An Update on the RHMC Algorithm with DWF

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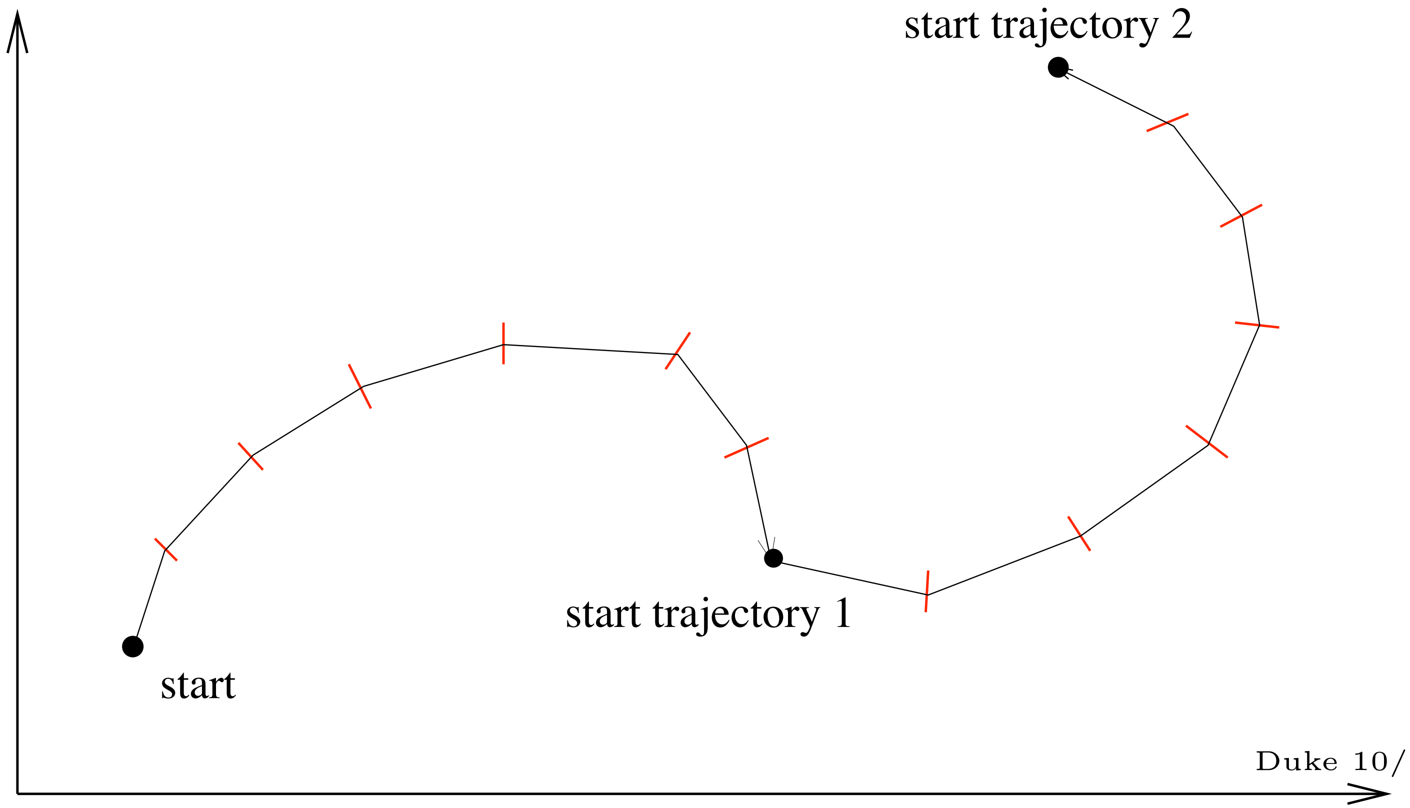
Lattice QCD Algorithms

- Fermion determinant represented by "pseudo fermion" fields

$$
\begin{aligned}
Z &= \int [dU]\,[d\psi]\,[d\bar\psi]\,\exp\left\{\sum_n\left[-\beta S_g + \bar\psi(\not{D}+m)\psi\right]\right\} = \int [dU]\,\det(\not{D}+m)\exp\left\{\sum_n(-\beta S_g)\right\} \\
&= \int [dU]\,[d\phi^*][d\phi]\exp\left\{\sum_n(-\beta S_g) + \phi^*(\not{D}+m)^{-1}\phi\right\} \\
&= \int [dU]\,[d\Pi]\,[d\phi^*][d\phi]\exp\left\{\sum_n(-\Pi^2 - \beta S_g) + \phi^*(\not{D}+m)^{-1}\phi\right\}
\end{aligned}
$$



RHMC

$$\det \mathcal{M} = \int D\bar{\phi} D\phi \exp(-\bar{\phi}\mathcal{M}^{-\alpha}\phi)$$

$$= \int D\bar{\phi} D\phi \, \exp(-\bar{\phi}\, r^2(\mathcal{M})\phi)$$

with $\quad r(x) \approx x^{-\alpha/2}$

and $\quad r(x) = \sum_{k=1}^{n} \frac{\alpha_k}{x + \beta_k}$

Mike Clark, PhD Thesis, U. Edinburgh, 2005

Clark, de Forcrand, Kennedy, hep-lat/0510004

Omelyan Integrator

Omelyan, Mryglod, Folk, 2003. Takaishi, de Forcrand, hep-lat/0505020

Symmetric, symplectic, second order integrator

λ controls coefficient of higher order terms

$$\hat{U}_{\mathrm{QPQPQ}}(\delta\tau) = e^{\lambda\delta\tau Q}\, e^{\delta\tau P/2}\, e^{(1-2\lambda)\delta\tau Q}\, e^{\delta\tau P/2}\, e^{\lambda\delta\tau Q},$$

Expected to be 50% better than leapfrog

Quotient RHMC

Can handle Pauli-Villars with separate stochastic field or use single field for light and Pauli-Villars determinants

Noted by Vranas, implemented for QCDSP by Dawson and for QCDOC and RHMC by Clark

$$\sqrt{\frac{\det M_{\mathrm{PF}}^{\dagger} M_{\mathrm{PF}}}{\det M_{\mathrm{PV}}^{\dagger} M_{\mathrm{PV}}}} = \det\left[(M_{\mathrm{PV}}^{\dagger} M_{\mathrm{PV}})^{-1/8} (M_{\mathrm{PF}}^{\dagger} M_{\mathrm{PF}})^{1/4} (M_{\mathrm{PV}}^{\dagger} M_{\mathrm{PV}})^{-1/8}\right]^2$$

$$S_{\mathrm{F}} = \bar{\phi}\left[(M_{\mathrm{PV}}^{\dagger} M_{\mathrm{PV}})^{1/4} (M_{\mathrm{PF}}^{\dagger} M_{\mathrm{PF}})^{-1/2} (M_{\mathrm{PV}}^{\dagger} M_{\mathrm{PV}})^{1/4}\right]^2 \phi$$

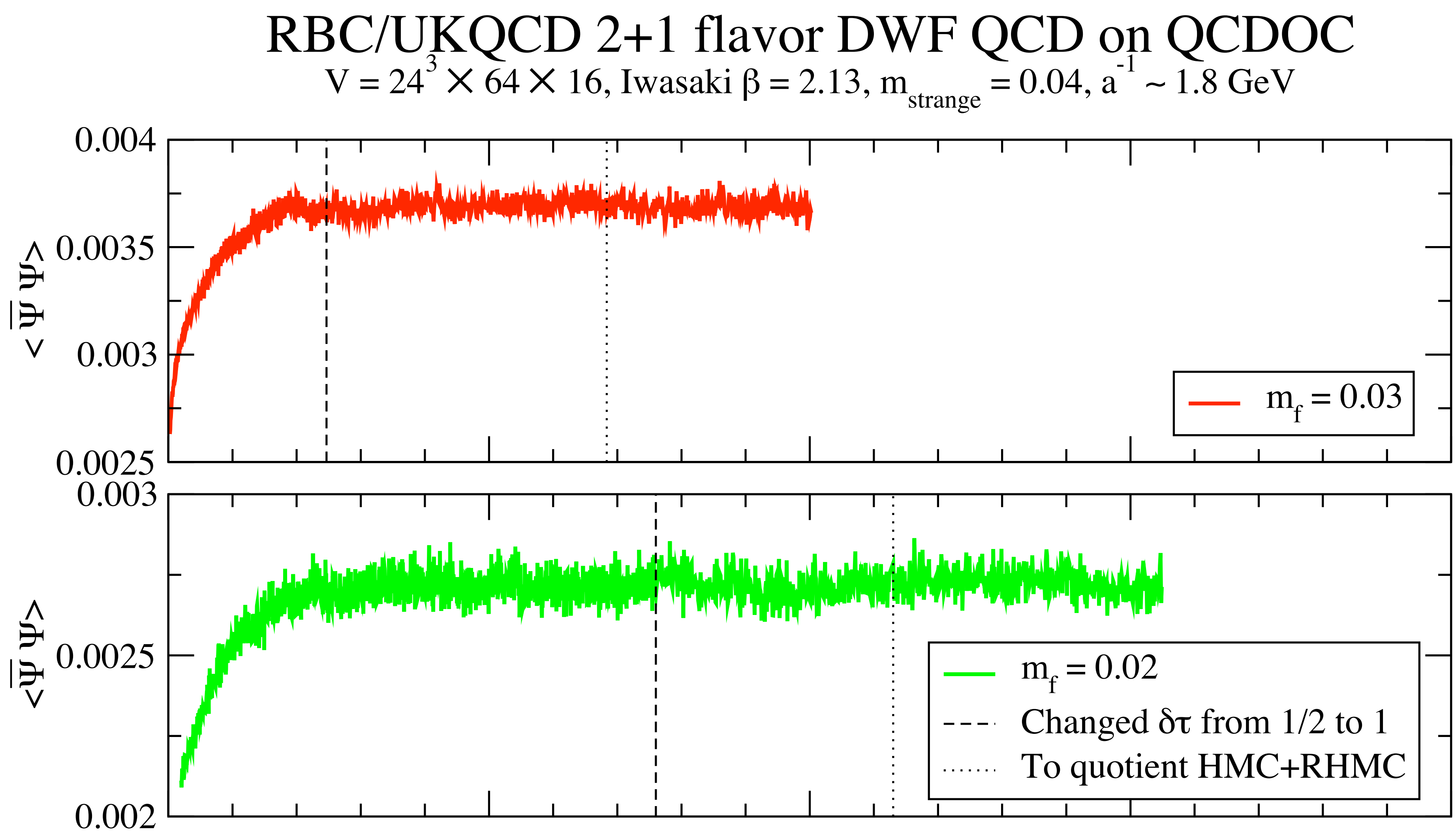
$$= \bar{\phi}\left[r_1(M_{\mathrm{PV}}^{\dagger} M_{\mathrm{PV}})\, r_2(M_{\mathrm{PF}}^{\dagger} M_{\mathrm{PF}})\, r_1(M_{\mathrm{PV}}^{\dagger} M_{\mathrm{PV}})\right]^2 \phi$$

Domain Wall Fermions

$$\frac{\det\left[D^\dagger(M_5,m_l)D(M_5,m_l)\right]\ \det^{1/2}\left[D^\dagger(M_5,m_s)D(M_5,m_s)\right]}{\det^{3/2}\left[D^\dagger(M_5,1.0)D(M_5,1.0)\right]}$$

$$=\frac{\det\left[D^\dagger(M_5,m_l)D(M_5,m_l)\right]}{\det\left[D^\dagger(M_5,m_s)D(M_5,m_s)\right]}\ \frac{\det^{3/2}\left[D^\dagger(M_5,m_s)D(M_5,m_s)\right]}{\det^{3/2}\left[D^\dagger(M_5,1.0)D(M_5,1.0)\right]}$$

Quotient force HMC	Quotient force RHMC
Hasenbusch preconditioned with strange quark	Pauli-Villars mass cancels bulk modes
Small force, but expensive to calculate	Large force, less expensive to calculate
	3, 1/2 power fields further reduce force



0.03/0.04 run: 30,308(7) CG iterations/traj

0.02/0.04 run: 31,616(8) CG iterations/traj