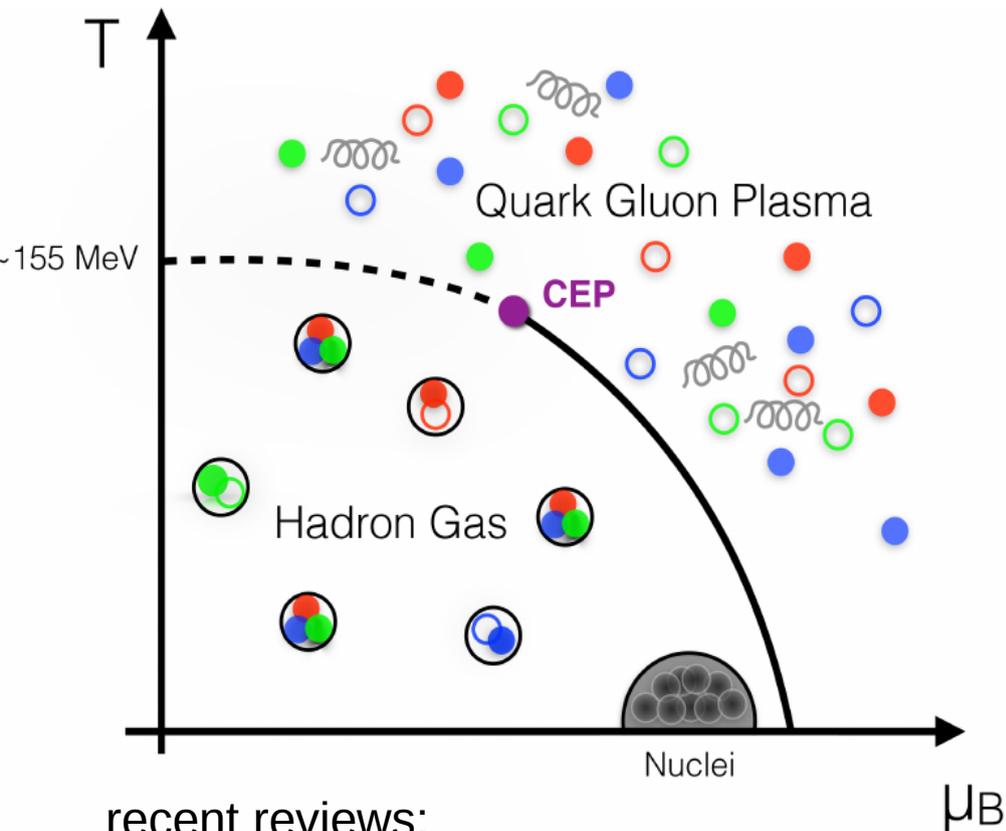


# QCD thermodynamics

Frithjof Karsch, BNL/Bielefeld



recent reviews:

H.-T. Ding, F. Karsch, S. Mukherjee,  
Thermodynamics of strong-interaction  
matter from Lattice QCD [arXiv:1504.05274](https://arxiv.org/abs/1504.05274)

U. Heinz et al., Exploring the properties of the phases of QCD matter  
- research opportunities and priorities for the next decade, [arXiv:1501.06477](https://arxiv.org/abs/1501.06477)

## Key Questions

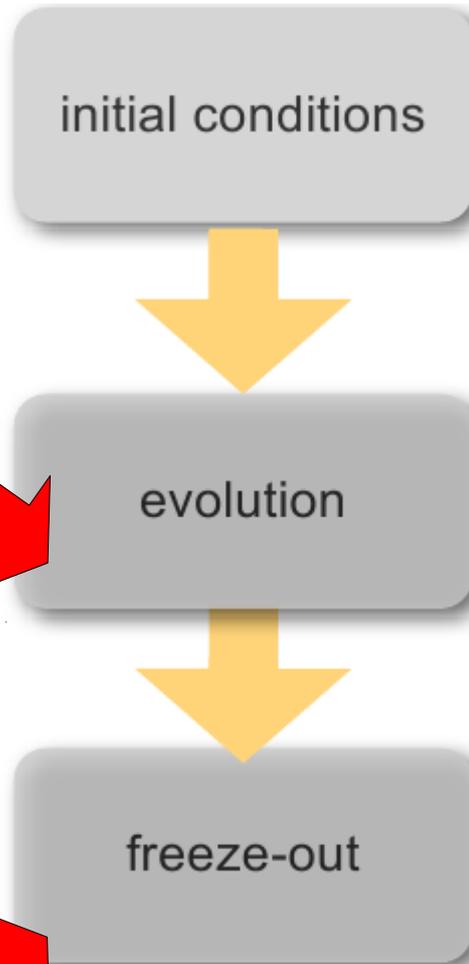
### NSAC Long Range Plan 2007

- What are the phases of strongly interacting matter, and what role do they play in the cosmos?
- What does QCD predict for the properties of strongly interacting matter?
- What governs the transition of quarks and gluons into pions and nucleons?



### NSAC Long Range Plan 2015

# Exploring the QCD phase diagram



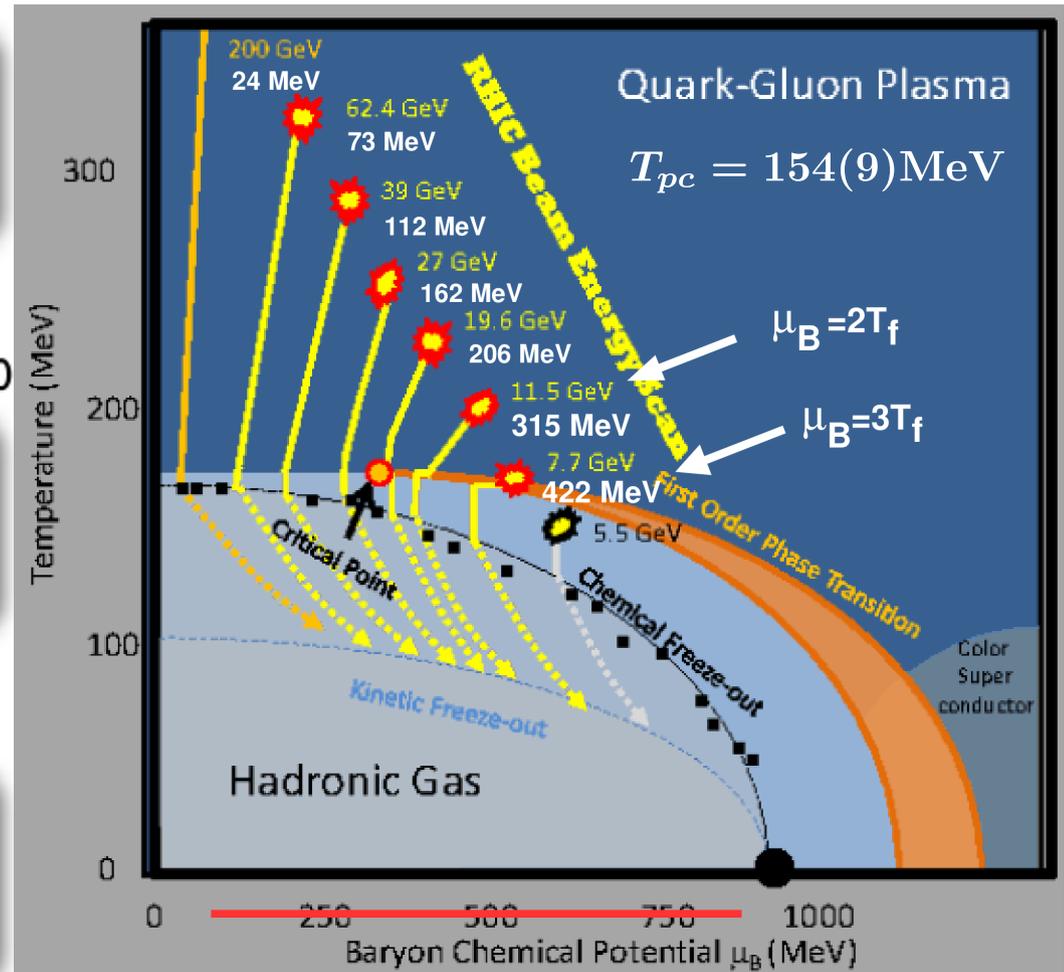
controlled by the QCD equation of state,  $T_c$

**expectation:** freeze-out close to QCD transition line

**observable consequences:**

freeze-out pattern of mesons and baryons, controlled by

$T_f, \mu_B, \mu_S$



- LHC: establish contact with the QCD PHASE transition
- RHIC: locate/provide evidence for the QCD critical point

# Outline

- QCD thermodynamics projects in 2014/15
- Equation of state and transition temperature
  - continuum extrapolated EoS at  $\mu_B = 0$ ;
  - insight into the role of topology close to  $T_c$
  - **EoS at non-zero net-baryon density and the BES@RHIC**
- Charge fluctuations and the RHIC search for the critical point
  - evidence for many new strange and charmed baryons
  - using electric charge fluctuations to search for the critical point
  - **characterization of freeze-out conditions using conserved charge fluctuations**

# Thermodynamics projects in 2014/15

## Equation of state and the transition temperature:

- A. Bazavov et al. (hotQCD), *Equation of State in (2+1)-flavor QCD*, *Phys. Rev. D90 (2014) 094503*
- V. Dick et al., *Microscopic Origin of UA(1) Symmetry Violation in the High Temperature Phase of QCD*, *Phys. Rev. D91 (2015) 094504*

## Fluctuation of conserved charges using Taylor expansions:

- A. Bazavov et al.(BNL-Bielefeld), *Additional strange hadrons from QCD thermodynamics and strangeness freeze-out in heavy ion collisions*, *Phys. Rev. Lett. 113 (2014) 072001*
- A. Bazavov et al.(BNL-Bielefeld), *The melting and abundance of open charm*, *Phys. Lett. B737 (2014) 210*.

## In-medium meson properties

- A. Bazavov et al., *In-medium modifications of open and hidden strange-charm mesons from spatial correlation functions*, *Phys. Rev. D91 (2015) 054503*
- Y. Maezawa et al., *Meson screening masses at finite temperature with Highly Improved Staggered Quarks*, arXiv:1312.4375



USQCD  
projects  
2014/15

# Lattice meets Experiment in 2014/15

## Workshops/Programs on Heavy Ion Physics:

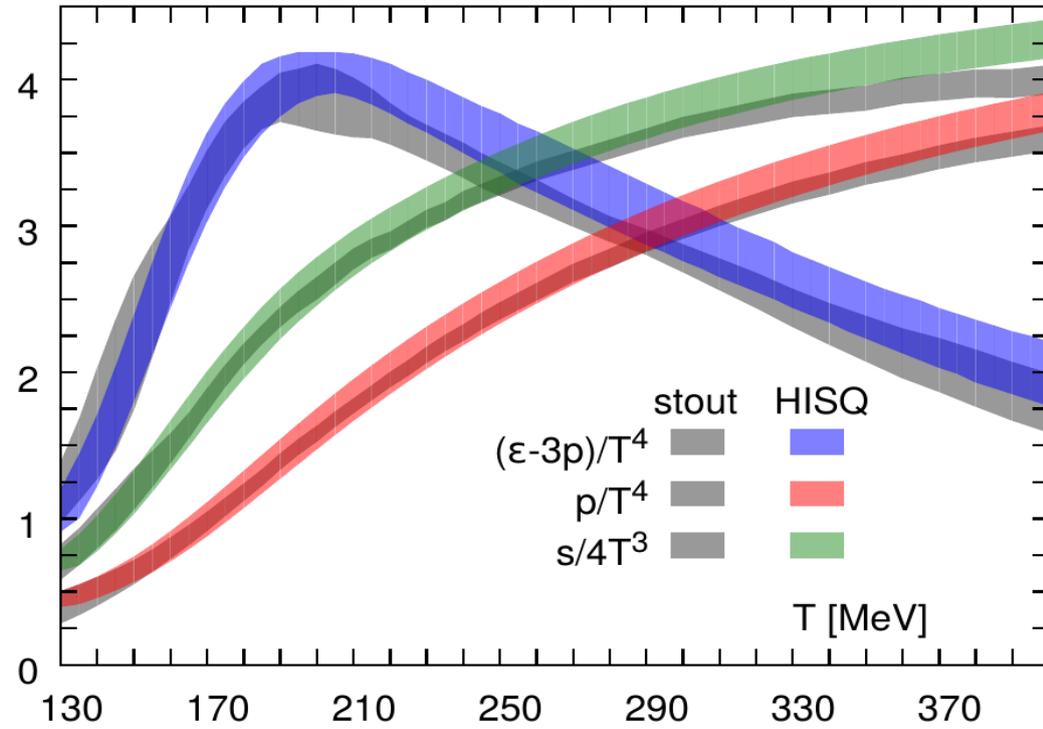
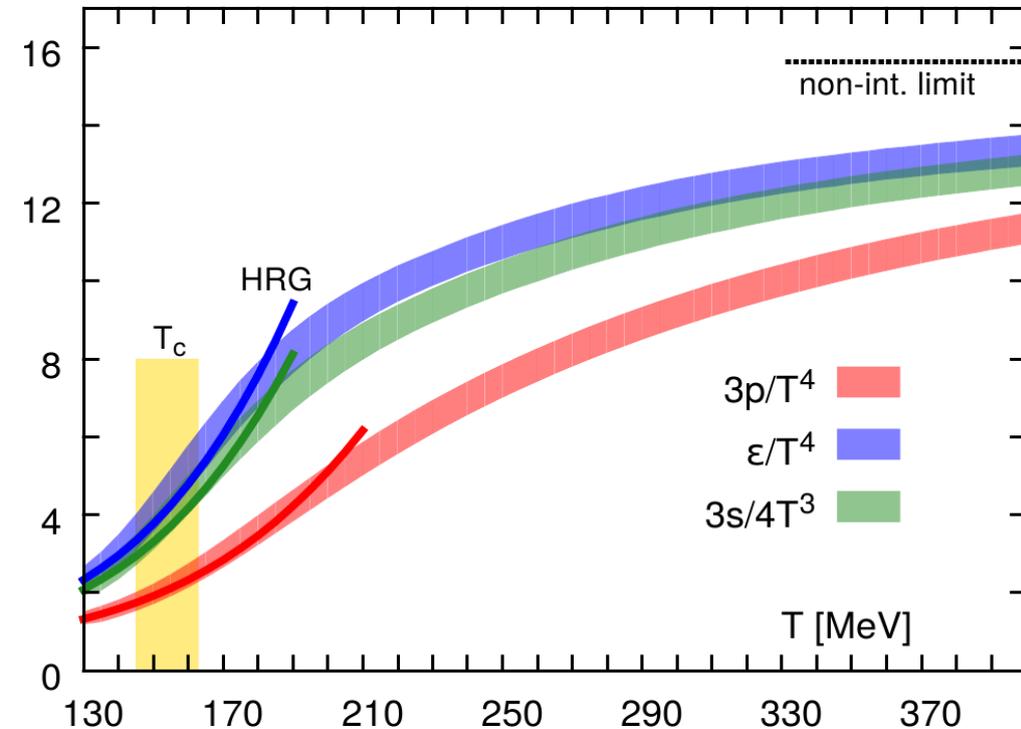
- Theory and Modeling for the Beam Energy Scan: from Exploration to Discovery, RIKEN/BNL Feb. 26-27, 2015, organizers: M. Lisa, S. Mukherjee, H. Petersen, M. Stephanov, P. Sorensen
- RHIC users' meeting: Beam Energy Scan workshop, BNL, June 10, 2015, organizers: S. Mukherjee, D. McDonald, J. Mitchell

## International Conference and Workshop Series:

- Extreme QCD  
2014: BNL&Stony Brook, June 19-21, 2014, organizers: F.Karsch, D.Kharzeev, J.Verbaarschot  
2015: CCNU Wuhan, China, Sept. 21-23, 2015, co-organizers: H.-T. Ding, F. Karsch.....
- Critical Point and Onset of Deconfinement  
2014: Bielefeld, Germany, Nov. 17-21, 2014, organizers: O. Kaczmarek, F. Karsch et al.

# Equation of state of (2+1)-flavor QCD

pressure, entropy & energy density



A. Bazavov et al. (hotQCD),  
Phys. Rev. D90 (2014) 094503

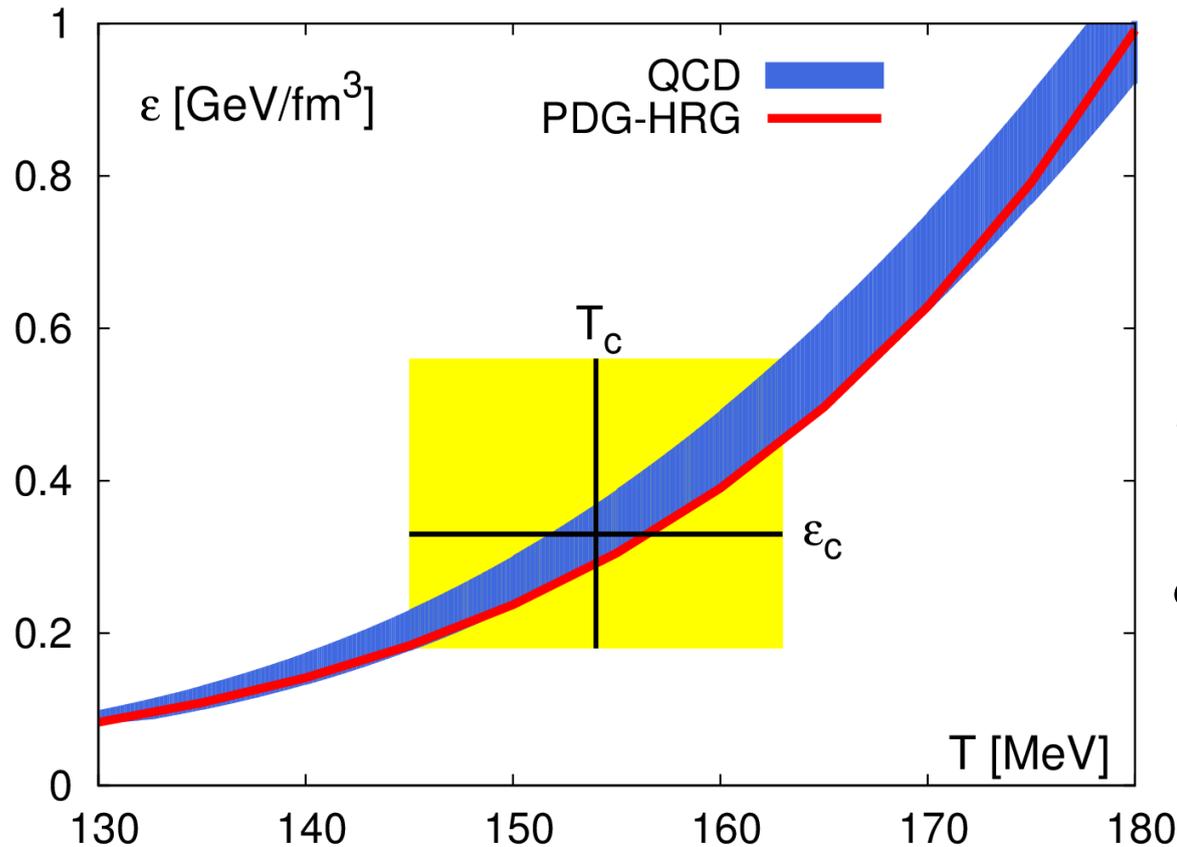
$$\mu_B = \mu_S = \mu_Q = 0$$

- improves over earlier hotQCD calculations:  
A. Bazavov et al., Phys. Rev. D80, 014504 (2009)
- consistent with results from Budapest-Wuppertal (stout): S. Borsanyi et al., PL B730, 99 (2014)

- up to the crossover region the QCD EoS agrees quite well with hadron resonance gas (HRG) model calculations; **However**, QCD results are systematically above HRG

# Crossover transition parameters

PDG: Particle Data Group hadron spectrum



$$T_c = (154 \pm 9) \text{ MeV}$$

$$\epsilon_c = (0.34 \pm 0.16) \text{ GeV}/\text{fm}^3$$

compare with:

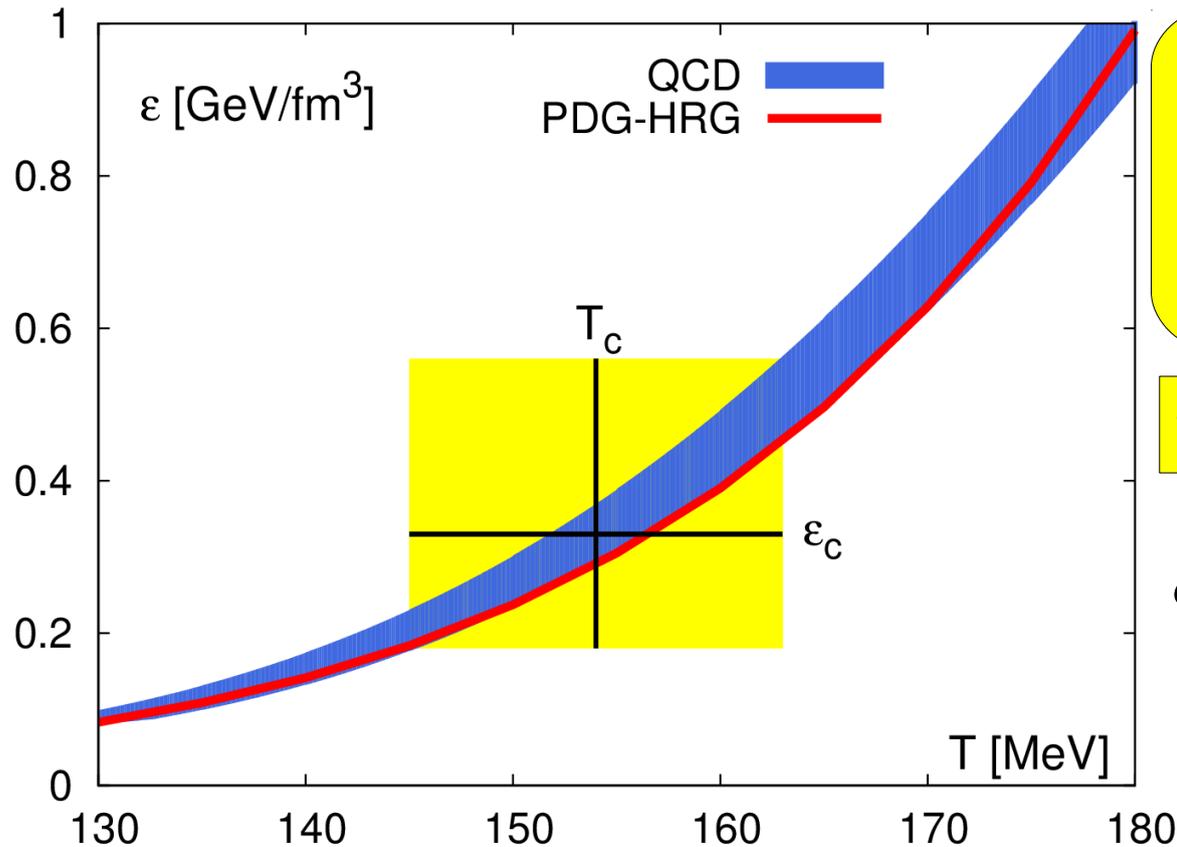
$$\epsilon^{\text{nucl. mat.}} \simeq 150 \text{ MeV}/\text{fm}^3$$

$$\epsilon^{\text{nucleon}} \simeq 450 \text{ MeV}/\text{fm}^3$$

A. Bazavov et al. (hotQCD),  
Phys. Rev. D90 (2014) 094503

# Crossover transition parameters

PDG: Particle Data Group hadron spectrum



#1 topcited hep-lat paper in 2014

A. Bazavov et al. (hotQCD), *The chiral and deconfining aspects of the QCD transition*, Phys. Rev. D85, 054503 (2012) (369 cit.)

$$T_c = (154 \pm 9) \text{ MeV}$$

$$\epsilon_c = (0.34 \pm 0.16) \text{ GeV}/\text{fm}^3$$

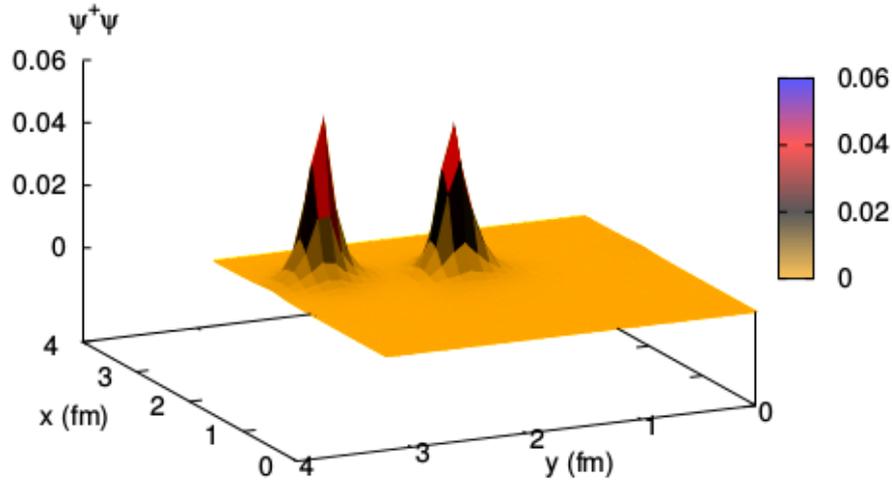
compare with:

$$\epsilon^{\text{nucl. mat.}} \simeq 150 \text{ MeV}/\text{fm}^3$$

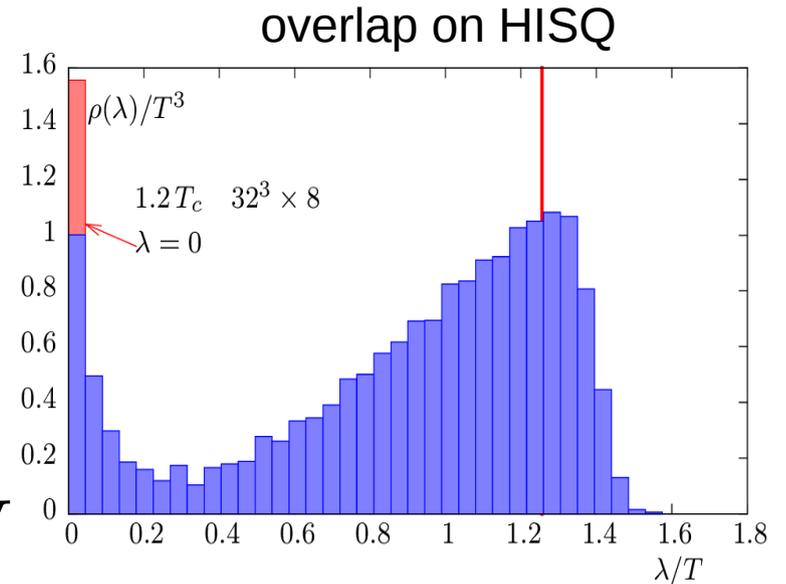
$$\epsilon^{\text{nucleon}} \simeq 450 \text{ MeV}/\text{fm}^3$$

A. Bazavov et al. (hotQCD),  
Phys. Rev. D90 (2014) 094503

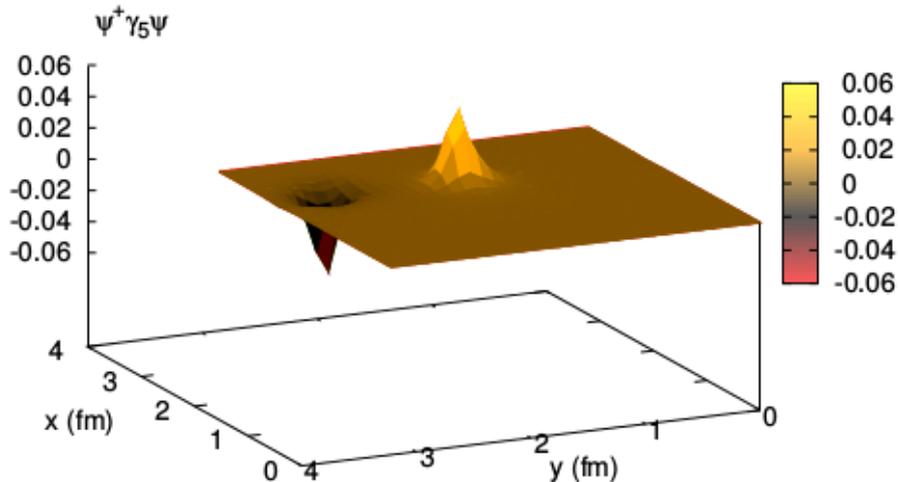
# Chiral Transition



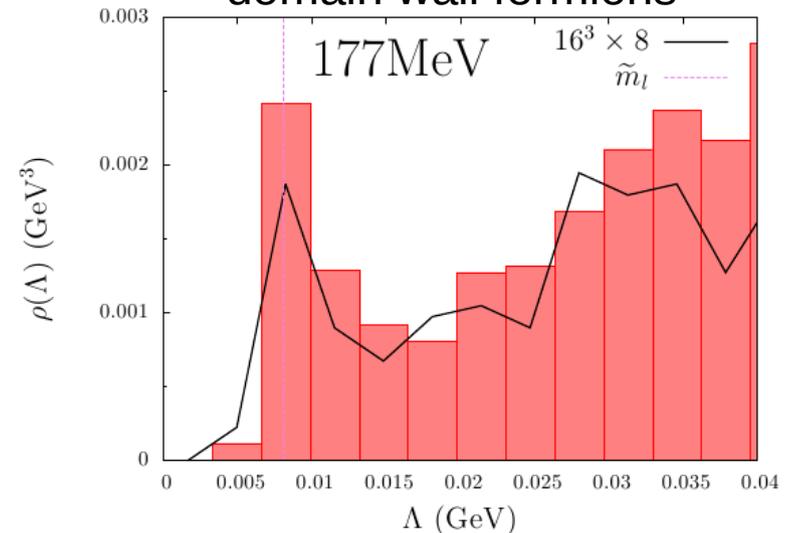
$$T_c = (154 \pm 9) \text{ MeV}$$



V. Dick et al, arXiv:1502.06190



domain wall fermions



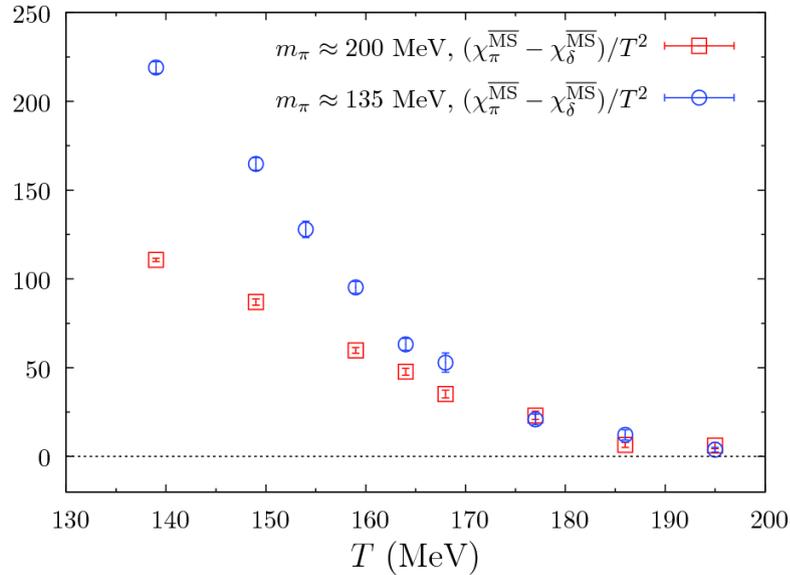
- near zero modes have well localized eigenstates
- consistent with instanton gas model
- UA(1) remains broken also above  $T_c$

M.I. Buchoff et al (hotQCD), PR D89, 094503 (2014)

# Chiral Transition

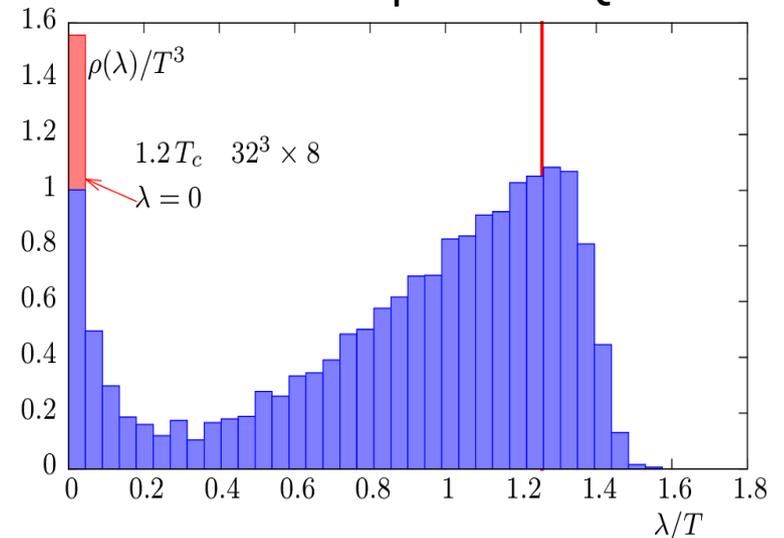
UA(1) remains broken at  $T_c$

T. Bhattacharya (hotQCD) PRL 113, 082001 (2014)



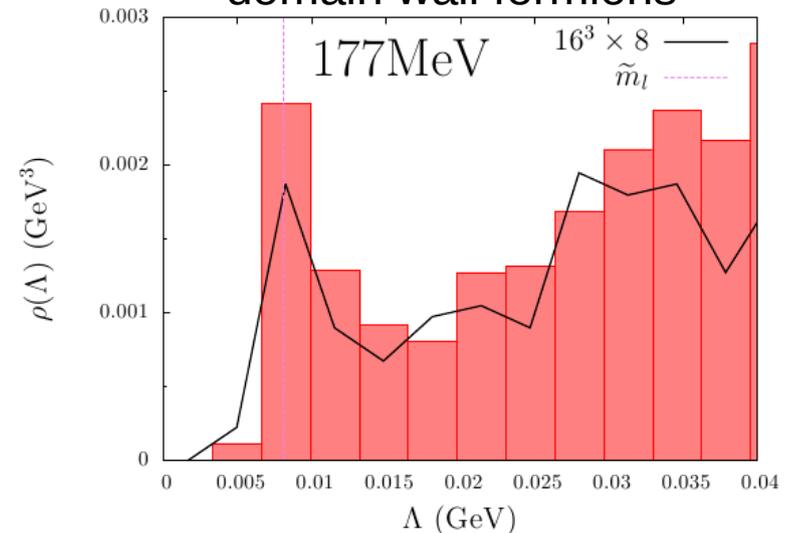
fate of UA(1) breaking and thus also the chiral phase transition in the massless (chiral) limit of QCD remains to be an unsolved textbook problem of QCD

overlap on HISQ



V. Dick et al, arXiv:1502.06190

domain wall fermions



M.I. Buchoff et al (hotQCD), PR D89, 094503 (2014)

Explore the **structure of matter** close to the QCD transition temperature using **fluctuations of conserved charges**

- probing the response of a thermal medium to an external field, i.e. variation of one of its external control parameters:  $T$ ,  $\mu$ ,  $m_q$

(generalized) response functions == (generalized) susceptibilities

pressure: 
$$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(V, T, \mu_{B,Q,S}, m_{u,d,s})$$

particle number density

$$\chi_1^q = \frac{1}{VT^3} \frac{\partial \ln Z}{\partial \mu_q/T}$$

$$\langle \delta N_X \rangle \equiv \langle N_X - N_{\bar{X}} \rangle$$

quark number susceptibility

$$\chi_2^q = \frac{1}{VT^3} \frac{\partial^2 \ln Z}{\partial (\mu_q/T)^2}$$

$$\sigma_X^2 \equiv \langle (\delta N_X)^2 \rangle - \langle \delta N_X \rangle^2$$

4<sup>th</sup> order cumulant

$$\chi_4^q = \frac{1}{VT^3} \frac{\partial^4 \ln Z}{\partial (\mu_q/T)^4}$$

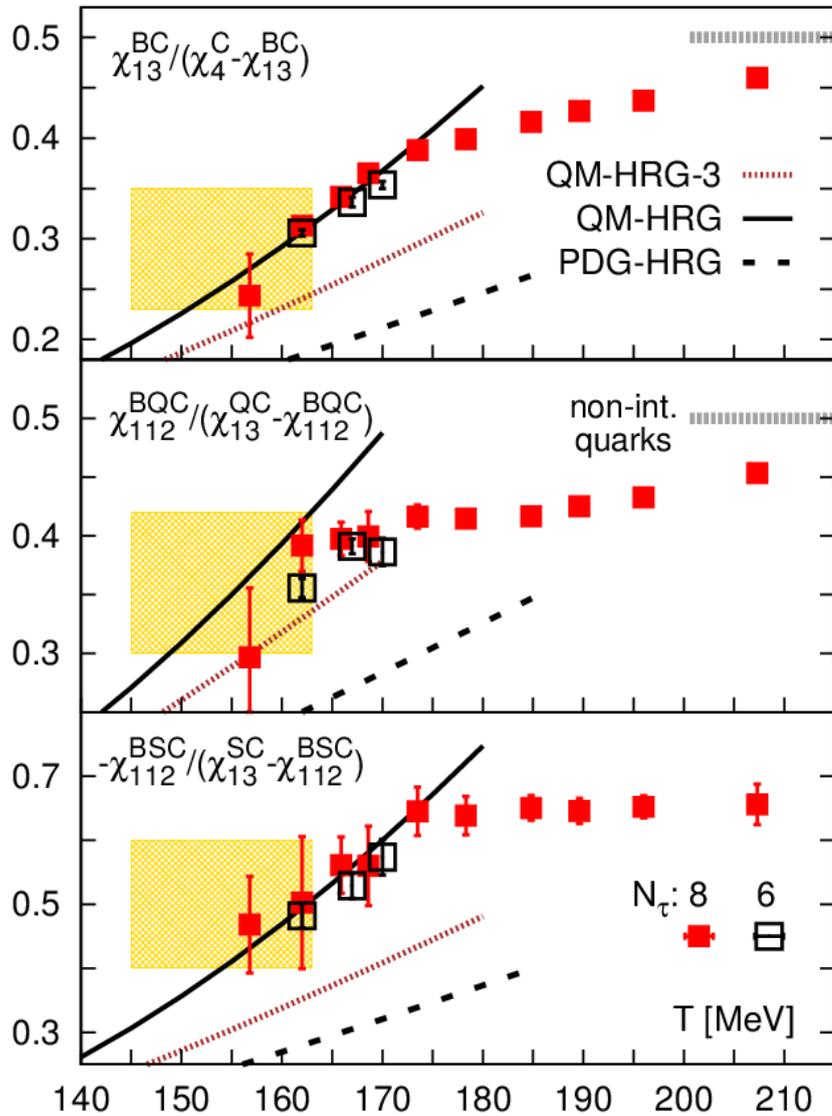
$$\kappa_X \sigma_X^4 \equiv \langle (\delta N_X)^4 \rangle - 3\sigma_X^4$$

generalized quark number susceptibilities:

$$\chi_{ijk}^{BQS} = \frac{\partial^{i+j+k} p/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}$$

$$\hat{\mu}_X \equiv \mu_X/T$$

# Evidence for many charmed baryons in thermodynamics



close to  $T_c$  charmed baryon fluctuations are about 50% larger than expected in a HRG based on known charmed baryon resonances (PDG-HRG)

all charmed baryons/mesons

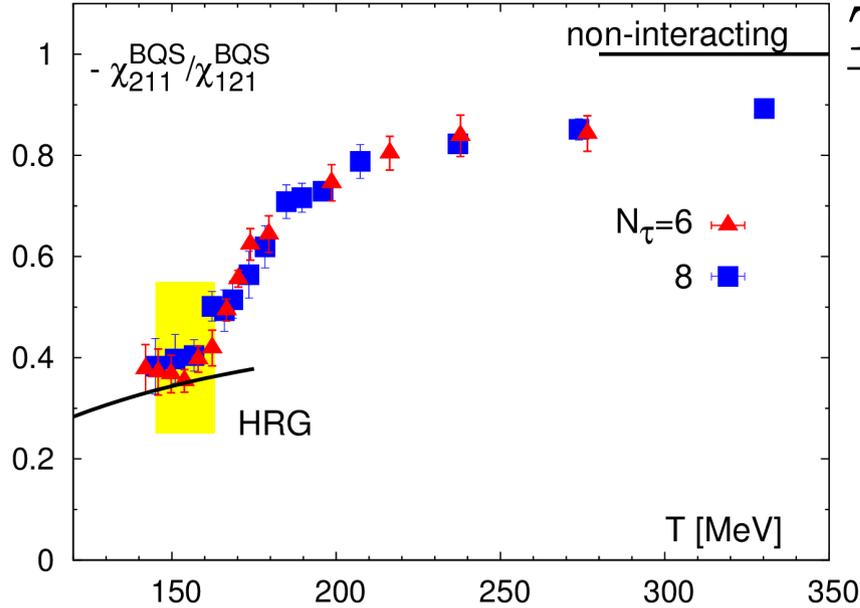
charged charmed baryons/mesons

strange charmed baryons/mesons

including resonances predicted in quark model calculations and observed in lattice QCD calculations allows for a HRG model (QM-HRG) description of lattice QCD results on conserved charge fluctuations and correlations

A. Bazavov et al., Phys.Lett. B737 (2014) 210

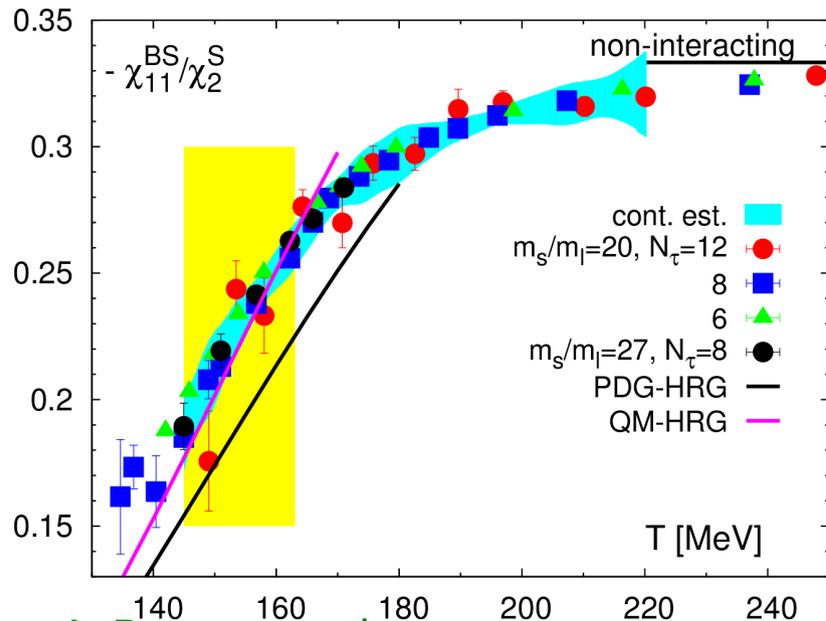
# Analyzing **strangeness carrier** with higher order cumulants



$T > T_c$  : Is strangeness carried by **quasi-particles** with quantum numbers of **quarks** ?

$$\frac{\chi_{211}^{BQS}}{\chi_{121}^{BQS}} = \begin{cases} -1, & T \rightarrow \infty \\ \text{HRG}, & T \leq T_c \end{cases}$$

↑  
ratio of properly weighted charged, strange baryons



$T < T_c$  : Who carries **strangeness in the HRG**?

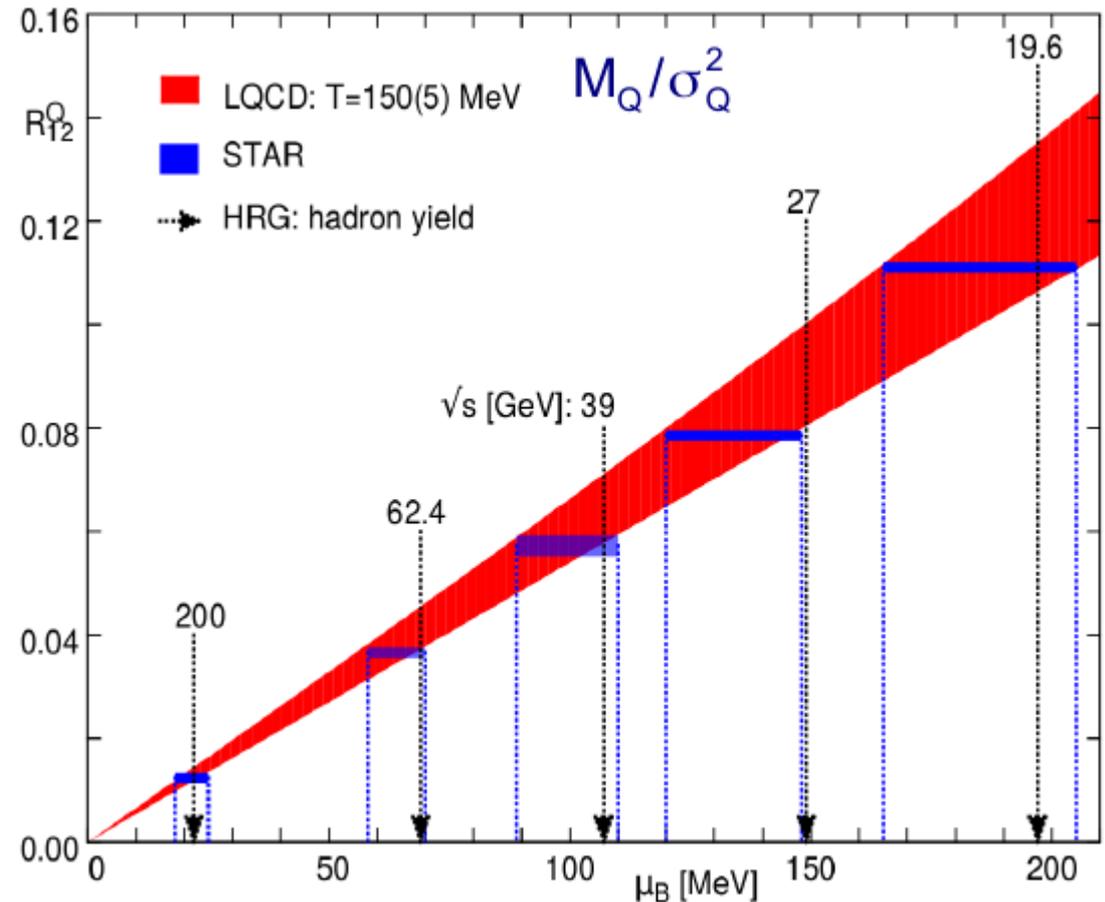
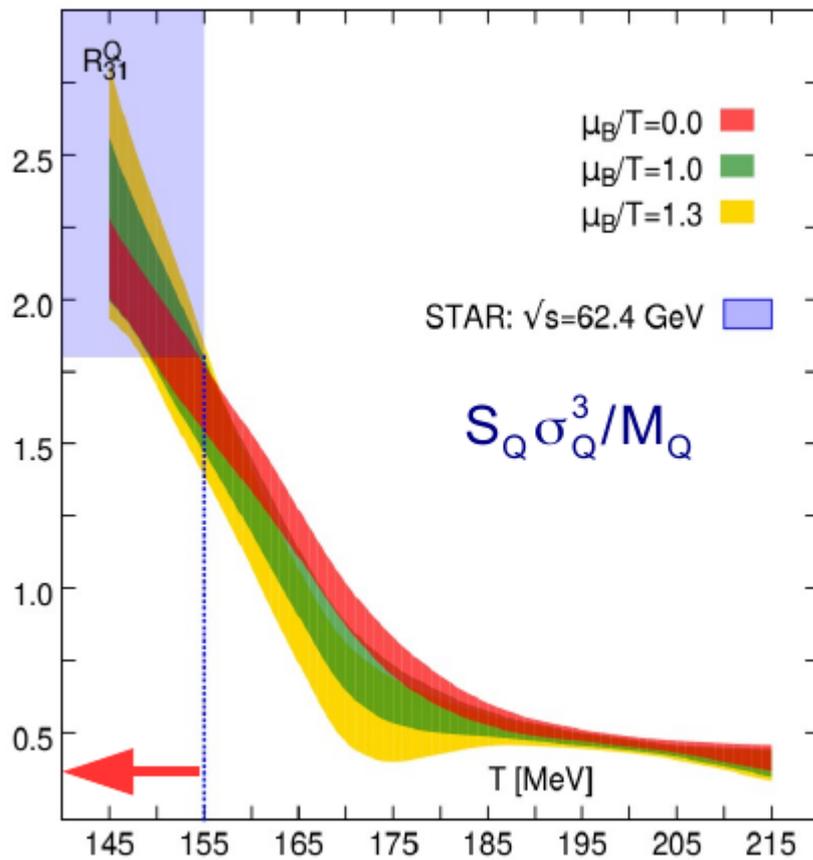
← significant deviations from a HRG based on known resonances (PDG-HRG)

**Enhanced strange baryon fluctuations:**  
indications for more strange hadron resonances as predicted in quark model calculations (QM-HRG)

A. Bazavov et al.,  
Phys. Rev. Lett. 113, 072001 (2014), arXiv:1404.6511

# Freeze-out parameter from conserved charge fluctuations

cumulant ratios of electric charge fluctuations



constrains freeze-out temperature

$$T_f \simeq (150 \pm 5) \text{ MeV}$$

determines freeze-out chemical potential

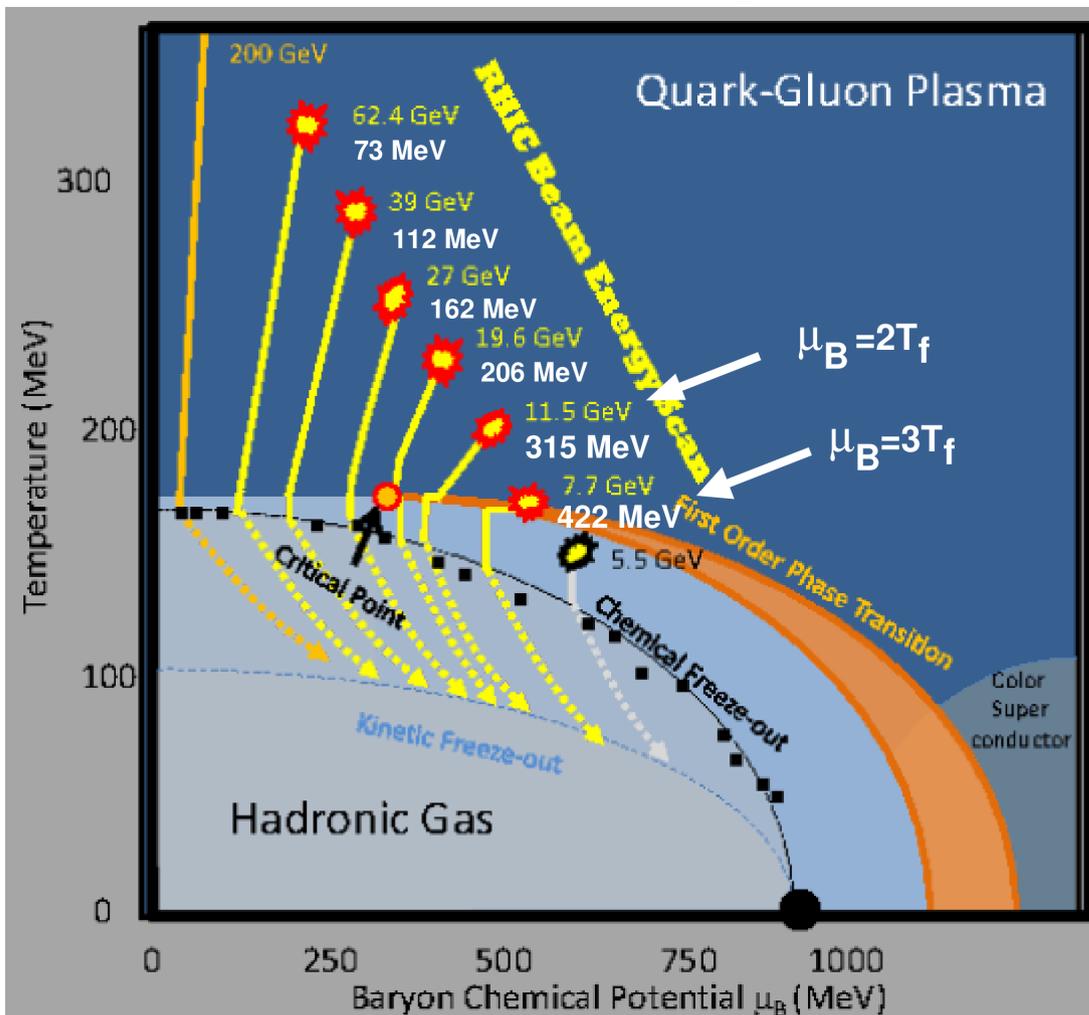
BI-BNL, PRL 109, 192302 (2012)

S. Mukherjee, M. Wagner, PoS CPOD2013 (2013) 039

# Exploring the QCD phase diagram with net charge fluctuations

RHIC beam energy scan:  $\sqrt{s} = (7.7 - 200)\text{GeV}/A$

**search for the critical point**



**preparing for BES-II: 2019/20**

want to understand properties of matter in the range

$$0.8 \leq T/T_c \leq 2$$

$$0 \leq \mu_B/T \leq 3$$

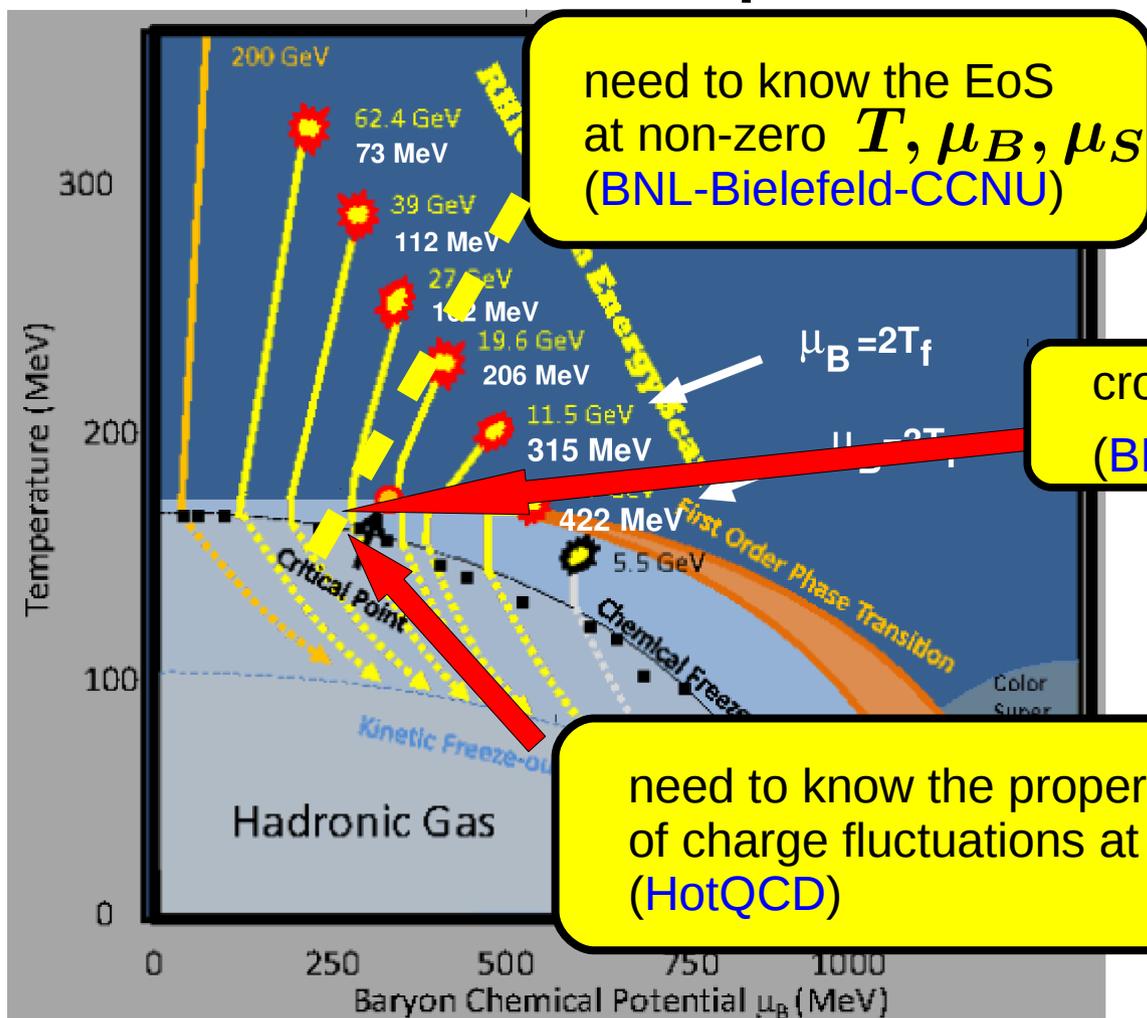
- equation of state
- crossover line
- critical point?
- freeze-out line
- transport properties
- in-medium hadron properties

# Exploring the QCD phase diagram with net charge fluctuations

RHIC beam energy scan:  $\sqrt{s} = (7.7 - 200) \text{ GeV} / A$

**search for the critical point**

**generalized susceptibilities**



$$\chi_n^q = \frac{\partial^n p / T^4}{\partial (\mu_q / T)^n}$$

$$q = B, Q, S$$

crossover line:  $T_c(\mu_B, \mu_S)$   
(BNL-Bielefeld-CCNU)

– coefficients in the Taylor expansion of the Equation of State

– higher order cumulants of net-charge (B,Q,S) fluctuations measured in the BES at RHIC

# Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

**Ambitious Goal: EoS in the entire parameter range accessible to the BES@RHIC**

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{i,j,k}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

the simplest case:  $\mu_S = \mu_Q = 0$

$$\frac{\Delta(T, \mu_B)}{T^4} = \frac{P(T, \mu_B) - P(T, 0)}{T^4} = \frac{\chi_2^B}{2} \left(\frac{\mu_B}{T}\right)^2 \left(1 + \frac{1}{12} \frac{\chi_4^B}{\chi_2^B} \left(\frac{\mu_B}{T}\right)^2\right)$$

variance of net-baryon  
number distribution

kurtosis\*variance

$$\kappa_B \sigma_B^2$$

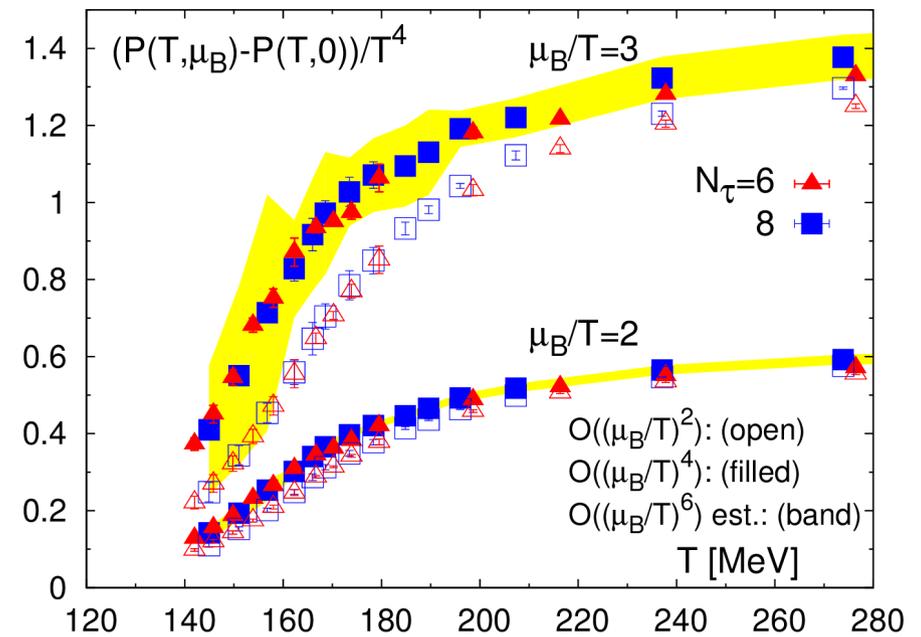
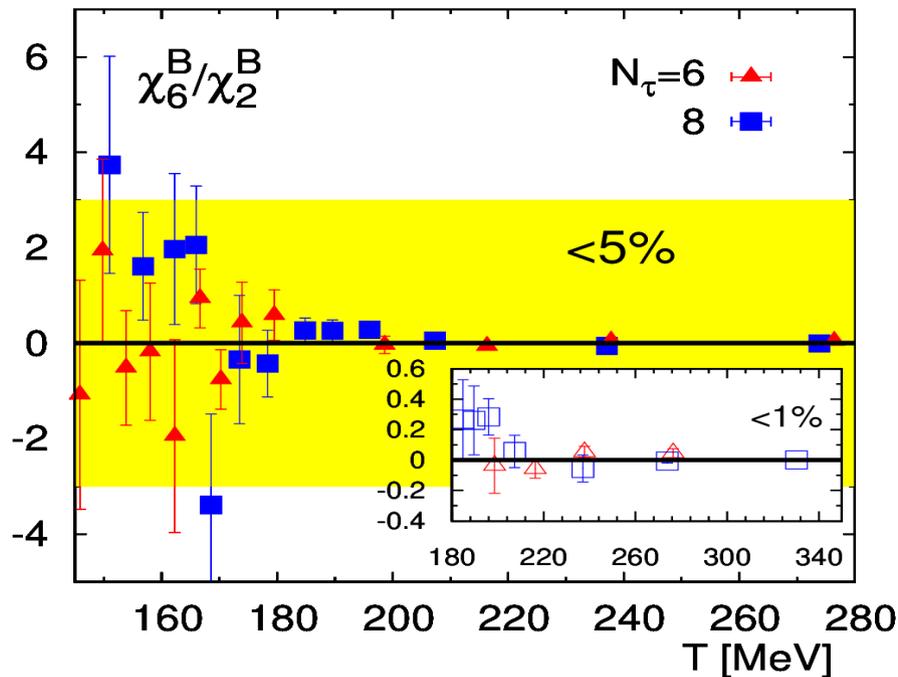
**work in progress:** report at Quark Matter 2014

P. Hegde (BNL-Bielefeld-CCNU), *The equation of state at  $\mathcal{O}(\mu_B^4)$*   
Nucl. Phys. A 931, 851 (2014)

# EoS at $\mu_B/T > 0$ : **Current status**

$$\frac{\Delta(T, \mu_B)}{T^4} = \frac{P(T, \mu_B) - P(T, 0)}{T^4} = \frac{\chi_2^B}{2} \left(\frac{\mu_B}{T}\right)^2 \left(1 + \frac{1}{12} \frac{\chi_4^B}{\chi_2^B} \left(\frac{\mu_B}{T}\right)^2\right)$$

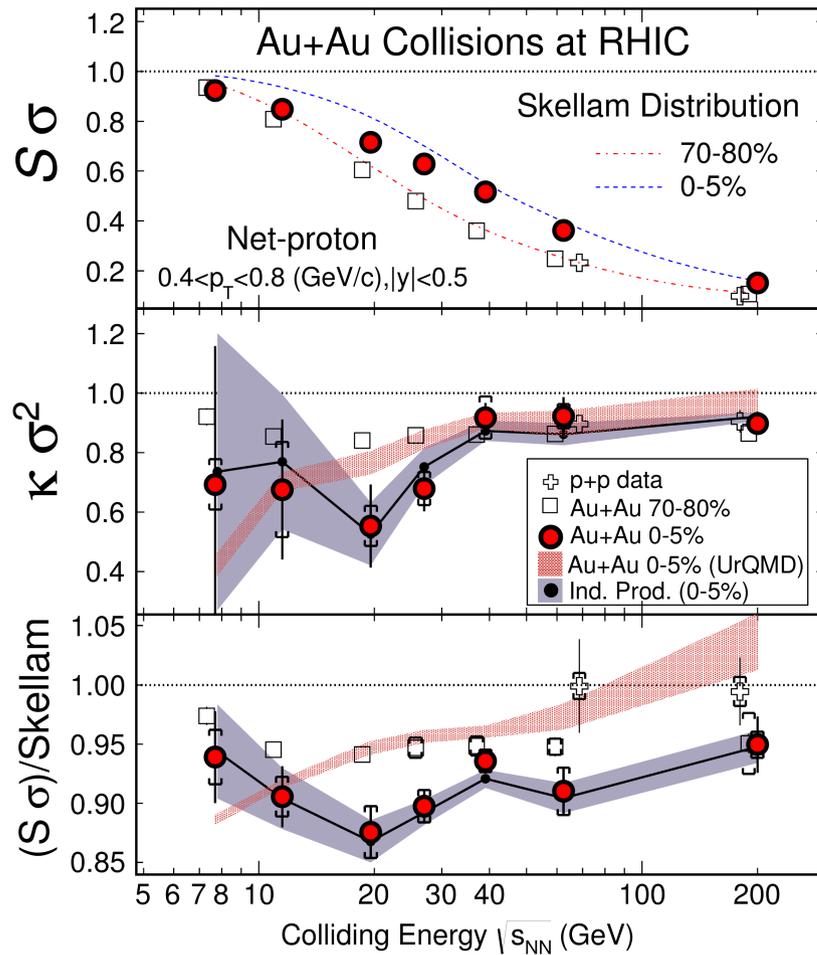
estimating the  $\mathcal{O}((\mu_B/T)^6)$  correction:  $\sim \frac{1}{720} \frac{\chi_6^B}{\chi_2^B} \left(\frac{\mu_B}{T}\right)^6$



The EoS is well controlled for  $\mu_B/T \leq 2$

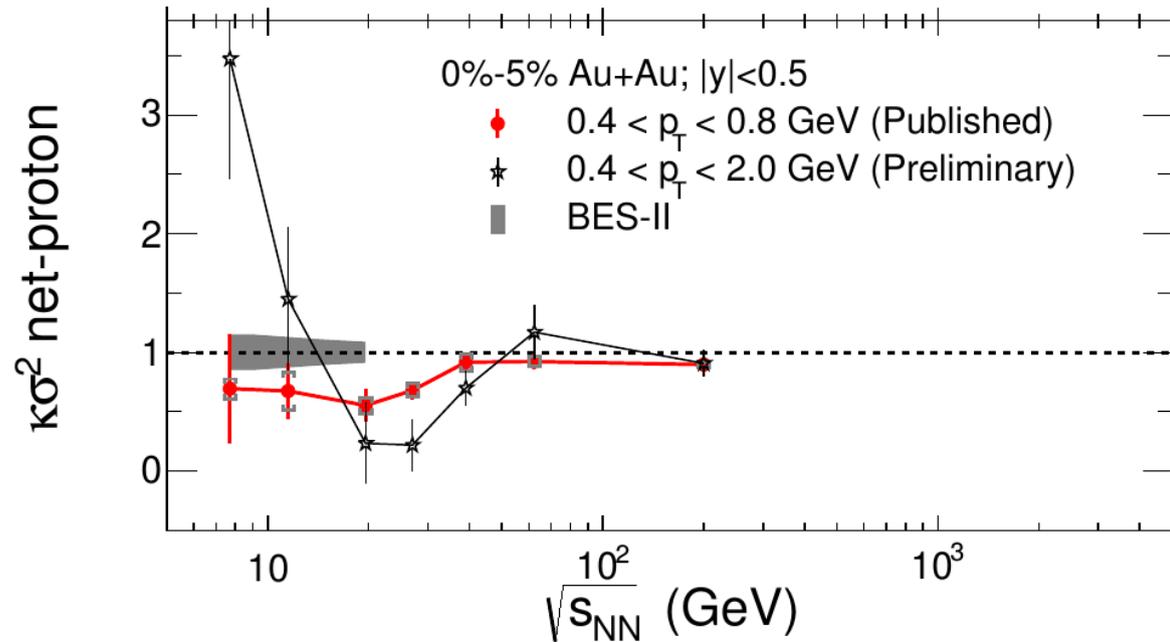
# Conserved charge fluctuations and freeze-out

## net baryon number fluctuations



STAR Collaboration, PRL 112, 032302 (2014)

## new STAR preliminary data and error projection for BES-II



X. Luo (for STAR), CPOD2014 (2015) 019

$$(S\sigma)_{proton} \equiv \frac{\chi_3^P}{\chi_2^P}$$

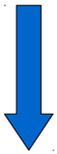
$$(\kappa\sigma^2)_{proton} \equiv \frac{\chi_4^P}{\chi_2^P}$$

# Conserved charge fluctuations and freeze-out

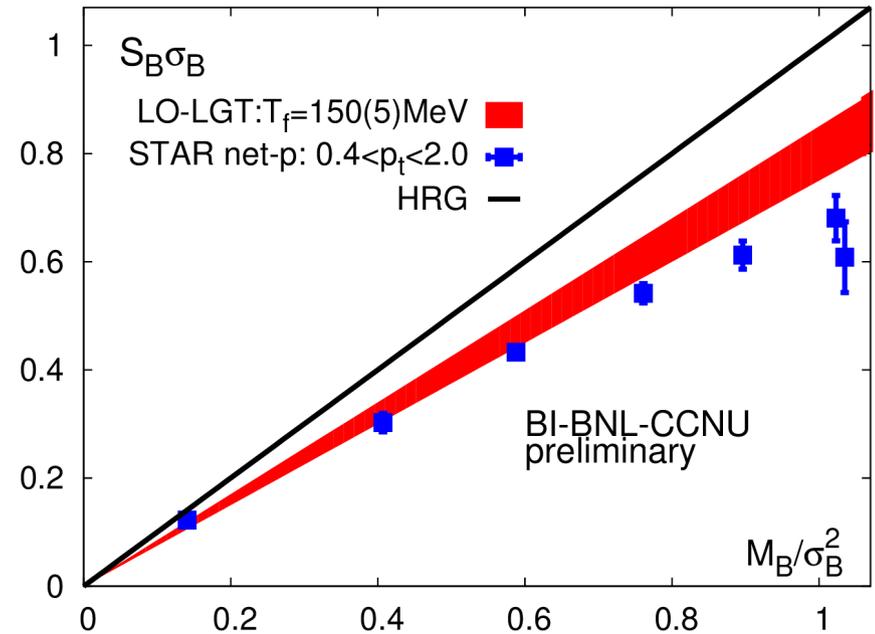
$$\frac{\Delta(T, \mu_B)}{T^4} = \frac{P(T, \mu_B) - P(T, 0)}{T^4} = \frac{\chi_2^B}{2} \left(\frac{\mu_B}{T}\right)^2 \left(1 + \frac{1}{12} \frac{\chi_4^B}{\chi_2^B} \left(\frac{\mu_B}{T}\right)^2\right)$$

$$\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

$$S_B \sigma_B = \frac{\mu_B}{T} \frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^3)$$



$$S_B \sigma_B = \frac{M_B}{\sigma_B^2} \frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^3)$$



warning: net-proton  $\neq$  net-baryon

M.Kitazawa et al, PR C86 (2012) 024904

A.Bzdak et al., PR C87 (2013) 014901

# Conserved charge fluctuations and freeze-out

**Next order:** depends on 6<sup>th</sup> order cumulants and requires knowledge on the parametrization of the freeze-out curve, eg.

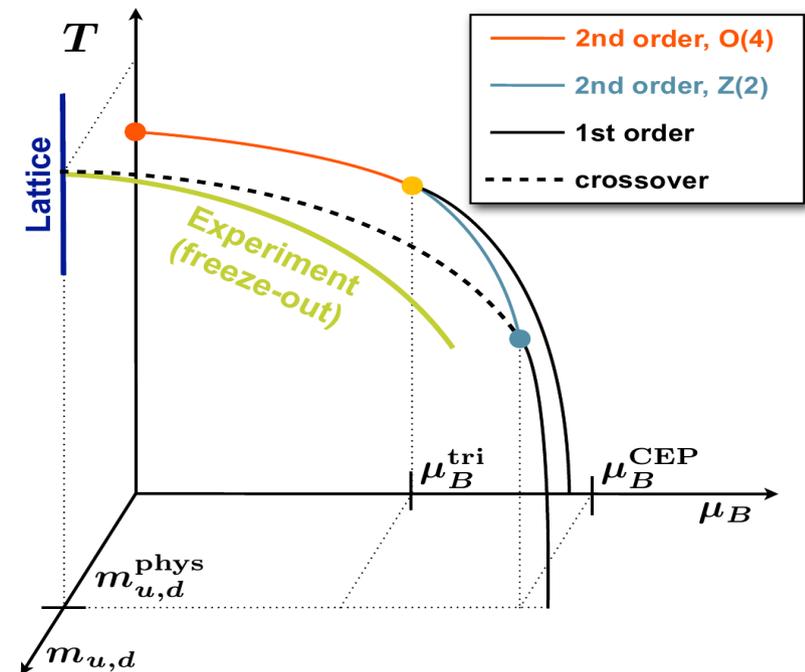
$$T_f(\mu_B) = T_f(0) \left( 1 - \kappa_f \left( \frac{\mu_B}{T} \right)^2 \right)$$

ratio of cumulants on "a line" in the  $(T, \mu_B)$  plane

$$\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} \frac{1 + \frac{1}{6} \frac{\chi_4^B}{\chi_2^B} \left( \frac{\mu_B}{T} \right)^2}{1 + \frac{1}{2} \frac{\chi_4^B}{\chi_2^B} \left( \frac{\mu_B}{T} \right)^2}$$

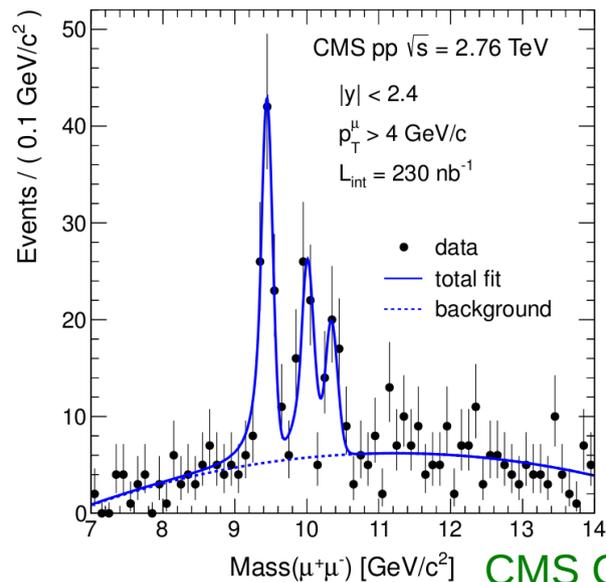
$$S_B \sigma_B = \frac{\mu_B}{T} \frac{\chi_4^B}{\chi_2^B} \frac{1 + \frac{1}{6} \frac{\chi_6^B}{\chi_4^B} \left( \frac{\mu_B}{T} \right)^2}{1 + \frac{1}{2} \frac{\chi_4^B}{\chi_2^B} \left( \frac{\mu_B}{T} \right)^2}$$

$$\updownarrow \equiv \left( \frac{\chi_4^B}{\chi_2^B} \right)_{T_f(0)} - \kappa_f T_f(0) \left( \frac{\chi_4^B}{\chi_2^B} \right)' \left( \frac{\mu_B}{T} \right)^2$$



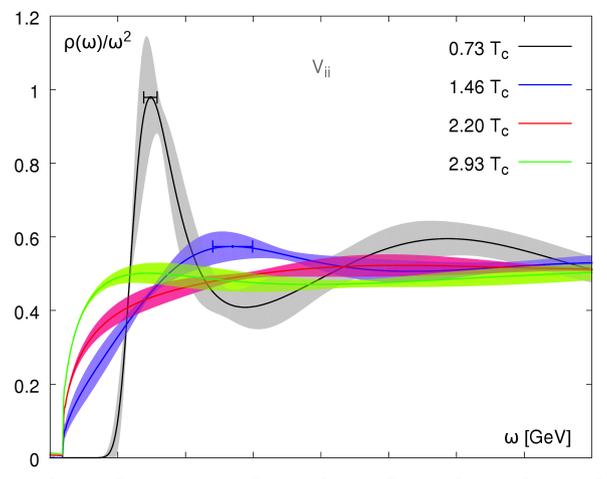
# Heavy Quark spectroscopy, thermal dileptons, transport coefficients

## bottomonium

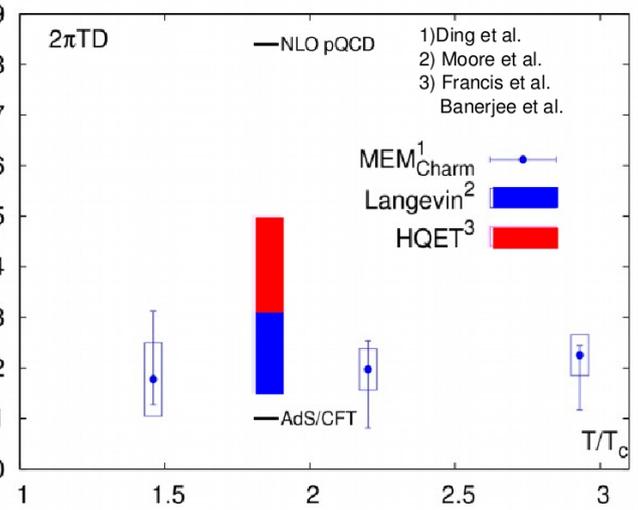


CMS Collaboration, PRL 109, 222301 (2012)

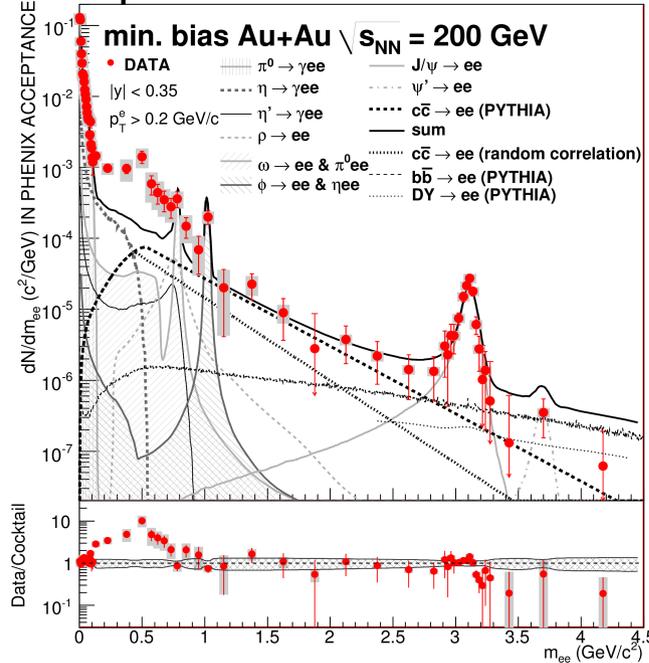
## heavy quark spectral functions



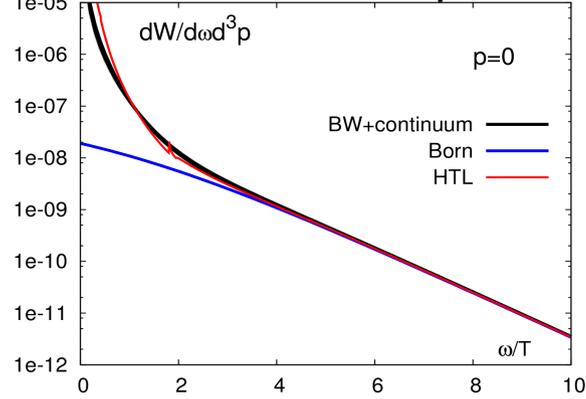
## heavy quark diffusion



## dilepton rates



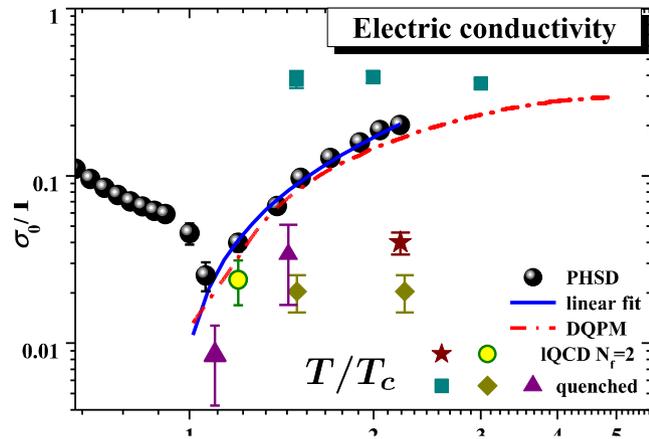
## thermal dileptons



BNL-Bi, arXiv:1301.7436

A. Adare et al. (PHENIX Collaboration), Phys. Rev. C81, 034911 (2010)

Bielefeld-BNL, PRD 86, 014509 (2012); H.-T. Ding, arXiv:1210.5442



W. Cassing et al., arXiv:1302.0906

# Advancing the development of software needed for QCD thermodynamics calculations

NP SciDAC-3 project: **Computing Properties of Hadrons, Nuclei and Nuclear Matter from Quantum Chromodynamics (LQCD)**  
(BNL, Jlab, LLNL, MIT, UNC, UoW, W&M)

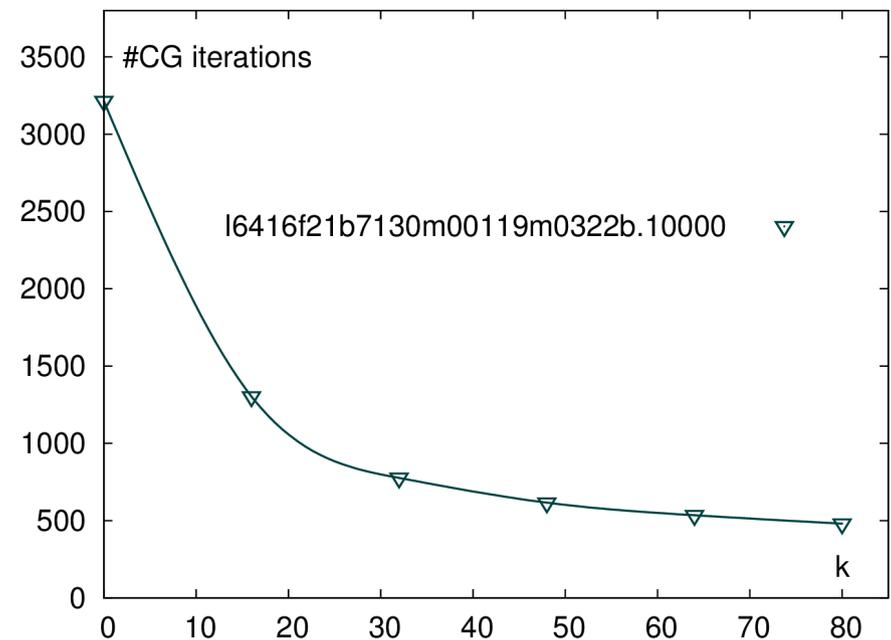
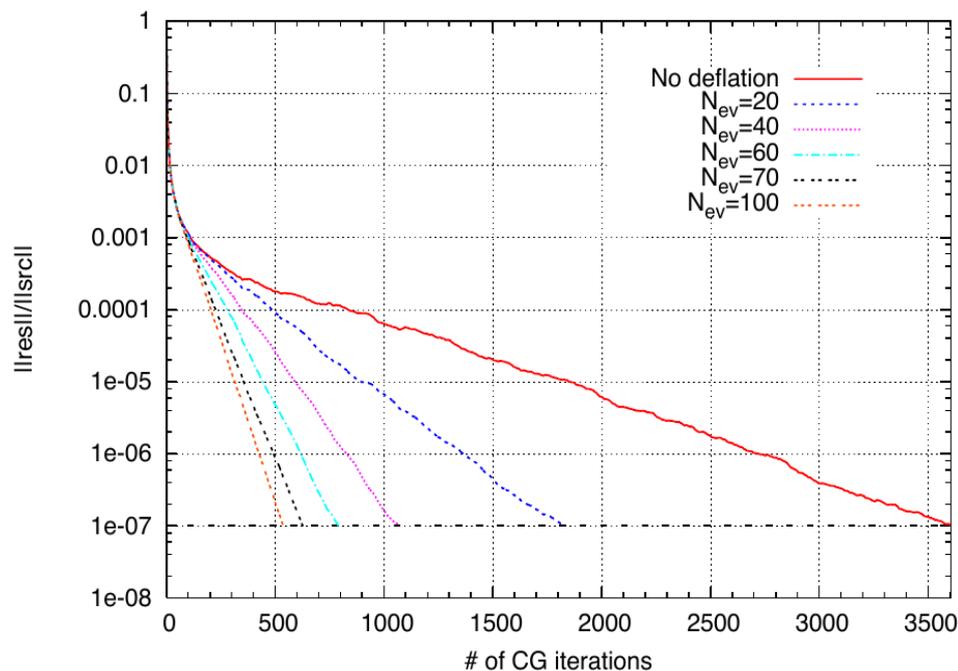
NERSC Exascale Science Application Program:  
**Domain Wall Fermions and Highly Improved Staggered Quarks for Lattice QCD**,  
N. Christ (Columbia U.), F. Karsch (BNL)

- optimization and improvement of inverters
- code optimization for BlueGene/Q, GPU-clusters (+Titan@ORNL), INTEL's Knights Corner and Knights Landing

# Calculating charge fluctuations on the lattice

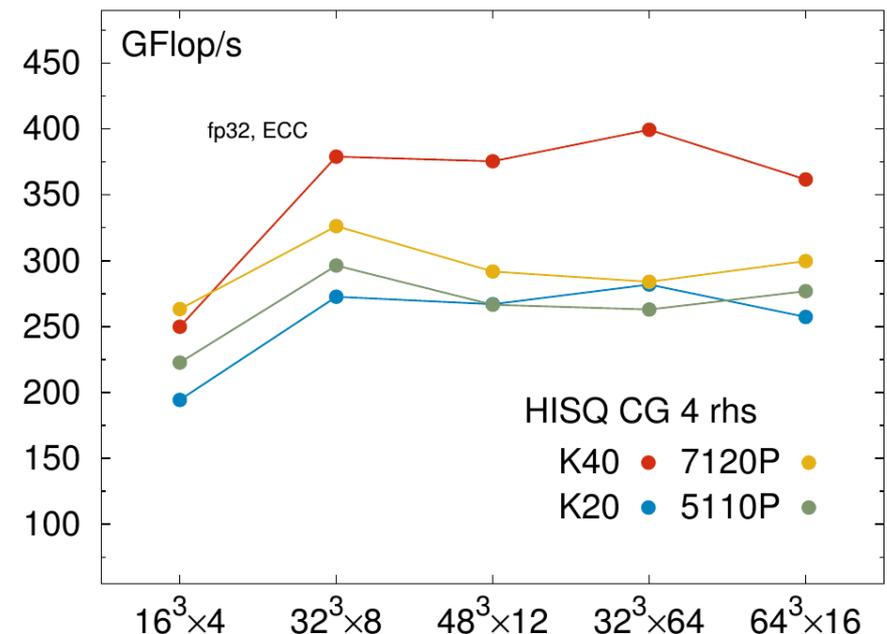
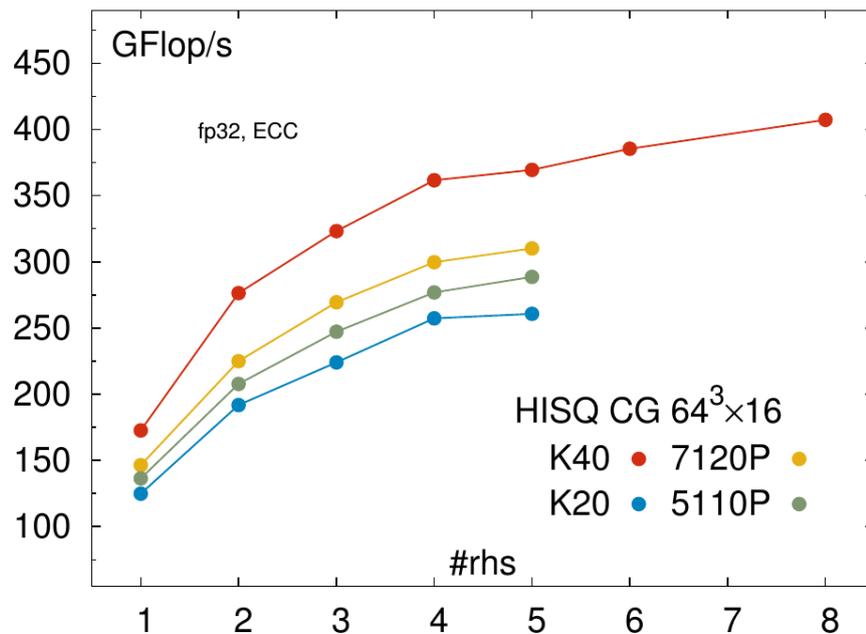
## recent advances:

- understood that higher order cumulants are free of divergences and can be calculated using the so-called **linear-mu formulation**:  
R.V. Gavai and S. Sharma, Phys. Rev. D85 (2012) 054508  
↙ now post-doc at BNL
- developed **efficient deflation code** for the evaluation of charge fluctuation observables on GPUs



# Calculating charge fluctuations on the lattice

- highly efficient CG inverter for HISQ action on GPUs (and also on MIC)



O. Kaczmarek, C. Schmidt, P. Steinbrecher, M. Wagner, arXiv:1411.4439



SciDAC-3 visiting grad.student at BNL

- gain by using **multiple right hand sides** – balance against #EV used
- **Titan specific**: shift eigenvector calculation for deflation to CPU; generation of eigenvectors "comes for free" and GPU can run with more right hand sides

# Conclusions

- During the last 5 years LGT calculations have achieved two important goals: **determination of  $T_c$  and calculation of the equation of state with physical quark masses in the continuum limit**
- LGT calculations start to produce quantitative predictions on QCD thermodynamics that provide input to the interpretation of heavy ion experiments

EoS,  $T_c$ , transport coefficients, spectral functions, phase boundary, fluctuations of conserved charges,....

## The next five years – before BES-II at RHIC

- **Lattice QCD will provide the EoS in the entire parameter range relevant for BES-II at RHIC (if no critical point exists in that range)**
- **Lattice QCD will strengthen evidence for a critical point (if it exists in the parameter range accessible to BES-II at RHIC)**