calculations of f_K/f_π and Kl3 form factor with unprecedented precision)

(2) Very fine lattice spacings

❖ Will enable simulat[...] HISQ bottom: anticipate dramati[...]ass, D- & B-meson decays

These improvements will become widespread over the next five years

(3) Dynamical charm quarks

❖ For calculations such as D decays & K_L-K_S mass difference (GIM cancellation)

(4) Systematic inclusion of isospin-breaking and electromagnetism

❖ Dynamical QED will help m_u/m_d and HLbL contribution to g-2

(5) Large physical volumes

❖ For nucleon matrix elements and muon g-2

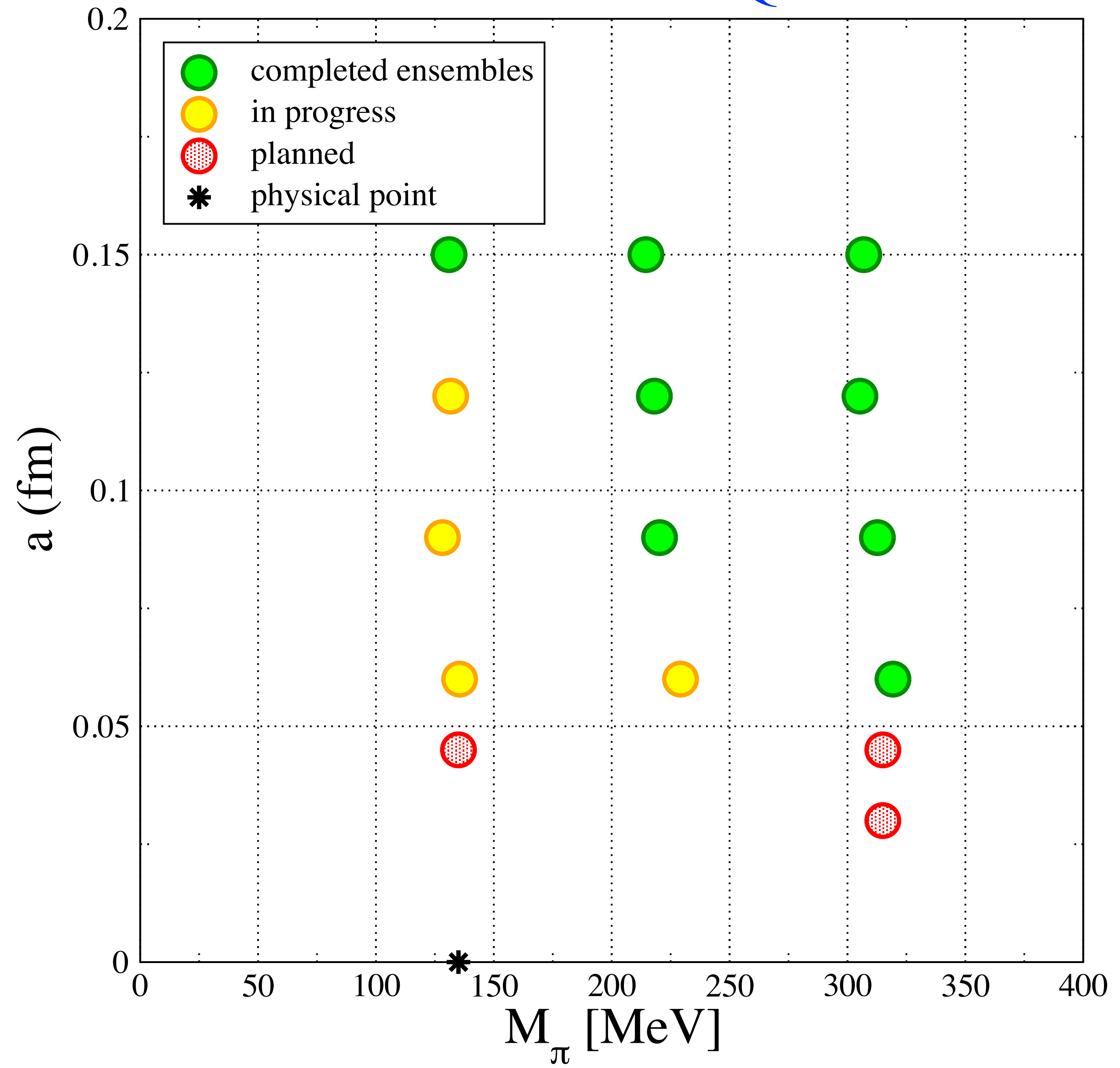
Gauge-field configurations

- USQCD work on precision matrix elements based primarily on two sets of ensembles using domain-wall fermions (RBC) and HISQ fermions (MILC)

  - Domain-wall fermions advantageous for calculations of quantities that require accurate control of chiral symmetry to suppress unphysical operator mixing or to replicate chiral structure of the standard model, e.g. kaon mixing or $K \to \pi\pi$ decay

  - HISQ ensembles being used for wide range of studies including precise calculations of SM parameters (strong coupling & quark masses), weak-interaction matrix elements needed to determine CKM matrix elements and test the Standard Model, and hadronic contributions to muon g-2

Planned Nf=2+1+1 HISQ Ensembles



Forecasts

- Planned simulations with physical-mass pions, finer lattice spacings, and larger volumes will improve the precision of "standard" matrix elements for CKM physics

[Snowmass Quark-flavor WG report, 1311.1076]

Quantity	CKM element	Present expt. error	2007 forecast lattice error	Present lattice error	2018 lattice error
f_K/f_π	$\|V_{us}\|$	0.2%	0.5%	0.4%	0.15%
$f_+^{K\pi}(0)$	$\|V_{us}\|$	0.2%	–	0.4%	0.2%
$D \to \pi\ell\nu$	$\|V_{cd}\|$	2.6%	–	4.4%	2%
$D \to K\ell\nu$	$\|V_{cs}\|$	1.1%	–	2.5%	1%
$B \to D^*\ell\nu$	$\|V_{cb}\|$	1.3%	–	1.8%	$< 1\%$
$B \to \pi\ell\nu$	$\|V_{ub}\|$	4.1%	–	8.7%	2%
f_B	$\|V_{ub}\|$	9%	–	2.5%	$< 1\%$
ξ	$\|V_{ts}/V_{td}\|$	0.4%	2-4%	4%	$< 1\%$
B_K	$\mathrm{Im}(V_{td}^2)$	0.5%	3.5–6%	1.3%	$< 1\%$

- For less mature calculations, cannot make quantitative uncertainty predictions

- Development of new algorithms and analysis methods also being pursued, some of which will likely lead to dramatic improvements

Summary and outlook

"[An] area of striking progress has been lattice gauge theory. … It is now possible to compute the spectrum of hadrons with high accuracy, and lattice computations have been crucial in the measurement of the properties of heavy quarks. Continuing improvements in calculational methods are anticipated in coming years."

– **Snowmass Executive Summary**

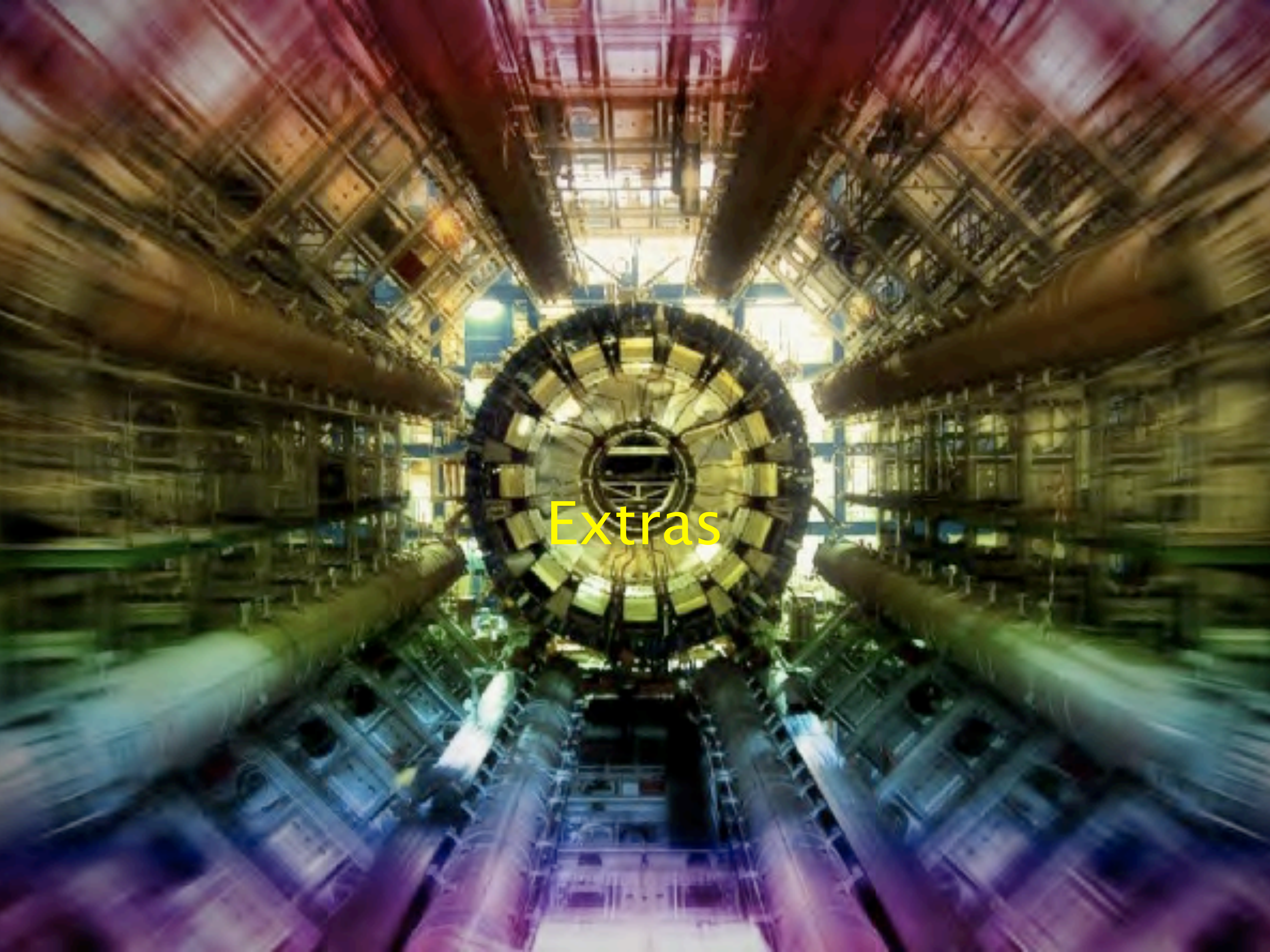
Outlook

- **Success of future experimental high-energy physics program hinges on reliable theoretical predictions on same time scale as experiments and with commensurate uncertainties**
- **Lattice-QCD calculations are needed throughout the HEP program**
  - For precision measurements of rare kaon and B decays, muon g-2, neutrino oscillation parameters, Higgs properties, ...
  - For searches for μ→e conversion, dark matter, proton decay, nucleon EDMs, ...
- USQCD is expanding program to meet needs of current and upcoming experiments:
  - **Increasing precision** in parameters of QCD Lagrangian and simplest quark flavor-changing and nucleon matrix elements
  - **Addressing new challenges** such as rare decays, muon g-2, long-distance amplitudes, and multi-hadron final states
- *Continued support of dedicated lattice-QCD hardware and software will be essential to accomplish our scientific goals*

Outlook

- Success of future experimental high-energy physics program hinges on reliable theoretical predictions on same time scale as experiments and with commensurate uncertainties
- Lattice-QCD calculations are needed throughout the HEP program
  - For p... trino osci...
  - For ... DMs, ...
- USQCD ... periments:
  - **Increasing precision** in parameters of QCD Lagrangian and simplest quark flavor-changing and nucleon matrix elements
  - **Addressing new challenges** such as rare decays, muon g-2, long-distance amplitudes, and multi-hadron final states
- *Continued support of dedicated lattice-QCD hardware and software will be essential to accomplish our scientific goals*

We are excited about the wealth of opportunities to help the current and future experimental program, and look forward to tightening the noose on the Standard Model and (*hopefully*) revealing evidence for new physics


Extras

2014 Highlight:
Update of $|V_{cb}|$ from $B \to D^* \ell \nu$ at zero recoil

- $B \to D^* \ell \nu$ semileptonic form factor allows determination of $|V_{cb}|$ via:

$$\frac{d\Gamma(B \to D^* l\nu)}{dw} = \frac{G_F^2}{48\pi^3} m_D^3 (m_B + m_D)^2 (w^2 - 1)^{3/2} |\mathbf{V_{cb}}|^2 |\mathcal{F}_{\mathbf{B \to D^*}}(\mathbf{w})|^2$$

$w \equiv v_B \cdot v_D$

- Only need one normalization point, so choose zero recoil (w=1) because it can be computed most precisely

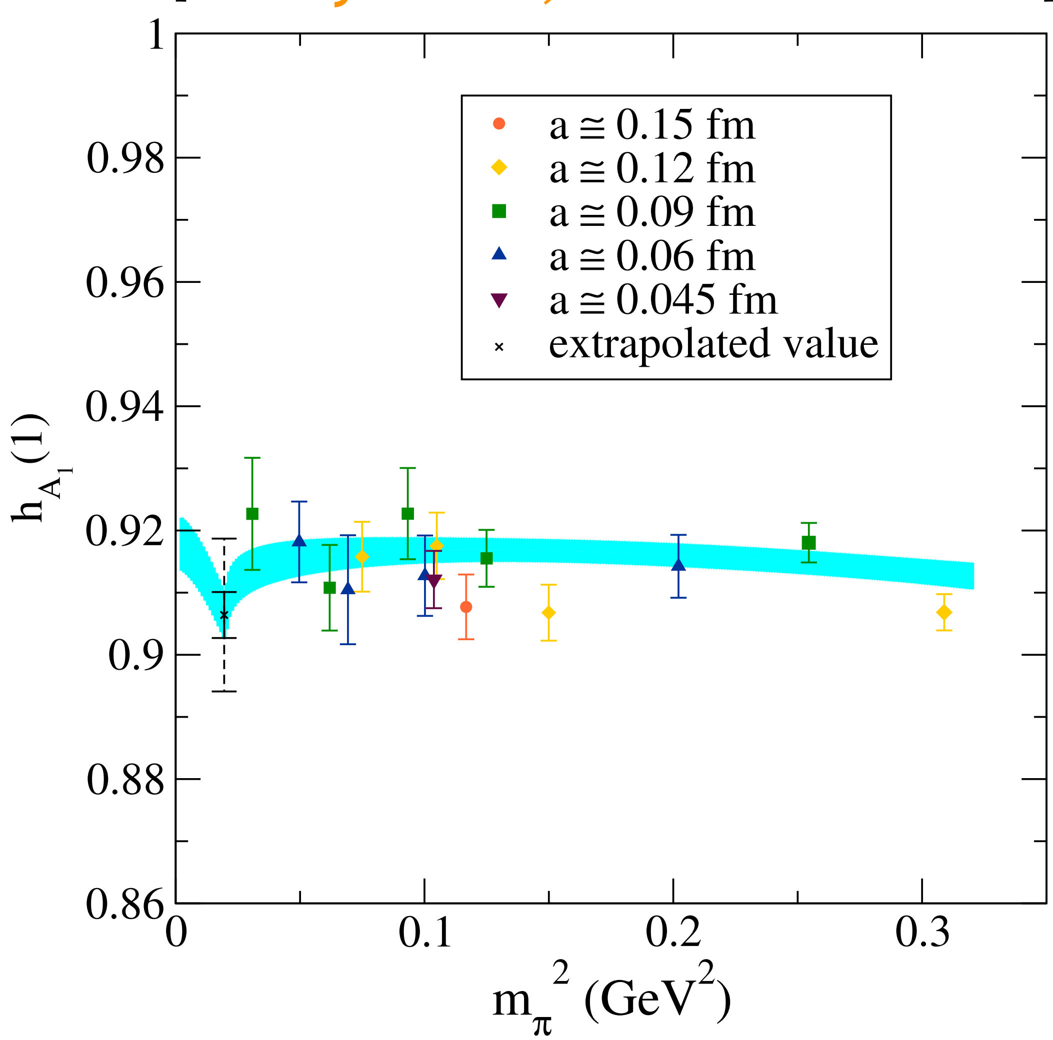
- Fermilab Lattice & MILC Collaborations recently **updated F(1) with increased statistics, lighter quark masses, & finer lattice spacings**, obtaining $|V_{cb}|$ to 1.9% precision

- **QCD error in $|V_{cb}|$ now commensurate with the experimental error**

$F(1) = 0.906(4)_{stat}(12)_{sys}$

$|V_{cb}| = [39.04(49)_{expt}(53)_{LQCD} \pm (19)_{QED}] \times 10^{-3}$

[Bailey *et al.*, arXiv:1403.0635]



2013 Highlight: f_K/f_π at the physical point

- The SU(3) flavor-breaking ratio f_K/f_π allows a determination of $|V_{ud}|/|V_{us}|$ [Marciano]

$$\frac{\Gamma(K\to l\bar\nu_l)}{\Gamma(\pi\to l\bar\nu_l)} = \left(\frac{|V_{us}|}{|V_{ud}|}\right)^2 \left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K\left(1-\frac{m_l^2}{m_K^2}\right)^2}{m_\pi\left(1-\frac{m_l^2}{m_\pi^2}\right)^2}\left[1+\frac{\alpha}{\pi}(C_K-C_\pi)\right]$$

- MILC collaboration recently obtained the **first lattice-QCD determination of f_K/f_π (1) including dynamical charm** and **(2) at the physical pion mass** with highly-improved staggered (HISQ) quarks [Bazavov *et al.* PRL **110**, 172003]
  - **Eliminate error from extrapolation to physical u- and d-quark masses**

Source	f_{K^+}/f_{π^+}
Monte-Carlo statistics	0.22%
Continuum extrapolation	0.28%
Finite-volume corrections	0.14%
EM corrections	0.02%
Total	0.38%

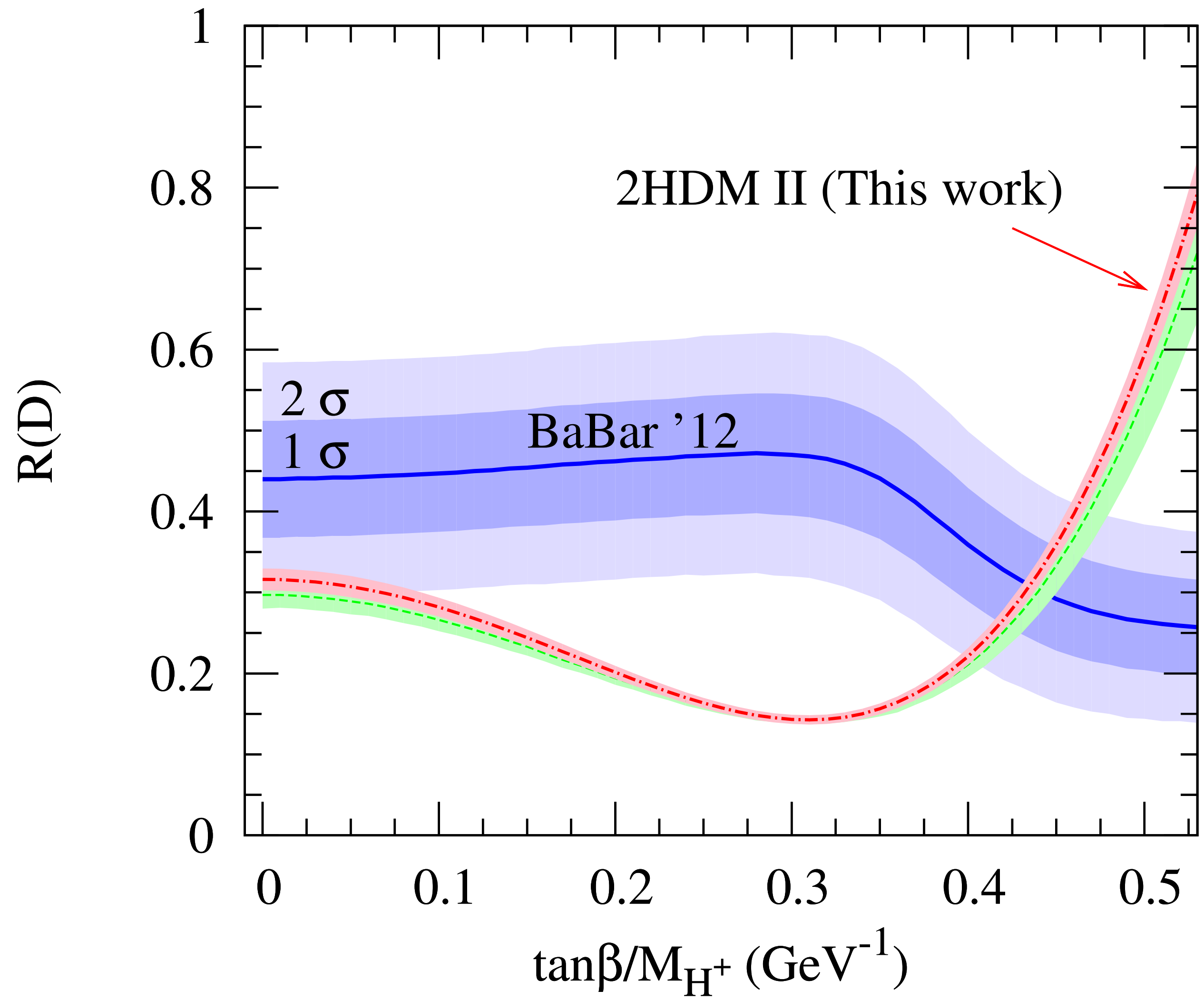
- Combined with $|V_{ud}|$ from nuclear β-decay, enables sub-percent test of unitarity of 1st row of CKM matrix

$$1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2 = 0.0003(6)$$

2012 Highlight:
R(D) from unquenched lattice QCD

- $B \to D^{(*)} \tau \nu$ decays sensitive to new-physics contributions such as from charged Higgs bosons

- Recently BaBar measured the ratios $R(D) = BR(B \to D\tau\nu)/BR(B \to Dl\nu)$, $R(D^*) = BR(B \to D^*\tau\nu)/BR(B \to D^*l\nu)$ and observed excesses in both channels that disagree with the Standard Model by 3.4σ [PRL 109 (2012) 101802]



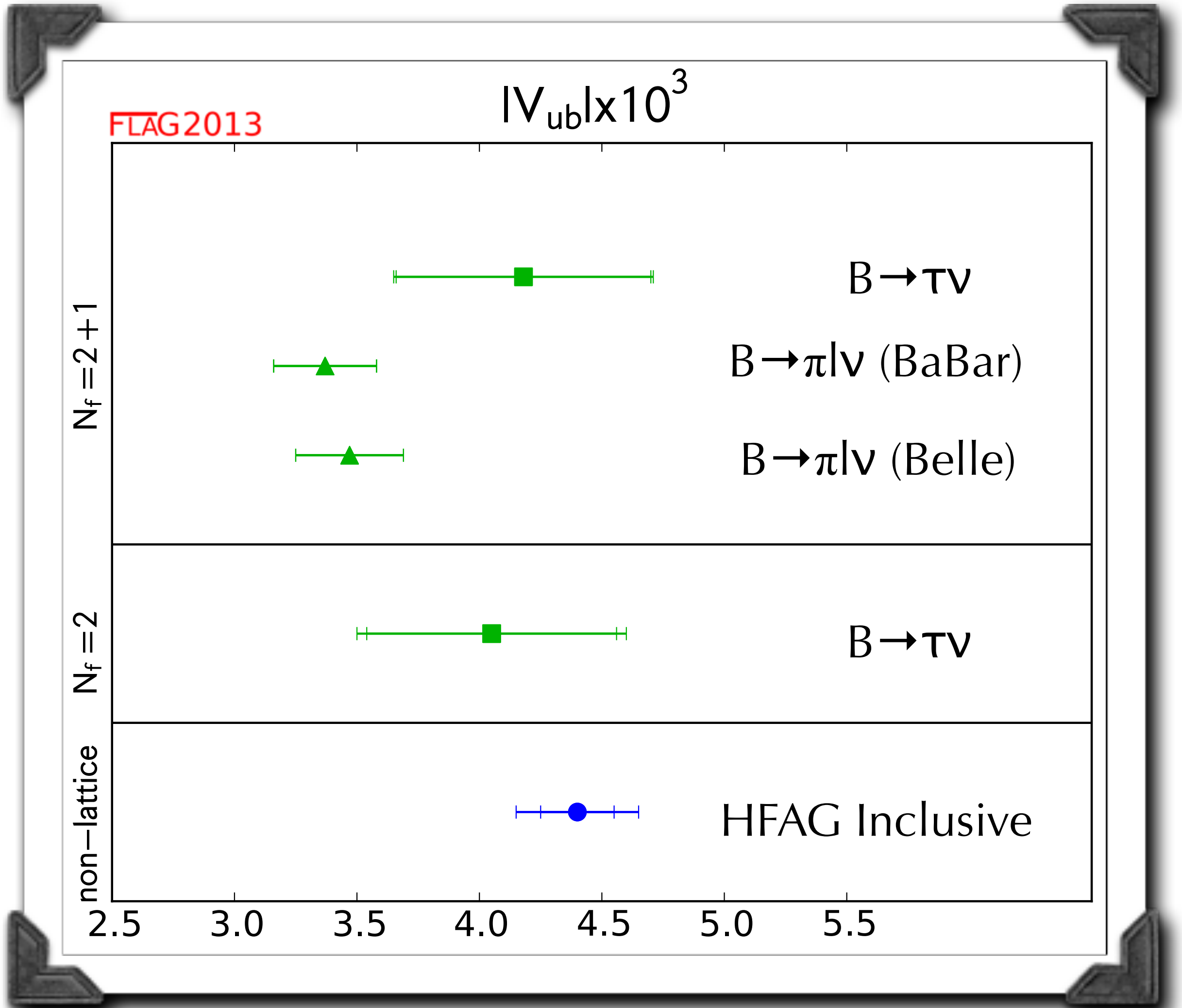
- Fermilab Lattice and MILC Collaborations quickly followed with first Standard-Model calculation of R(D) from ab initio lattice-QCD [PRL 109 (2012) 071802]

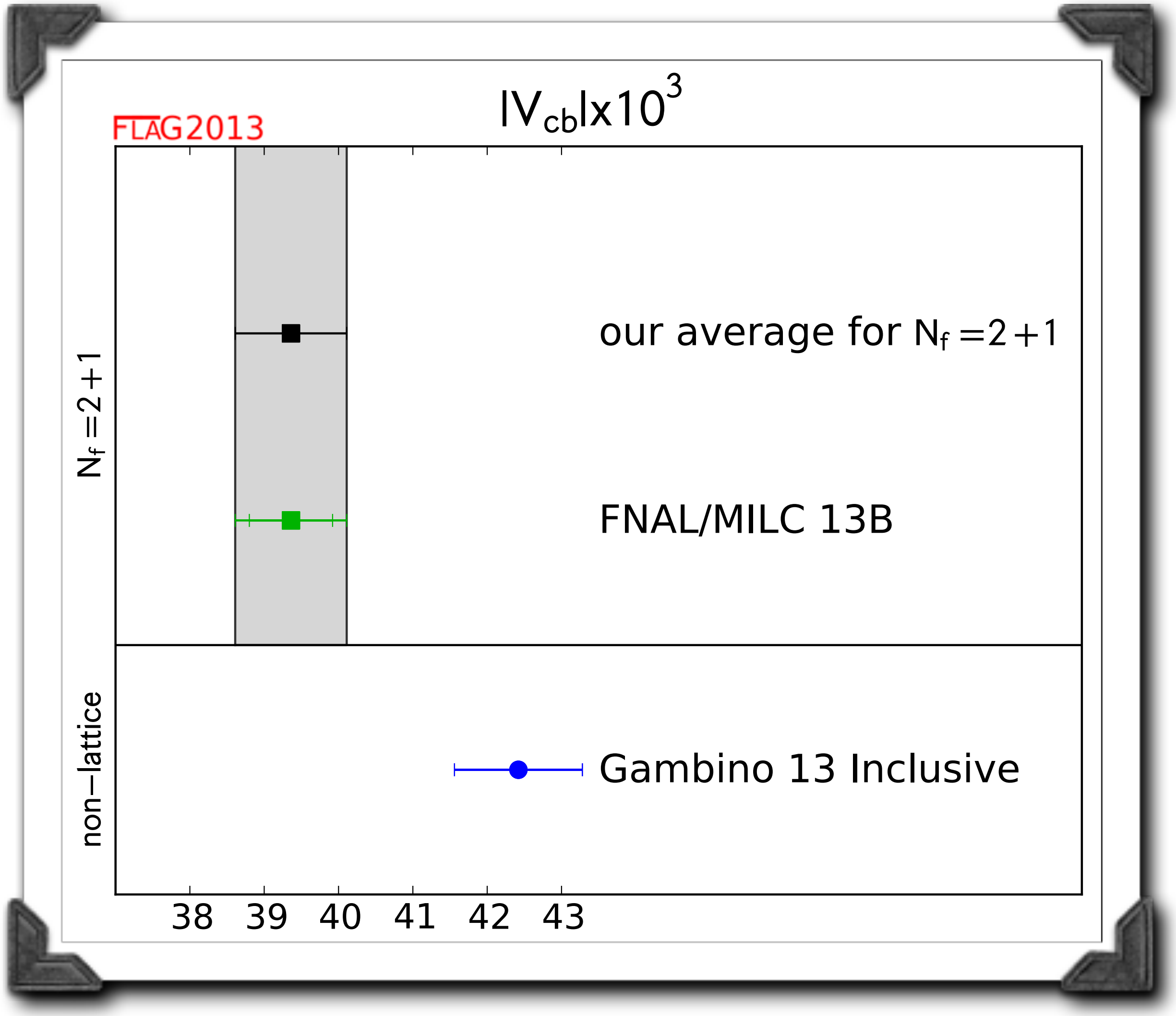
- Uncertainty smaller than previous model estimate from dispersive bounds, heavy-quark symmetry, and quenched lattice QCD

- *Lattice calculation of $R(D^*)$ in progress...*

Outstanding puzzles

- Long-standing ~3σ tension between determinations of $|V_{ub}|$ and $|V_{cb}|$ from inclusive and exclusive semileptonic B-decays still needs resolution





- **Determinations from other exclusive decays will provide important checks**:
  - Lattice-QCD calculations underway of form factors for **$B_s \to K\mu\nu$ to obtain $|V_{ub}|$** (will be measured @ LHCb) and **$B \to Dl\nu$ to obtain $|V_{cb}|$** (N_f=2+1 result coming soon)

Rare kaon decays

- Standard-Model branching ratios for **"GOLDEN" MODES** $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ known to a **precision unmatched by any other quark FCNC processes**

- Within this decade, NA62 @ CERN SPS will measure $\mathcal{O}(100)$ K^+ events (assuming the SM), and KOTO @ J-PARC will collect first K^0_L events

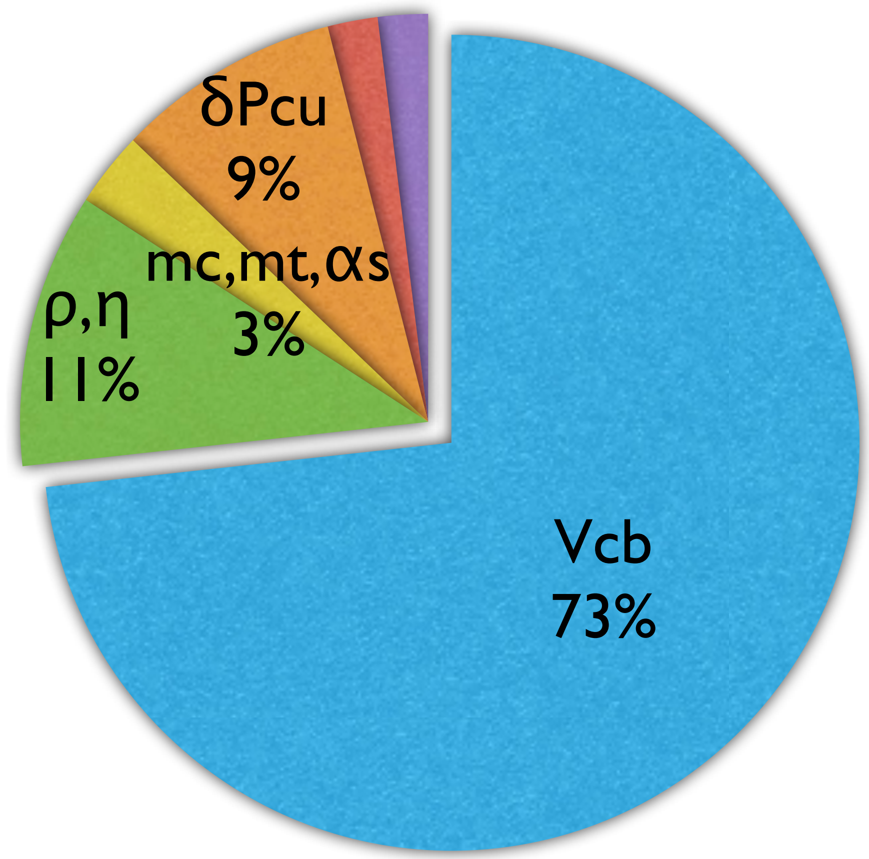
- Hadronic form factor can be obtained precisely using experimental $K \to \pi \ell \nu$ data and chiral perturbation theory [Mescia & Smith, PRD76 (2007) 034017]

  ➡ Limited by parametric uncertainty in $A^4 \propto |V_{cb}|^4$

- With calculations of $B \to D^{(*)} l \nu$ at *nonzero* recoil in the next few years, expect to reduce error in $|V_{cb}|$ to ~1.5%, and in the Standard-Model branching fractions to ~6%

  ➡ Theory error in Standard-Model predictions will be commensurate with expected experimental error

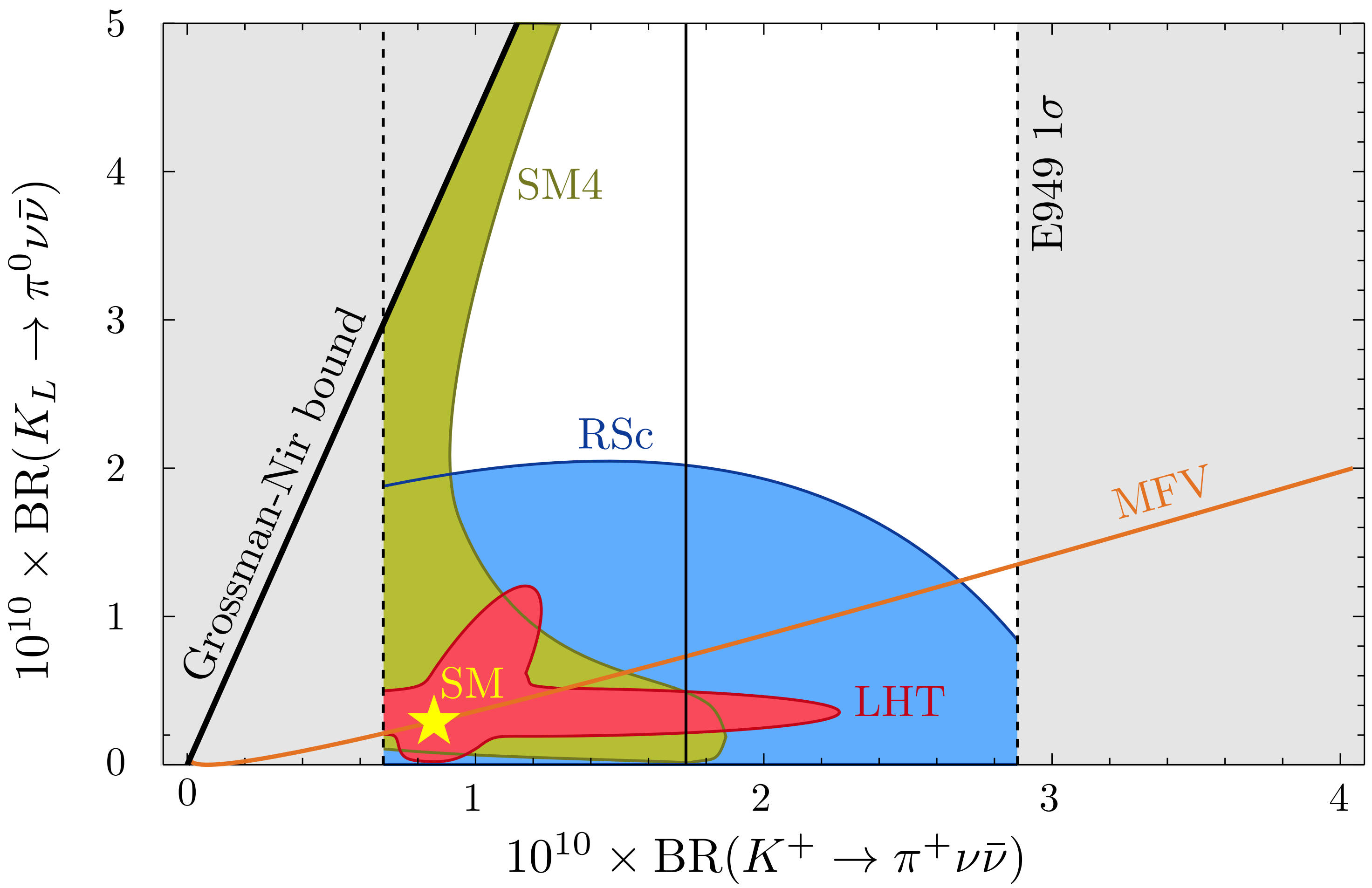
$BR(K^+ \to \pi^+ \nu\bar{\nu})$



[Brod & Gorbahn PRD83 (2011) 034030]

Room for new physics

- Sensitive to Little Higgs models, warped extra dimensions, and 4th generation [Buras, Acta Phys.Polon.B41:2487-2561,2010]



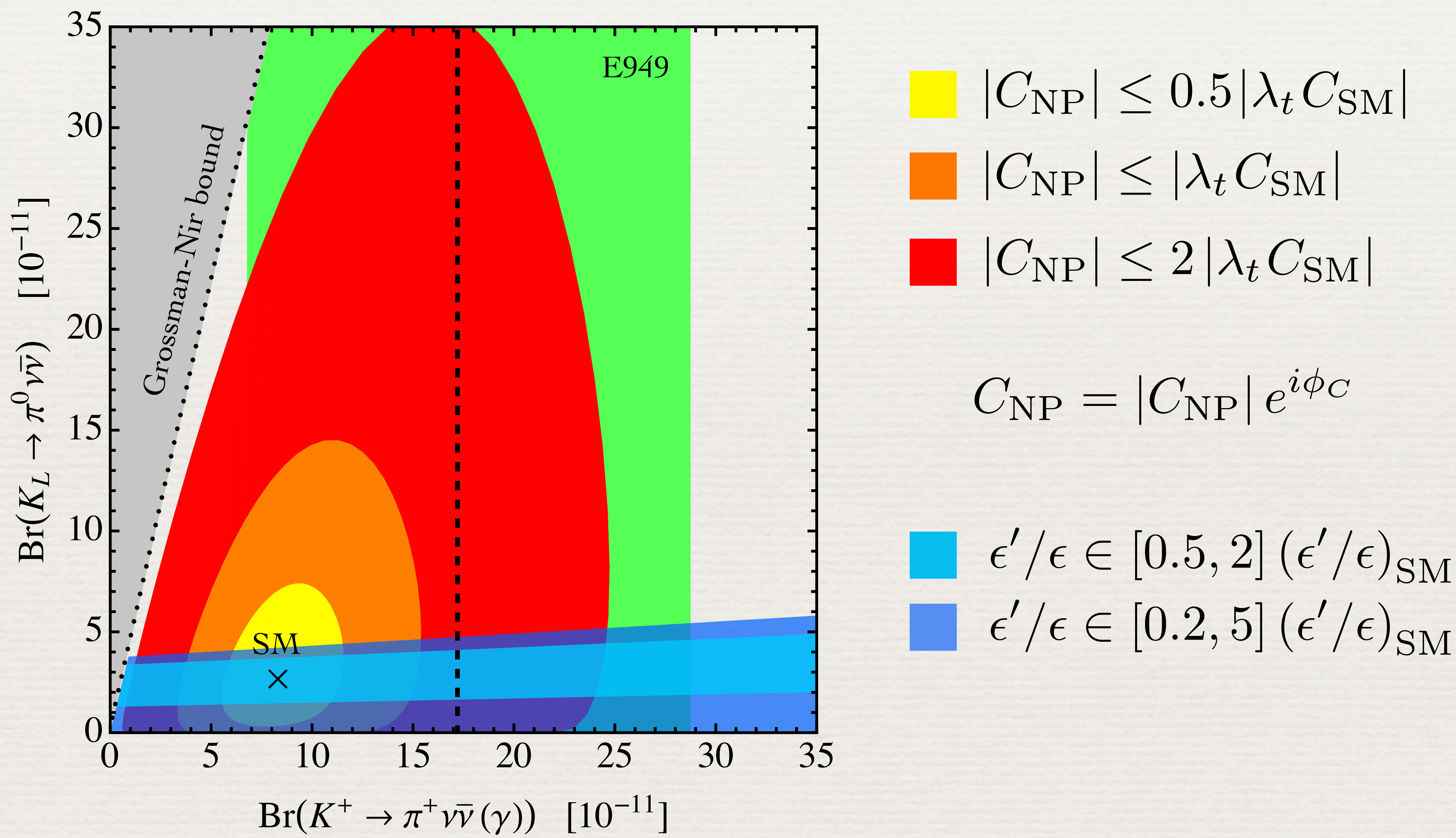
[D. Straub, arXiv:1012.3893 (CKM 2010)]

- **Spectacular deviations from the Standard Model are possible in many new physics scenarios**

- Correlations between the two channels can help distinguish between models

ε′/ε Strikes Back

[U. Haisch, 2012 Project X Physics Study]



[see S. Jäger, talk at NA62 Physics Handbook Workshop; M. Bauer et al., arXiv:0912.1625 [hep-ph]]

- With the anticipated lattice-QCD improvements from ongoing K→ππ calculation by RBC/UKQCD, combining the pattern of results $\varepsilon'_K/\varepsilon_K$ with K→πνν decays can further distinguish between new-physics scenarios [Buras *et al.*, Nucl.Phys. B566 (2000)]

Lattice efforts on a_μ^{HVP}

- Several independent efforts ongoing (plus additional ones without quotable results…):

Collaboration	N_f	Fermion action	$a_\mu^{HVP} \times 10^{10}$
HPQCD	2+1+1	HISQ	strange: $53.41(59)_{tot}$
			charm: $14.42(39)_{tot}$
ETMC	2+1+1	twisted-mass	$674(21)_{stat}(18)_{sys}$
Aubin & Blum	2+1	Asqtad staggered	$713(15)_{stat}(31)_{\chi PT}(??)_{other}$
Edinburgh	2+1	domain-wall	$641(33)_{stat}(32)_{sys}$
ETMC	2	twisted-mass	$572(16)_{tot}$
Mainz	2	$\mathcal{O}(a)$ improved Wilson	$618(64)_{tot}$

- Most use the same general approach

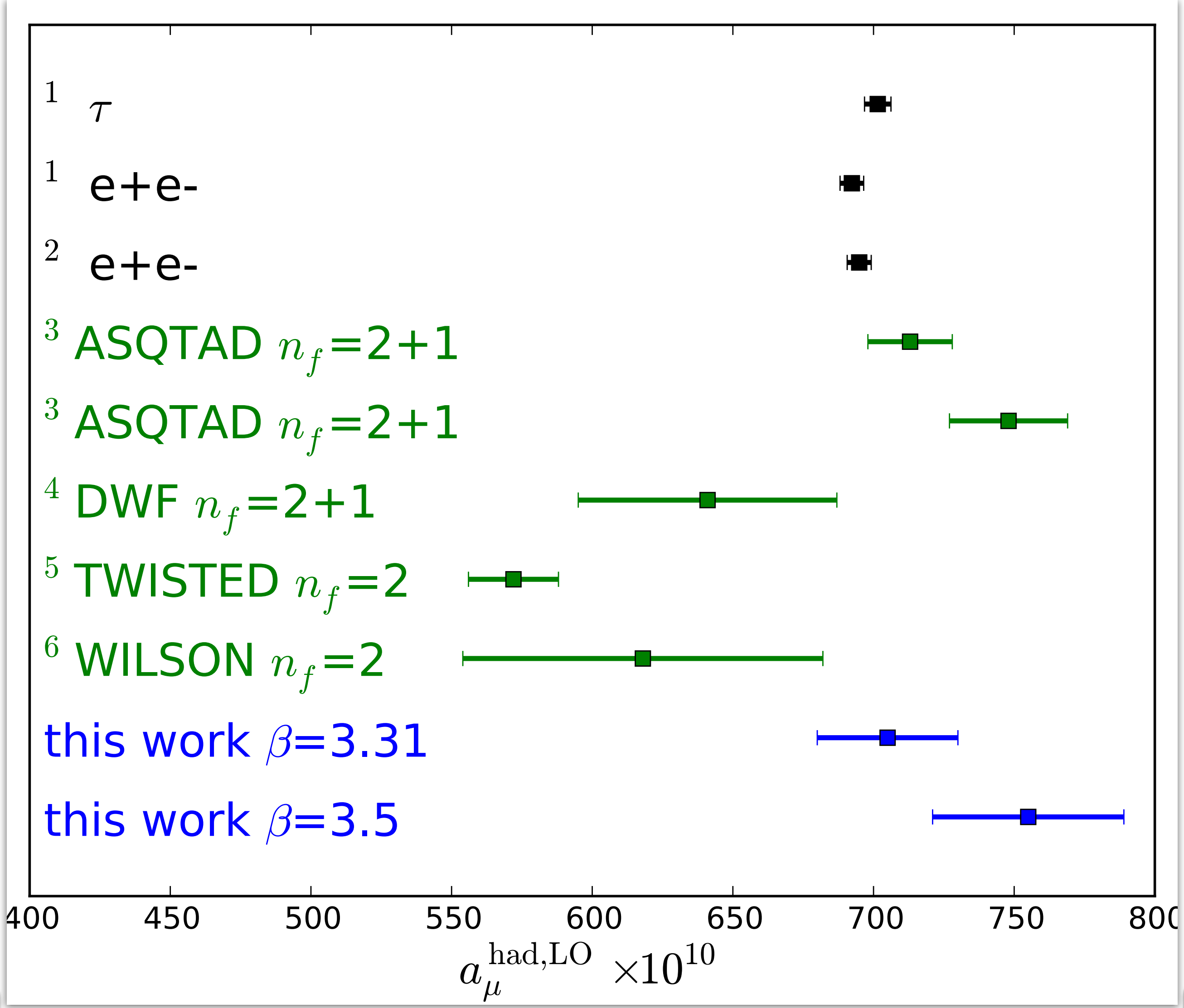
[1] Chakraborty *et al.*, JHEP 1402 (2014) 099
[2] Feng *et al.*, JHEP 1402 (2014) 099
[3] Aubin & Blum, PRD 75 (2007) 114502
[4] Boyle *et al.*, PRD 85 (2012) 074504
[5] Feng *et al.*, PRL 107 (2011) 081802
[6] Della Morte *et al.*, JHEP 1203 (2012) 055

Lattice efforts on a_μ^{HVP}

- Several independent efforts ongoing (plus additional ones without quotable results…):

Collaboration	N_f	Fermi
HPQCD	2+1+1	H
ETMC	2+1+1	twist
Aubin & Blum	2+1	Asqtad
Edinburgh	2+1	dom
ETMC	2	twist
Mainz	2	$\mathcal{O}(a)$ impr

[BMW,1311.4446]



- Most use the same general approach
- Errors typically in the 5–10% percent range, and (mostly) neglect quark-disconnected contributions

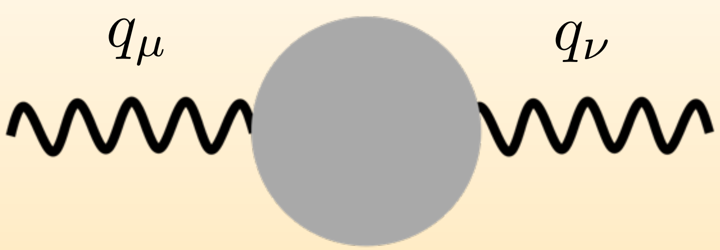
Standard lattice method for a_μ^{HVP}

[Blum, Phys.Rev.Lett. 91 (2003) 052001]

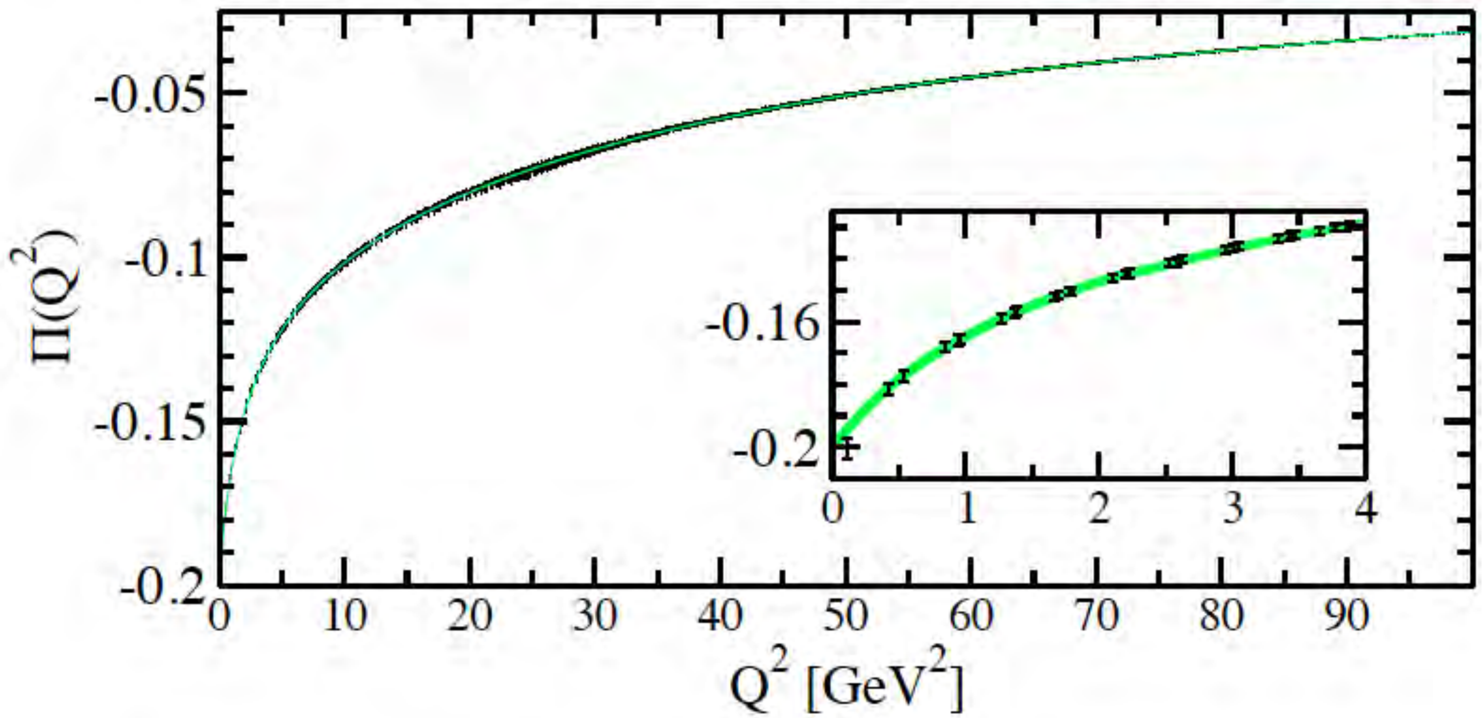
- Calculate a_μ^{HVP} directly from Euclidean space vacuum polarization function
- $\Pi(Q^2)$ a simple correlation function of two electromagnetic currents
- In Euclidean space, $\Pi(Q^2)$ has a **smooth Q^2 dependence with no resonance structure**

$$a_\mu^{\mathrm{HVP(LO)}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 f(Q^2)\left[\Pi(Q^2) - \Pi(0)\right]$$

$$i\Pi_{\mu\nu}(q^2) =$$



[plot from Dru Renner]

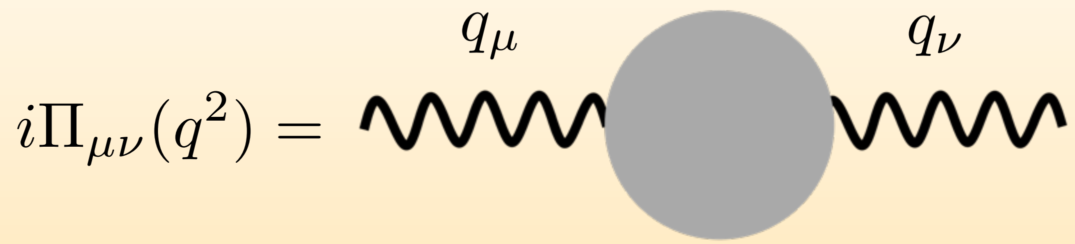


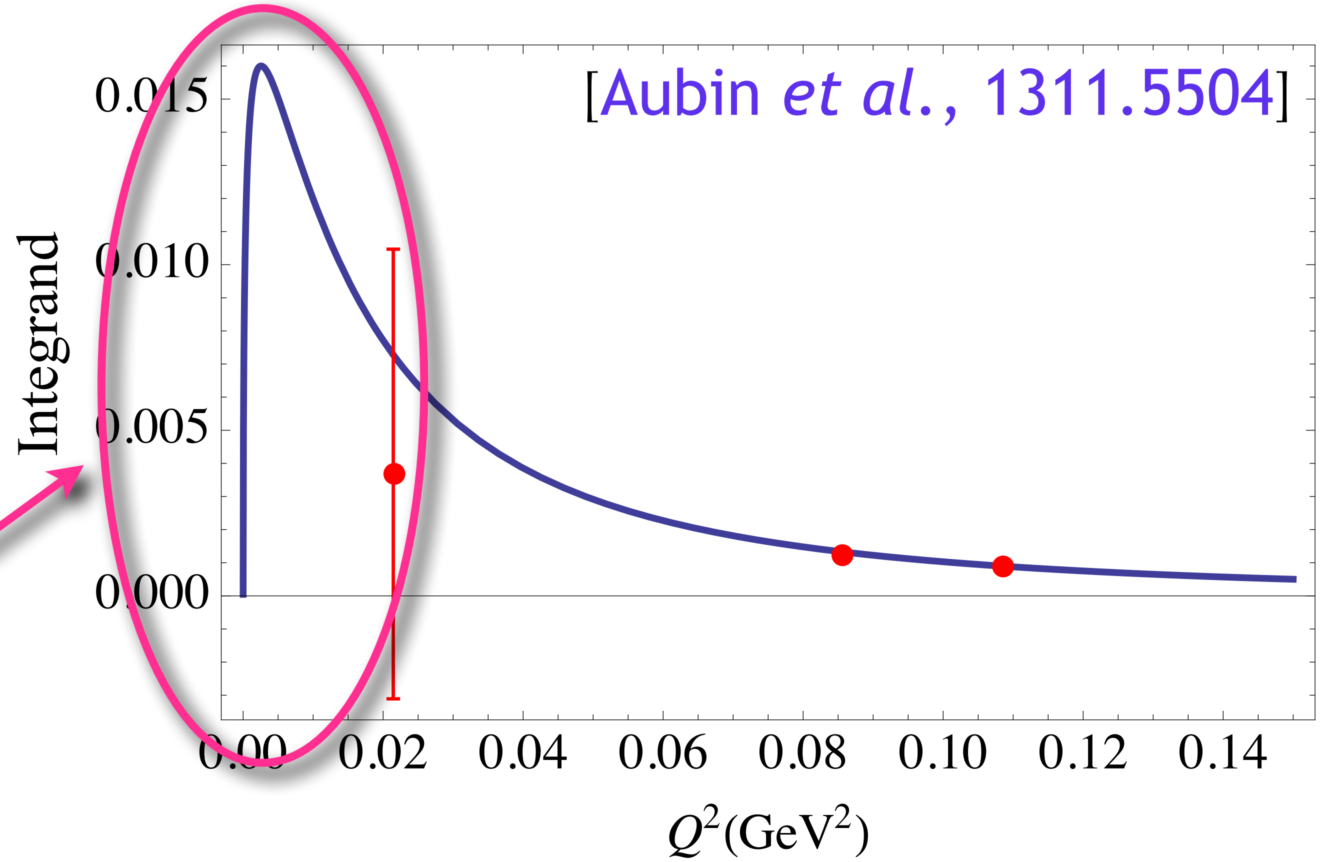
Standard lattice method for a_μ^{HVP}

[Blum, Phys.Rev.Lett. 91 (2003) 052001]

- Calculate a_μ^{HVP} directly from Euclidean space vacuum polarization function
- $\Pi(Q^2)$ a simple correlation function of two electromagnetic currents
- In Euclidean space, $\Pi(Q^2)$ has a smooth Q^2 dependence with no resonance structure
- Integrand $f(Q^2)[\Pi(Q^2)-\Pi(0)]$, however, peaks around $Q^2 \approx (m_\mu/2)^2$, where lattice data is sparse and noisy ➜ need precise determination of $\Pi(Q^2)$ in this region to obtain precise result for a_μ^{HVP}

$$a_\mu^{\mathrm{HVP(LO)}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 f(Q^2) \left[\Pi(Q^2) - \Pi(0)\right]$$





Expected precision of SM Higgs couplings

- Uncertainties in m_c, m_b, and α_s have led some to conclude that (sub)percent measurements of Higgs properties may never be useful [Almeida *et al.*, PRD89 (2014) 033006]

- In fact, however, **lattice calculations have already determined m_c, m_b, and α_s more precisely than is currently being assumed in discussions of Higgs decay channels**

	Higgs X-section Working Group	PDG	non-lattice	**Lattice (2013)**	Lattice (2018)
$\Delta\alpha_s$	0.002	0.0007	0.0012	**0.0006**	0.0004
Δm_c (GeV)	0.03	0.025	0.013	**0.006**	0.004
Δm_b (GeV)	0.06	0.03	0.016 [21]	**0.023**	0.011

[Snowmass Higgs WG Report, 1310.8361]

- Lepage, Mackenzie, & Peskin [1404.0319] use toy Monte-Carlo calculations to estimate how much the uncertainties in m_c, m_b, and α_s from lattice QCD could be decreased over the next decade given the anticipated ~100x growth in computing resources
  - Show that **reducing lattice spacing to 0.023 fm with current analysis methods sufficient to bring parametric errors in SM Higgs couplings to below errors expected from full ILC**

Computing resources

Year	ANL LCF (BG/P + BG/Q core-hours)	ORNL LCF (Cray core-hours)	Dedicated Capacity Hardware (core-hours)
2010	187M	53.6M	125M
2011	182M	49.8M	205M
2012	143M	77.9M	330M
2013	290M (allocated)	140M (allocated)	971M (planned)

- Projected resources assume sustained level of funding for dedicated hardware and constant percentage of leadership-class facilities

Year	Leadership Class (Tflop/sec-yrs)	Dedicated Capacity Hardware (Tflop/sec-yrs)
2015	430	325
2016	680	520
2017	1080	800
2018	1715	1275
2019	2720	1900

[Snowmass LFT WG report, arXiv: 1310.6087]