The RBC/UKQCD $g-2$ project

Christoph Lehner (BNL)

April 28, 2017 – USQCD All Hands Meeting

Collaborators

RBC/UKQCD

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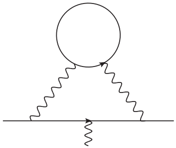
Peter Boyle (Edinburgh)
Norman Christ (Columbia)
Vera Guelpers (Southampton)
Masashi Hayakawa (Nagoya)
James Harrison (Southampton)
Taku Izubuchi (BNL/RBRC)
Christoph Lehner (BNL)
Chulwoo Jung (BNL)
Andreas Jüttner (Southampton)
Luchang Jin (Columbia)
Antonin Portelli (Edinburgh)
Matt Spraggs (Southampton)

Theory status – summary

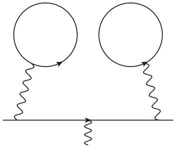
Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED (5 loops)	11 658 471.895	0.008
EW	15.4	0.1
HVP LO	692.3	**4.2**
HVP NLO	-9.84	0.06
HVP NNLO	1.24	0.01
Hadronic light-by-light	10.5	**2.6**
Total SM prediction	11 659 181.5	4.9
BNL E821 result	11 659 209.1	6.3
FNAL E989/J-PARC E34 goal		$\approx$ **1.6**

A reduction of uncertainty for HVP and HLbL is needed. A systematically improvable first-principles calculation is desired.

First-principles approach to HVP LO

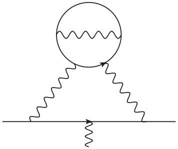


Quark-connected piece with by far dominant part from up and down quark loops, $\mathcal{O}(700 \times 10^{-10})$

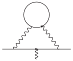


Quark-disconnected piece, $-9.6(4.0) \times 10^{-10}$

Phys.Rev.Lett. 116 (2016) 232002



QED corrections, $\mathcal{O}(10 \times 10^{-10})$

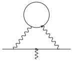


HVP quark-connected contribution

Biggest challenge to direct calculation at physical pion masses is to control statistics and potentially large finite-volume errors.

Statistics: for strange and charm solved issue, for up and down quarks existing methodology less effective

Finite-volume errors are exponentially suppressed in the simulation volume but may be sizeable



HVP quark-connected contribution

Starting from the vector current

$$J_\mu(x) = i \sum_f Q_f \overline{\Psi}_f(x) \gamma_\mu \Psi_f(x)$$

we may write

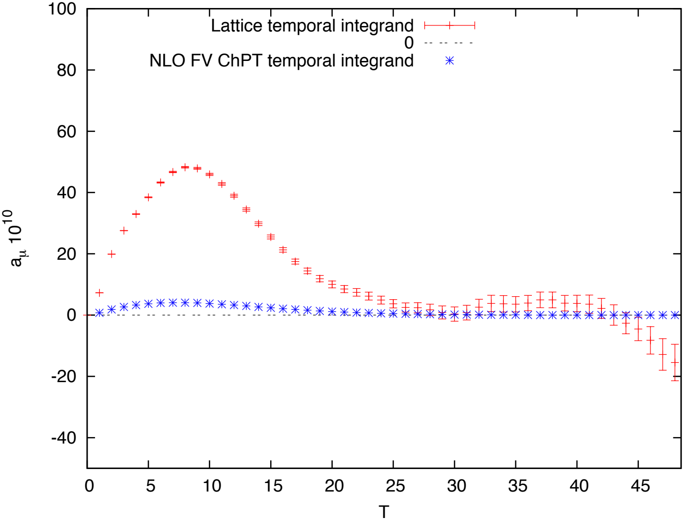
$$a_\mu^{\mathrm{HVP}} = \sum_{t=0}^{\infty} w_t C(t)$$

with

$$C(t) = \frac{1}{3} \sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x}, t) J_j(0) \rangle$$

and w_t capturing the photon and muon part of the diagram (Bernecker-Meyer 2011).

Integrand $w_T C(T)$ for the light-quark connected contribution:



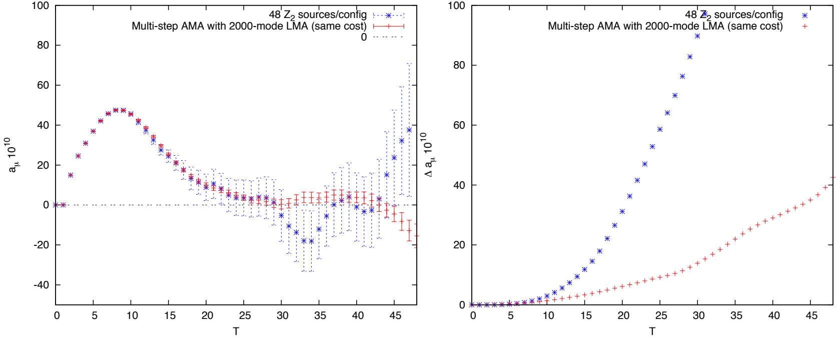
$m_\pi = 140$ MeV, $a = 0.11$ fm (RBC/UKQCD 48^3 ensemble)

Statistical noise from long-distance region

Addressing the long-distance noise problem

- Replace $C(t)$ for large t with model; multi-exponentials for $t \geq 1.5$ fm was recently used to compute $a_\mu^{\mathrm{HVP\ LO\ CON}} = 666(6) \times 10^{-10}$ arXiv:1601.03071.

- Our recent improvement: Improved stochastic estimator (hierarchical approximations including exact treatment of low-mode space; DeGrand & Schäfer 2004):



Complete first-principles analysis

- Currently the statistical uncertainty for a pure first-principles analysis in the continuum limit is at the $\Delta a_\mu \approx 15 \times 10^{-10}$ level

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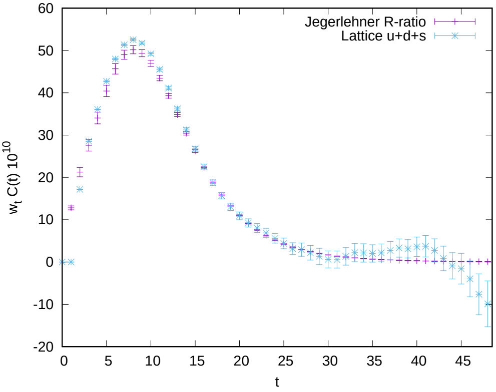
Fermilab E989 target		$\approx$ **1.6**

- Sub-percent statistical error achievable with a few more months of running

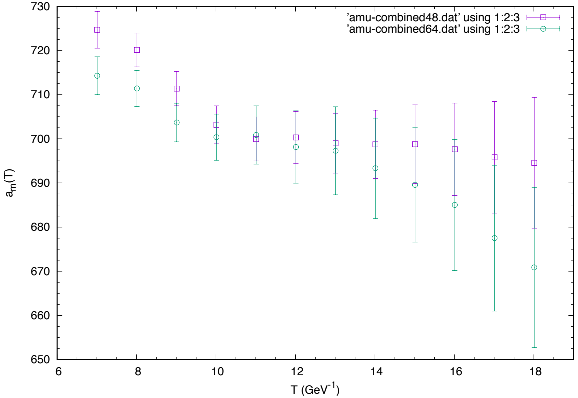
- While we are waiting for more statistics ...

Combined lattice and dispersive analysis

We can use the dispersion relation to overlay experimental e^+e^- scattering data (Bernecker, Meyer 2011). Below the experimental result is taken from Jegerlehner 2016:



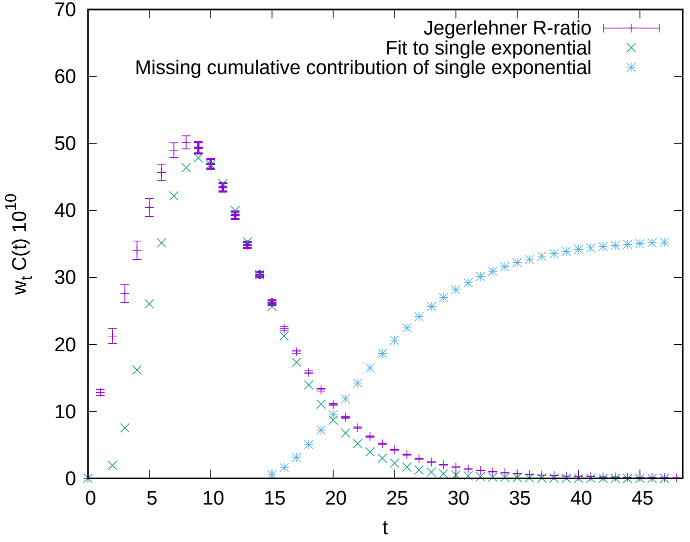
The lattice data is precise at shorter distances and the experimental data is precise at longer distances. We can do a combined analysis with lattice and experimental data: $a_\mu = \sum_{t=0}^{T} w_t C^{\mathrm{lattice}}(t) + \sum_{t=T+1}^{\infty} w_t C^{\mathrm{exp}}(t)$



Errors range from ~ 0.5 to 1.2 % for T $\lesssim$ 12 (GeV^{-1})

This is a promising way to reduce the overall uncertainty on a short time-scale.

We can also learn about the validity of long-distance modelling from using the R-ratio data



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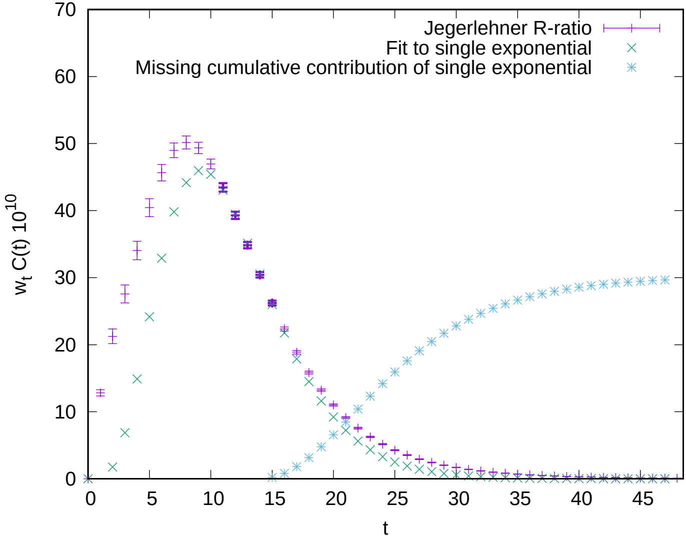

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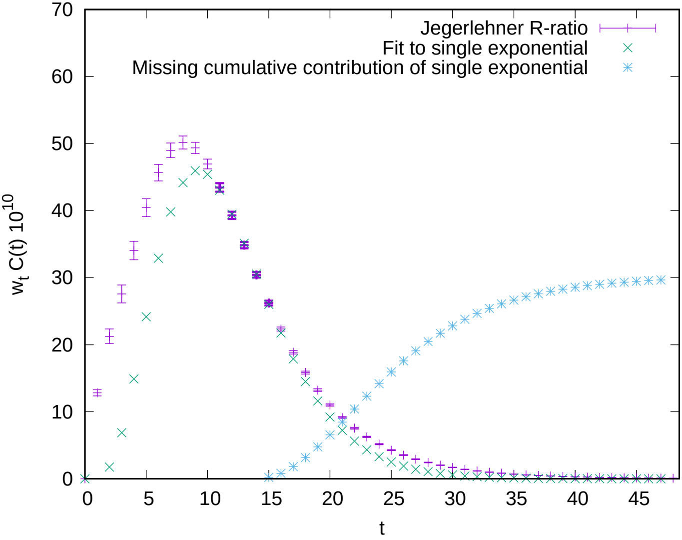
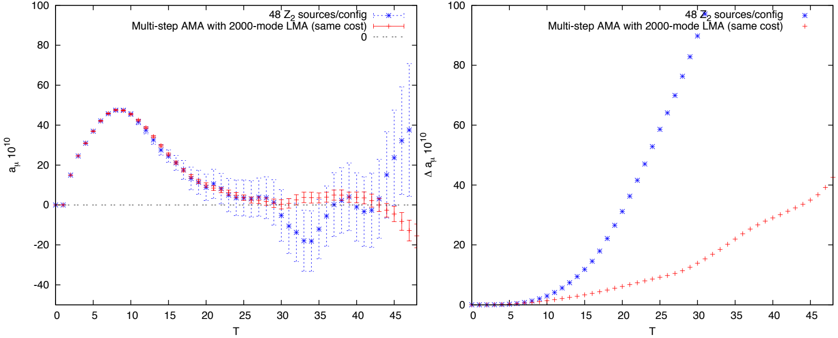


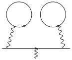
If only data $t \leq 1.5$ fm is used to constrain such a model, it is conceivable to systematically undershoot the true HVP by $O(30 \times 10^{-10})$.

Addressing the long-distance noise problem

- Replace $C(t)$ for large t with model; multi-exponentials for $t \geq 1.5$ fm was recently used to compute $a_\mu^{\mathrm{HVP\ LO\ CON}} = 666(6) \times 10^{-10}$ arXiv:1601.03071. Difficult to control systematics of modelling.

- Our recent improvement: Improved stochastic estimator (hierarchical approximations including exact treatment of low-mode space):





HVP quark-disconnected contribution

First results at physical pion mass with a statistical signal
Phys.Rev.Lett. 116 (2016) 232002

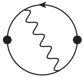
Statistics is clearly the bottleneck; calculation was a potential road-block of a first-principles calculation for a long time; due to very large pion-mass dependence calculation at physical pion mass is crucial.

New stochastic estimator allowed me to get result

$$a_\mu^{\mathrm{HVP\ (LO)\ DISC}} = -9.6(3.3)_{\mathrm{stat}}(2.3)_{\mathrm{sys}} \times 10^{-10}$$

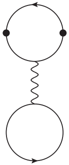
from a modest computational investment ($\approx$ 1M core hours).

HVP QED contribution

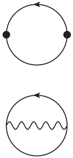


(a) V

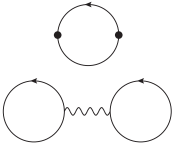
(b) S



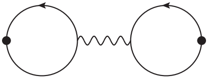
(c) T



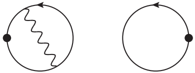
(d) D1



(e) D2



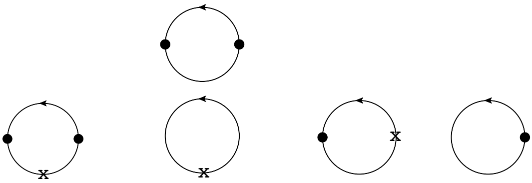
(f) F



(g) D3

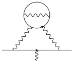
New method: use importance sampling in position space and local vector currents

HVP strong IB contribution



(a) M (b) R (c) O

Calculate strong IB effects via insertions of mass corrections in an expansion around isospin symmetric point



HVP QED+strong IB contributions

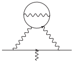
Strategy

1. Re-tune parameters for QCD+QED simulation (m_u, m_d, m_s, a)

2. Verify simple observables ($m_{\pi^+} - m_{\pi^0}$, ...)

3. Calculate QED and strong IB corrections to HVP LO

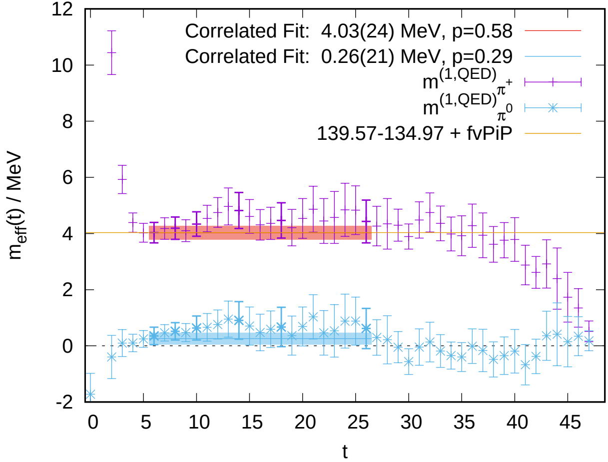
All results shown below are preliminary! For now focus on diagrams S, V, F; preliminary study below does not yet include re-tuning of a.

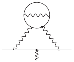


HVP QED+strong IB contributions

RBC 2017

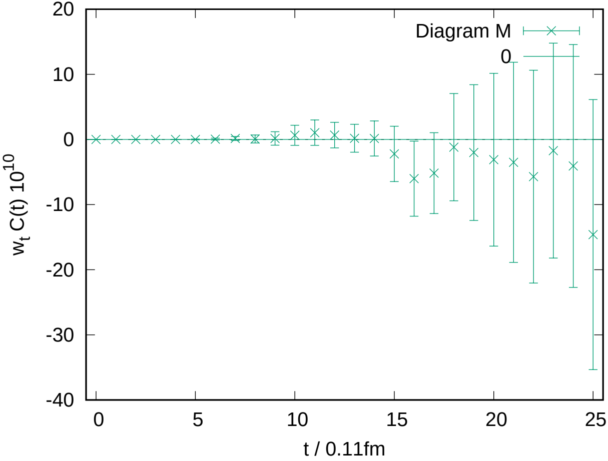
Diagrams S, V for pion mass:

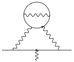




HVP QED+strong IB contributions

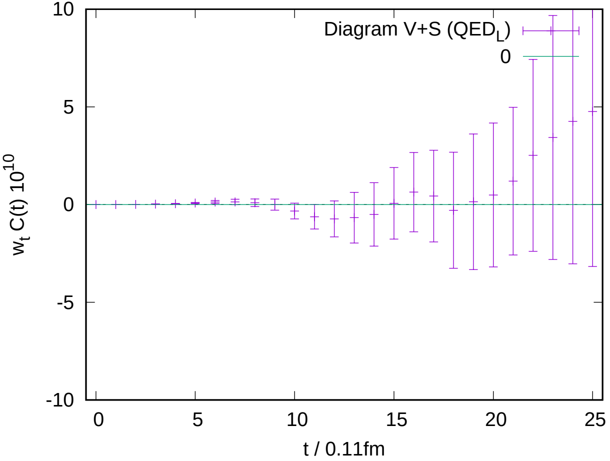
HVP strong IB effect

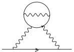




HVP QED+strong IB contributions

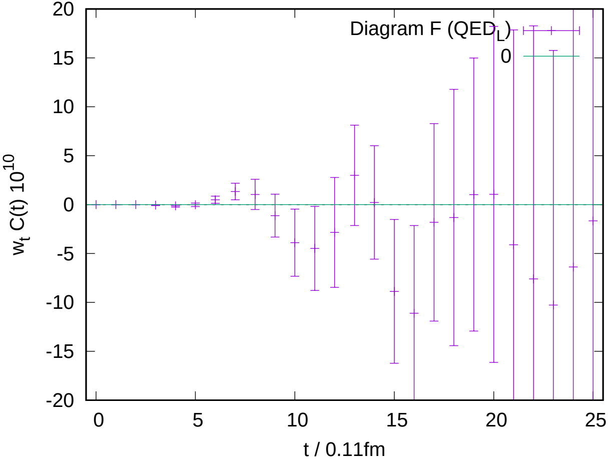
HVP QED diagram V+S





HVP QED+strong IB contributions

HVP QED diagram F

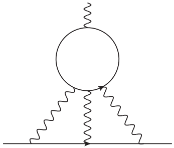


Straightforward improvements of statistics available, too late for this talk

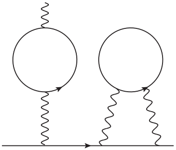
Status and outlook for the HVP LO

- New methods: improved statistical estimators both for connected light and disconnected contributions at physical point.
- For the connected light contribution the new method reduces noise in the long-distance part of the correlator by an order of magnitude compared to previous method.
- For the disconnected contributions the new method allowed for a precise calculation at physical pion mass. Phys.Rev.Lett. 116 (2016) 232002
- Combination with e^+e^- scattering data should allow for a significant improvement over current most precise estimate within the next 6 months.
- Leading QED corrections at physical pion mass under active investigation

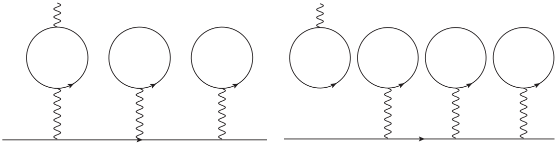
The Hadronic Light-by-Light contribution



Quark-connected piece (charge factor of up/down quark contribution: $\frac{17}{81}$)



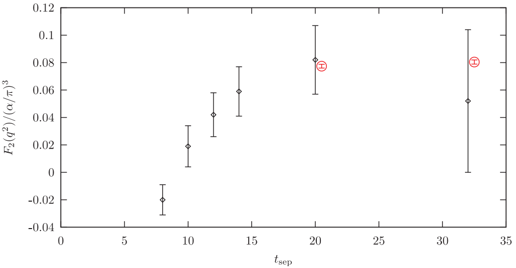
Dominant quark-disconnected piece (charge factor of up/down quark contribution: $\frac{25}{81}$)



Sub-dominant quark-disconnected pieces (charge factors of up/down quark contribution: $\frac{5}{81}$ and $\frac{1}{81}$)

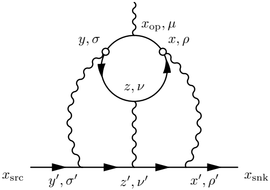
All results below are from: T. Blum, N. Christ, M. Hayakawa, T. Izubuchi, L. Jin, and C.L., Phys. Rev. D 93, 014503 (2016)

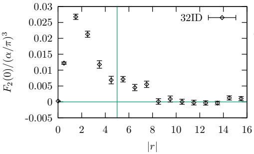
Compute quark-connected contribution with new computational strategy

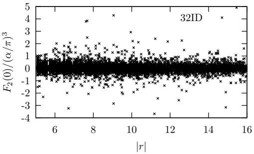


yields more than an order-of-magnitude improvement (red symbols) over previous method (black symbols) for a factor of ≈ 4 smaller cost.

New stochastic sampling method



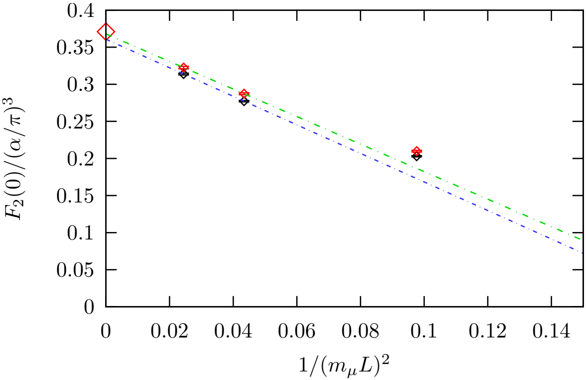




Stochastically evaluate the sum over vertices x and y:

- Pick random point x on lattice
- Sample all points y up to a specific distance $r = |x - y|$, see vertical red line
- Pick y following a distribution $P(|x - y|)$ that is peaked at short distances

Cross-check against analytic result where quark loop is replaced by muon loop



Current status of the HLbL

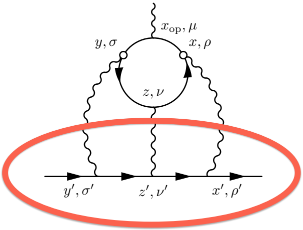
T. Blum, N. Christ, M. Hayakawa, T. Izubuchi, L. Jin, and C.L., PRL118(2017)022005

$$a_\mu^{\rm cHLbL} = \left.\frac{g_\mu - 2}{2}\right|_{\rm cHLbL} = (0.0926 \pm 0.0077)\left(\frac{\alpha}{\pi}\right)^3$$
$$= (11.60 \pm 0.96) \times 10^{-10} \quad (11)$$
$$a_\mu^{\rm dHLbL} = \left.\frac{g_\mu - 2}{2}\right|_{\rm dHLbL} = (-0.0498 \pm 0.0064)\left(\frac{\alpha}{\pi}\right)^3$$
$$= (-6.25 \pm 0.80) \times 10^{-10} \quad (12)$$
$$a_\mu^{\rm HLbL} = \left.\frac{g_\mu - 2}{2}\right|_{\rm HLbL} = (0.0427 \pm 0.0108)\left(\frac{\alpha}{\pi}\right)^3$$
$$= (5.35 \pm 1.35) \times 10^{-10} \quad (13)$$

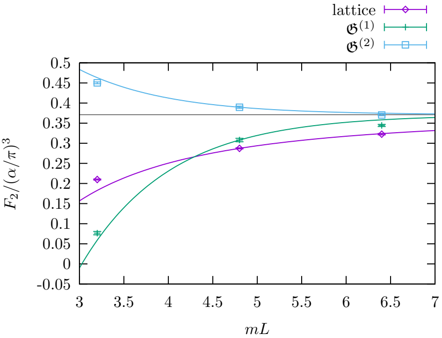
Makes HLbL an unlikely candidate to explain the discrepancy!

Next: finite-volume and lattice-spacing systematics; sub-leading diagrams

Finite-volume errors of the HLbL



Remove power-law like finite-volume errors by computing the muon-photon part of the diagram in infinite volume (C.L. talk at lattice 2015 and Green et al. 2015, PRL115(2015)222003; Asmussen et al. 2016, PoS,LATTICE2016 164)



Now completed arXiv:1704.XXXX with improved weighting function.

Next step: combine weighting function with existing QCD data

Summary and outlook

New methods allow for a substantial reduction in uncertainty of the theory calculation of the $(g-2)_\mu$.

A reduction of uncertainty over the currently most precise value within the next year seems possible.

Over the next five years should allow for a reduction of uncertainty commensurate with the Fermilab E989 target precision.

The Fermilab experiment may have first results in 2018?

Thank you

The setup:

$$C(t) = \frac{1}{3V} \sum_{j=0,1,2} \sum_{t'} \langle \mathcal{V}_j(t+t') \mathcal{V}_j(t') \rangle_{\mathrm{SU(3)}} \tag{1}$$

where V stands for the four-dimensional lattice volume, $\mathcal{V}_\mu = (1/3)(\mathcal{V}_\mu^{u/d} - \mathcal{V}_\mu^s)$, and

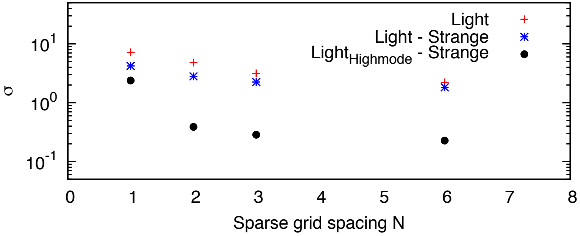
$$\mathcal{V}_\mu^f(t) = \sum_{\vec{x}} \operatorname{Im} \operatorname{Tr}[D^{-1}_{\vec{x},t;\vec{x},t}(m_f)\gamma_\mu] \,. \tag{2}$$

We separate 2000 low modes (up to around m_s) from light quark propagator as $D^{-1} = \sum_n v^n (w^n)^\dagger + D^{-1}_{\mathrm{high}}$ and estimate the high mode stochastically and the low modes as a full volume average Foley 2005.

We use a sparse grid for the high modes similar to Li 2010 which has support only for points x_μ with $(x_\mu - x_\mu^{(0)}) \bmod N = 0$; here we additionally use a random grid offset $x_\mu^{(0)}$ per sample allowing us to stochastically project to momenta.

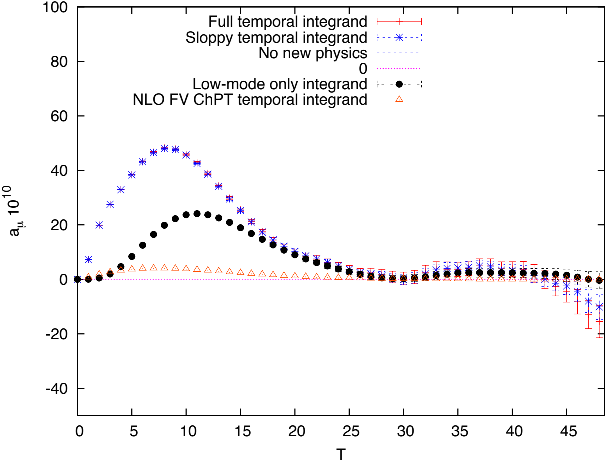
Combination of both ideas is crucial for noise reduction at physical pion mass!

Fluctuation of $\mathcal{V}_\mu$ (σ):

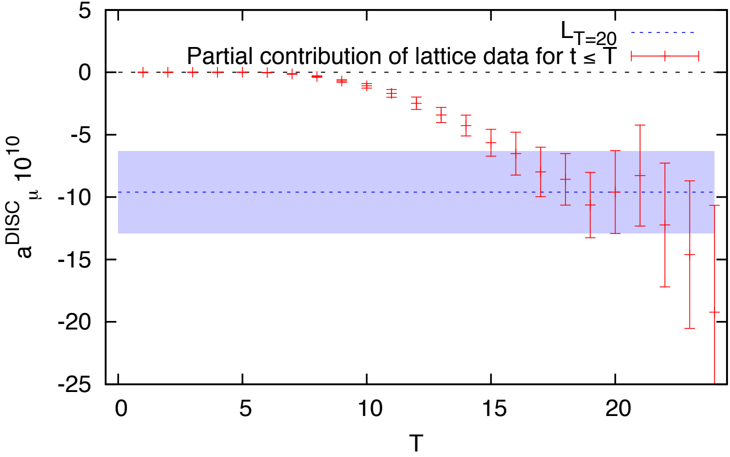


Since $C(t)$ is the autocorrelator of $\mathcal{V}_\mu$, we can create a stochastic estimator whose noise is potentially reduced linearly in the number of random samples, hence the normalization in the lower panel

Low-mode saturation for physical pion mass (here 2000 modes):

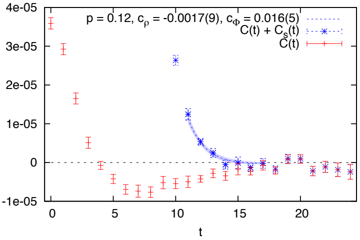


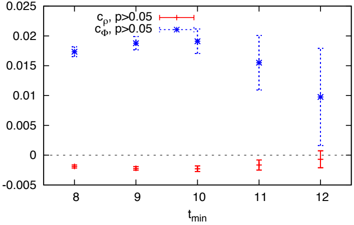
Result for partial sum $L_T = \sum_{t=0}^{T} w_t C(t)$:



For $t \geq 15$ $C(t)$ is consistent with zero but the stochastic noise is t-independent and $w_t \propto t^4$ such that it is difficult to identify a plateau region based only on this plot

Resulting correlators and fit of $C(t) + C_s(t)$ to $c_\rho e^{-E_\rho t} + c_\phi e^{-E_\phi t}$ in the region $t \in [t_{\min}, \dots, 17]$ with fixed energies $E_\rho = 770$ MeV and $E_\phi = 1020$. $C_s(t)$ is the strange connected correlator.

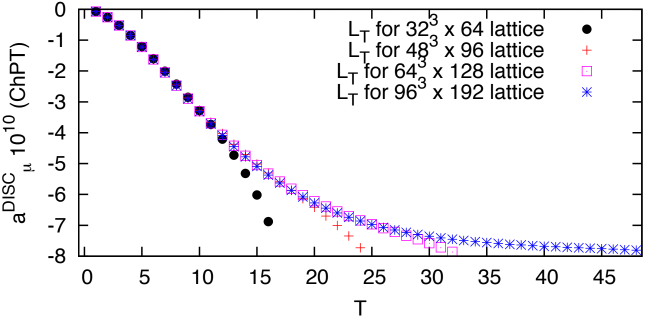




We fit to $C(t) + C_s(t)$ instead of $C(t)$ since the former has a spectral representation.

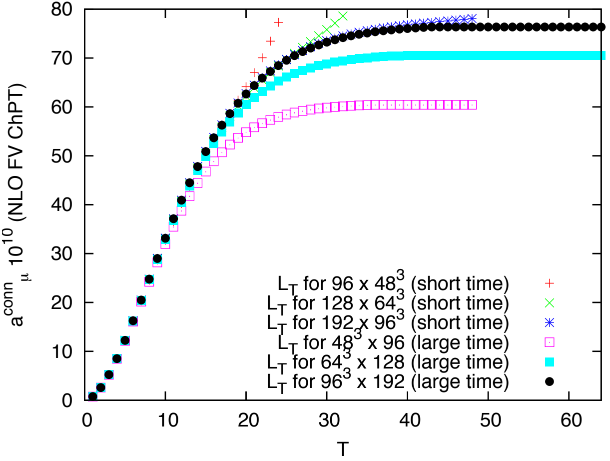
We could use this model alone for the long-distance tail to help identify a plateau but it would miss the two-pion tail

We therefore additionally calculate the two-pion tail for the disconnected diagram in ChPT:



A closer look at the NLO FV ChPT prediction (1-loop sQED):

We show the partial sum $\sum_{t=0}^{T} w_t C(t)$ for different geometries and volumes:



The dispersive approach to HVP LO

The dispersion relation

$$\begin{aligned} \Pi_{\mu\nu}(q) &= i\left(q_\mu q_\nu - g_{\mu\nu} q^2\right)\Pi(q^2) \\ \Pi(q^2) &= -\frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds}{s}\frac{\mathrm{Im}\Pi(s)}{q^2-s}\,. \end{aligned}$$

allows for the determination of a_μ^{HVP} from experimental data via

$$a_\mu^{\mathrm{HVP\ LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left[\int_{4m_\pi^2}^{E_0^2} ds\,\frac{R_\gamma^{\mathrm{exp}}(s)\hat{K}(s)}{s^2} + \int_{E_0^2}^{\infty} ds\,\frac{R_\gamma^{\mathrm{pQCD}}(s)\hat{K}(s)}{s^2}\right],$$

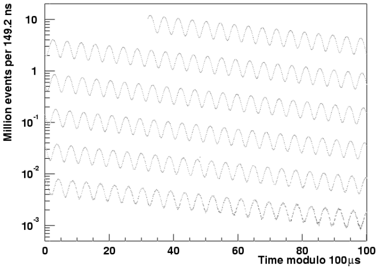
$$R_\gamma(s) = \sigma^{(0)}(e^+e^- \to \gamma^* \to \mathrm{hadrons})/\frac{4\pi\alpha^2}{3s}$$

Experimentally with or without additional hard photon (ISR: $e^+e^- \to \gamma^*(\to \mathrm{hadrons})\gamma$)

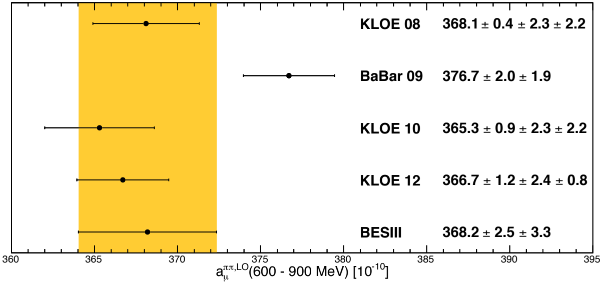
Experimental setup: muon storage ring with tuned momentum of muons to cancel leading coupling to electric field

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

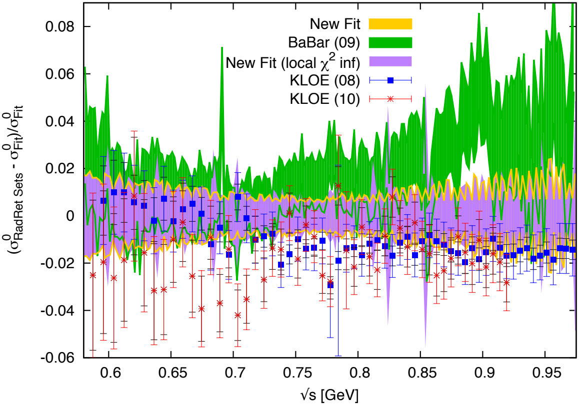
Because of parity violation in weak decay of muon, a correlation between muon spin and decay electron direction exists, which can be used to measure the anomalous precession frequency ω_a:

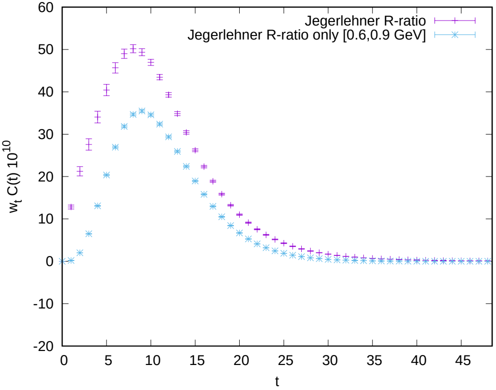


BESIII 2015 update:



Hagiwara et al. 2011:





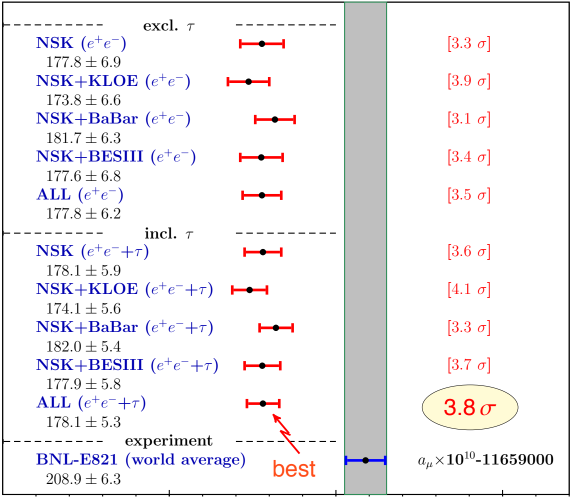
Problematic experimental region can readily be replaced by precise lattice data. Lattice also can be arbiter regarding different experimental data sets.

Jegerlehner FCCP2015 summary:

final state	range (GeV)	$a_\mu^{\mathrm{had}(1)} \times 10^{10}$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	507.55 (0.39) (2.68)[2.71]	0.5%	39.9%
ω	(0.42, 0.81)	35.23 (0.42) (0.95)[1.04]	3.0%	5.9%
ϕ	(1.00, 1.04)	34.31 (0.48) (0.79)[0.92]	2.7%	4.7%
J/ψ		8.94 (0.42) (0.41)[0.59]	6.6%	1.9%
Υ		0.11 (0.00) (0.01)[0.01]	6.8%	0.0%
had	(1.05, 2.00)	60.45 (0.21) (2.80)[2.80]	4.6%	42.9%
had	(2.00, 3.10)	21.63 (0.12) (0.92)[0.93]	4.3%	4.7%
had	(3.10, 3.60)	3.77 (0.03) (0.10)[0.10]	2.8%	0.1%
had	(3.60, 9.46)	13.77 (0.04) (0.01)[0.04]	0.3%	0.0%
had	(9.46,13.00)	1.28 (0.01) (0.07)[0.07]	5.4%	0.0%
pQCD	(13.0,∞)	1.53 (0.00) (0.00)[0.00]	0.0%	0.0%
data	(0.28,13.00)	687.06 (0.89) (4.19)[4.28]	0.6%	0.0%
total		688.59 (0.89) (4.19)[4.28]	0.6%	100.0%

Results for $a_\mu^{\mathrm{had}(1)} \times 10^{10}$. Update August 2015, incl SCAN[NSK]+ISR[KLOE10,KLOE12,BaBar,BESIII]

Jegerlehner FCCP2015 summary ($\tau \leftrightarrow e^+e^-$):



Our setup:

$$C(t) = \frac{1}{3V} \sum_{j=0,1,2} \sum_{t'} \langle \mathcal{V}_j(t+t')\mathcal{V}_j(t') \rangle_{\mathrm{SU(3)}} \tag{3}$$

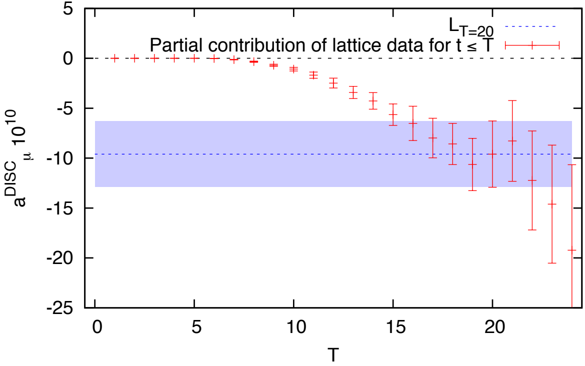
where V stands for the four-dimensional lattice volume, $\mathcal{V}_\mu = (1/3)(\mathcal{V}_\mu^{u/d} - \mathcal{V}_\mu^s)$, and

$$\mathcal{V}_\mu^f(t) = \sum_{\vec{x}} \mathrm{Im}\,\mathrm{Tr}[D^{-1}_{\vec{x},t;\vec{x},t}(m_f)\gamma_\mu]\,. \tag{4}$$

We separate 2000 low modes (up to around m_s) from light quark propagator as $D^{-1} = \sum_n v^n(w^n)^\dagger + D^{-1}_{\mathrm{high}}$ and estimate the high mode stochastically and the low modes as a full volume average Foley 2005.

We use a sparse grid for the high modes similar to Li 2010 which has support only for points x_μ with $(x_\mu - x_\mu^{(0)}) \bmod N = 0$; here we additionally use a random grid offset $x_\mu^{(0)}$ per sample allowing us to stochastically project to momenta.

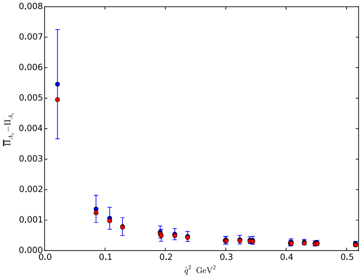
Study $L_T = \sum_{t=T+1}^{\infty} w_t C(t)$ and use value of T in plateau region (here $T = 20$) as central value. Use a combined estimate of a resonance model and the two-pion tail to estimate systematic uncertainty.

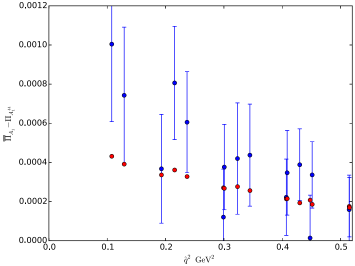


Combined with an estimate of discretization errors, we find

$$a_\mu^{\mathrm{HVP\ (LO)\ DISC}} = -9.6(3.3)_{\mathrm{stat}}(2.3)_{\mathrm{sys}} \times 10^{-10}\,. \qquad (5)$$

From Aubin et al. 2015 (arXiv:1512.07555v2)



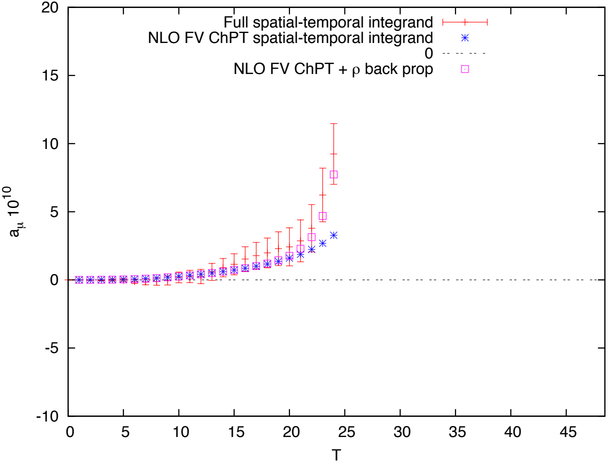


MILC lattice data with $m_\pi L = 4.2$, $m_\pi \approx 220$ MeV; Plot difference of $\Pi(q^2)$ from different irreps of 90-degree rotation symmetry of spatial components versus NLO FV ChPT prediction (red dots)

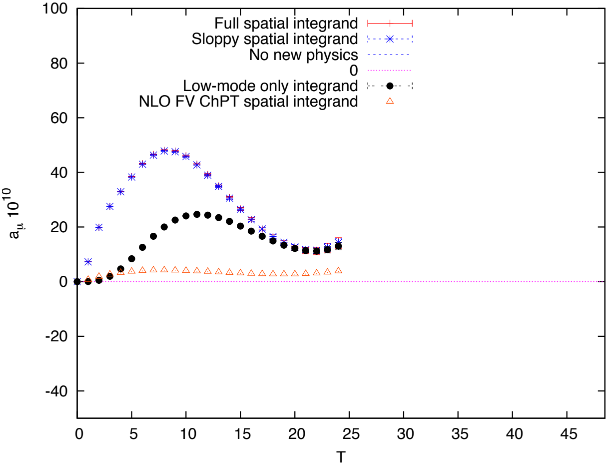
While the absolute value of a_μ is poorly described by the two-pion contribution, the volume dependence may be described sufficiently well to use ChPT to control FV errors at the 1% level; this needs further scrutiny

Aubin et al. find an $O(10\%)$ finite-volume error for $m_\pi L = 4.2$ based on the $A_1 - A_1^{44}$ difference (right-hand plot)

Compare difference of integrand of $48 \times 48 \times 96 \times 48$ (spatial) and $48 \times 48 \times 48 \times 96$ (temporal) geometries with NLO FV ChPT $(A_1 - A_1^{44})$:



$m_\pi = 140$ MeV, $p^2 = m_\pi^2/(4\pi f_\pi)^2 \approx 0.7\%$



Full spatial integrand
Sloppy spatial integrand
No new physics
0
Low-mode only integrand
NLO FV ChPT spatial integrand
$a_\mu\ 10^{10}$
T
100
80
60
40
20
0
-20
-40
0
5
10
15
20
25
30
35
40
45