Computing Pion Parton Distribution Function on Fine Lattice

by Luchang Jin

BNL

Jiunn-Wei Chen, Tomomi Ishikawa, Taku Izubuchi,

Swagato Mukherjee, Peter Petreczky, Sergey Syritsyn

USQCD All HANDS MEETING

28 Apr 2017

Jefferson Lab

Outline

- Quasi-PDF
- Background
- Proposal
- Conclusion

Parton Distribution Function (PDF) is one of the structure functions of hadrons.

$$\mathrm{DIS} \;=\; \mathrm{Perturbatively\,Computable\,Quantities} \otimes \mathrm{PDF} + \mathrm{higher\,twist\,contribution}$$

PDF is universal! Above factorization applies to many other observables as well.

To compute PDF on the lattice, one should find one such observable which can be easily measured with lattice QCD.

Quasi-PDF seems to be a good choice:

$$\begin{aligned}
\tilde{q}(x, \tilde{\mu}^2, P_z) &= \int_{-\infty}^{\infty} \frac{d\delta z}{4\pi}\, e^{-ix\,\delta z\,P_z} \big\langle P \big| \tilde{O}_{\delta z, \tilde{\mu}^2}(0) \big| P \big\rangle \\
\tilde{O}_{\delta z, \tilde{\mu}^2}(z) &= Z(\delta z, a, \tilde{\mu}^2) \left[\bar{\psi}(z + \delta z)\, \gamma_z \exp\left(-i\, g \int_z^{z+\delta z} d\, z'\, A^z(z') \right) \psi(z) \right]
\end{aligned}$$

$Z(\delta z, a, \tilde{\mu}^2)$ renormalizes the lattice operator into some continuum scheme at scale $\tilde{\mu}$.

[X. Ji, PhysRevLett.110.262002] [X. Xiong et al, PhysRevD.90.014051]

[Y.Q. Ma, J.W. Qiu, 1404.6860]

When P_z is large, Quasi-PDF is related with normal PDF by:

$$\tilde{q}(x, \tilde{\mu}^2, P_z) \ = \ \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \tilde{\mu}^2, \mu^2, P_z\right) q(y, \mu^2) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{P_z^2}\right)$$

Quasi-PDF $\tilde{q}$ can be factorized into PDF q and perturbatively computable function C.

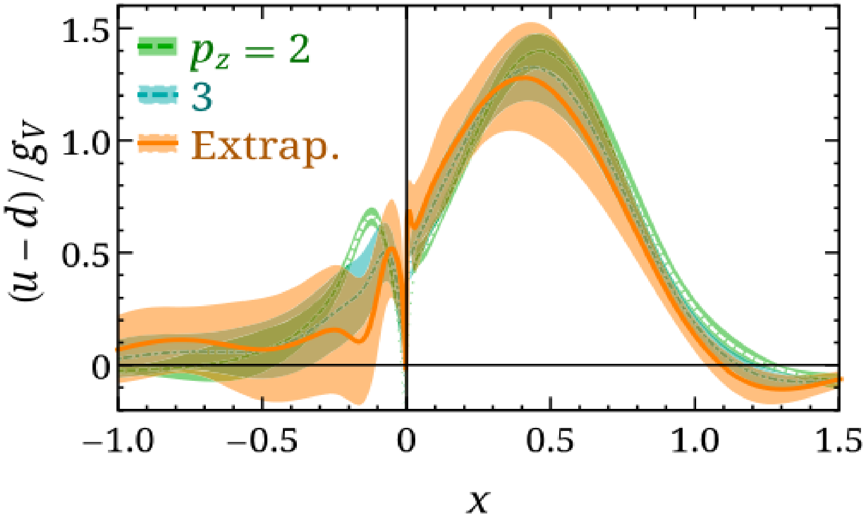
$$\mathrm{DIS} \ = \ \mathrm{Perturbatively\,Computable\,Quantities} \otimes \mathrm{PDF} + \mathrm{higher\,twist\,contribution}$$

We can see the forms of the two equations match.

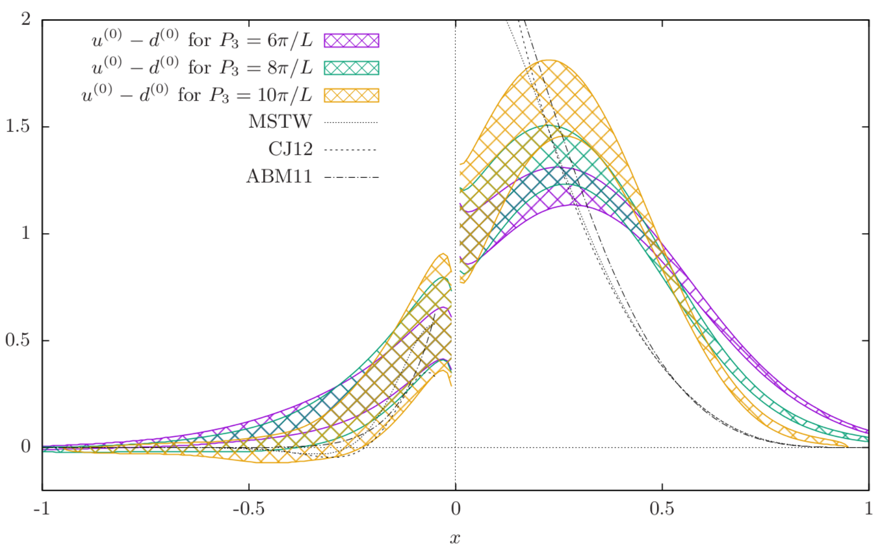
[X. Ji, PhysRevLett.110.262002] [X. Xiong et al, PhysRevD.90.014051]

[Y.Q. Ma, J.W. Qiu, 1404.6860]

Two existing studies for proton, iso-vector PDF, corresponds to $u - d$.



[H.W. Lin, 1603.06664]
max $P_z = 1.3\,\mathrm{GeV}$
$a = 0.12\,\mathrm{fm}$
$m_\pi = 310\,\mathrm{MeV}$
$L = 3\,\mathrm{fm}$



[ETMC, 1610.03689]
max $P_z = 2.4\,\mathrm{GeV}$
$a = 0.082\,\mathrm{fm}$
$m_\pi = 370\,\mathrm{MeV}$
$L = 2.6\,\mathrm{fm}$

Proposal

Current systematics includes

- **Proton is not moving fast enough? That is, P_z is not large enough?**
- **Lattice spacing is not small enough?**
- Pion mass is not physical.
- Finite volume effects.
- **Renormalization of the lattice operator not accurately performed?**
- Higher order in α_s effects in the matching kernel $C\left(\frac{x}{y}, \tilde{\mu}^2, \mu^2, P_z\right)$?

Trying to study these systematics, we propose

- Compute **pion PDF**.

  Pion correlation functions are generally easier to compute than proton. We plan to reach $P_z = 3\ \mathrm{GeV}$, which means very fast moving pion because of its smaller rest mass. Momentum smearing would be a big help in order to reach large momentum. [G. S. Bali, 1602.05525]

- Use **fine lattice**. Compute with two lattice spacings $a = 0.06\,\mathrm{and}\,0.04\,\mathrm{fm}$.
- Carefully perform the **renormalization**.

Reasons for fine lattices

- If one perform the OPE for the quasi-PDF operator, both leading twist effects (from the 2nd or higher order moment of PDF) and higher twist effects will be suppressed by δz^2. However, leading twist effects are enhanced by $P_z^2/\Lambda_{\rm QCD}^2$.
- One should extract PDF from the lattice result in the region where $P_z^2 \gg \Lambda_{\rm QCD}^2$ to enhance the leading twist effects, and $\delta z^2 \lesssim 1/\Lambda_{\rm QCD}^2$ to suppress the higher twist effects.
- Since the range of δz is limited, smaller lattice spacing means more usable data points.

$$
\begin{aligned}
q(x,\mu^2) = q(x) &= \int_{-\infty}^{\infty} \frac{d\zeta}{4\pi} e^{-ix\zeta} Q(\zeta) \\
\tilde{q}(x,\tilde{\mu}^2,P_z) = \tilde{q}(x) &= \int_{-\infty}^{\infty} \frac{d\zeta}{4\pi} e^{-ix\zeta} \tilde{Q}(\zeta = P_z\,\delta z)
\end{aligned}
$$

$$
\begin{aligned}
\tilde{q}(x) &= \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}\right) q(y) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{P_z^2}\right) \\
\tilde{Q}(\zeta = P_z\,\delta z) &= \int_{-\infty}^{\infty} d\alpha\, C(\alpha) Q(\alpha\,\zeta) + \mathcal{O}\left(\delta z^2 \Lambda_{\rm QCD}^2 = \zeta^2 \frac{\Lambda_{\rm QCD}^2}{P_z^2}\right)
\end{aligned}
$$

Renormalization of Quasi-PDF

$$\tilde{q}(x,\tilde{\mu}^2,P_z) \;=\; \int_{-\infty}^{\infty}\frac{d\delta z}{4\pi}\,e^{-ix\delta z P_z}\big\langle P\big|\tilde{O}_{\delta z,\tilde{\mu}^2}(0)\big|P\big\rangle$$

$$\tilde{O}_{\delta z,\tilde{\mu}^2}(z) \;=\; Z(\delta z,a,\tilde{\mu}^2)\left[\bar{\psi}(z+\delta z)\,\gamma_z\exp\left(-i\,g\int_z^{z+\delta z}d\,z'\,A^z(z')\right)\psi(z)\right]$$

→ [A. Bazavov, 1603.06637] Wilson line renormalization through Polyakov loop.

→ [T. Ishikawa, 1609.02018] Lattice perturbation theory + Wilson line self energy.

→ [J.H. Zhang, 1609.08102] Momentum cutoff regularization + Wilson line self energy.

→ [C. Monahan, 1612.01584] Wilson flow.

→ [Y. Zhao, to appear] RI/MOM scheme.

Summary

- Compute on two HISQ ensemble using smeared Wilson fermions:

  Same pion mass $m_\pi = 300\,\mathrm{MeV}$ for two ensembles.

  $48^3 \times 64$ with $a = 0.06\,\mathrm{fm}$ and $L = 2.9\,\mathrm{fm}$.

  64^4 with $a = 0.04\,\mathrm{fm}$ and $L = 2.6\,\mathrm{fm}$.

- Plan to compute with three momenta

  $P_z = 1, 2, 3\,\mathrm{GeV}$.

- The **GOAL** is: reliable determination of pion PDF at $m_\pi = 300\,\mathrm{MeV}$.

  Or, find out how hard it might be.

- In this proposal, we request **12.17 M Jpsi core hours on Pi0**, including 0.55 M J/psi core hour for storage.

Table 2: Resource estimates for the propagators

Size	$48^3 \times 64$	64^4
N_{conf}	400	400
$N_{\mathrm{samples/conf}}$ (exact+sloppy)	2+64	2+64
Costs in M J/psi core h	3.45	8.17
Total costs in M J/psi core h: 11.62		

Thank You!

1) SPC: Besides the renormalization issues, a primary issue is the range of x accessible with the P_z in your calculation. Do you have plans to study how large a P_z is accessible for a given lattice, and the values of x of the light-cone distributions that can be faithfully reproduced?

Response: We plan to compute many different P_z for the first few configurations and choose three typical P_z which are not too noisy to compute, but large enough to suppress the higher twist and other effects. For now, we estimate $P_z = 1$ GeV, 2 GeV, 3 GeV might be sensible choices.

The larger the P_z the smaller is the value of x at which PDF can be determined reliably. The formula that relates the quasi-PDF to PDF is beased on perturbation theory. Therefore it is important that the large values of δz do not contribute significantly to the quasi-PDF. This is the case when x is not too small and P_z is large enough. Therefore, using larger values of P_z allows us to consider smaller value of x. However, even for the smallest lattice spacing proposed in our study we do not expect to get reliable results for $x<0.1$.

2) SPC: Does smaller lattice spacing guarantee large physical P_z?

Response: Small lattice spacing makes calculation with large P_z possible. We understand that large momentum usually leads to the signal to noise problem. We plan to investigate how to improve the signal to noise ratio in the large momentum case. The recent momentum smearing technique introduced in the paper "Novel quark smearing for hadrons with high momenta in lattice QCD" (PhysRevD.93.094515) would be a good start.

Q&A

3) SPC: Instead of working on two lattices with different lattice spacings, is it worthwhile extending 3 values of proposed to P_z to, say, 6 on one lattice to study the large P_z behavior?

Response: Having two lattice spacings is very important for understanding the discretization error, which is the major source of the uncertainty we wish to control. Also, the coarse lattice requires much fewer resources compare with the finer 64^4 lattice.

Relations in position space

$$q(x,\mu^2) = q(x) \;=\; \int_{-\infty}^{\infty} \frac{d\zeta}{4\pi}\, e^{-ix\zeta}\, Q(\zeta)$$

$$\tilde{q}(x,\tilde{\mu}^2,P_z) = \tilde{q}(x) \;=\; \int_{-\infty}^{\infty} \frac{d\zeta}{4\pi}\, e^{-ix\zeta}\, \tilde{Q}(\zeta = P_z\,\delta z)$$

$$\tilde{Q}(\zeta = P_z\,\delta z) \;=\; Z(\delta z, a, \tilde{\mu}^2)\,\tilde{Q}^{\rm bare}(\zeta)$$

$$\tilde{Q}^{\rm bare}(\zeta = P_z\,\delta z) \;=\; \frac{1}{P_z}\left\langle P\middle|\tilde{O}^{\rm bare}_{\delta z,\tilde{\mu}^2}(0)\middle|P\right\rangle$$

$$\tilde{O}^{\rm bare}_{\delta z,\tilde{\mu}^2}(0) \;=\; \bar{\psi}(z+\delta z)\,\gamma_z \exp\left(-i\,g\int_z^{z+\delta z} d\,z'\,A^z(z')\right)\psi(z)$$

$$\tilde{q}(x) \;=\; \int_{-1}^{1} \frac{dy}{|y|}\, C\!\left(\frac{x}{y}\right) q(y) + \mathcal{O}\!\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2}\right)$$

$$\tilde{Q}(\zeta = P_z\,\delta z) \;=\; \int_{-\infty}^{\infty} d\alpha\, C(\alpha)\, Q(\alpha\,\zeta) + \mathcal{O}\!\left(\delta z^2\Lambda^2_{\rm QCD} = \zeta^2\,\frac{\Lambda^2_{\rm QCD}}{P_z^2}\right)$$