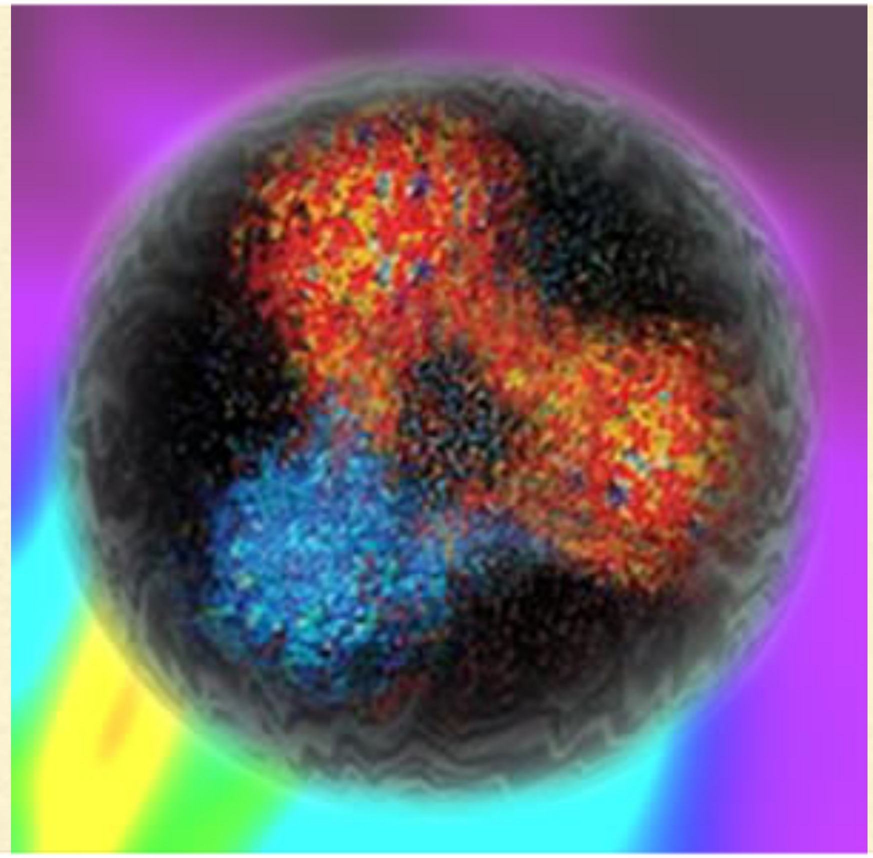


PION properties from Lattice QCD

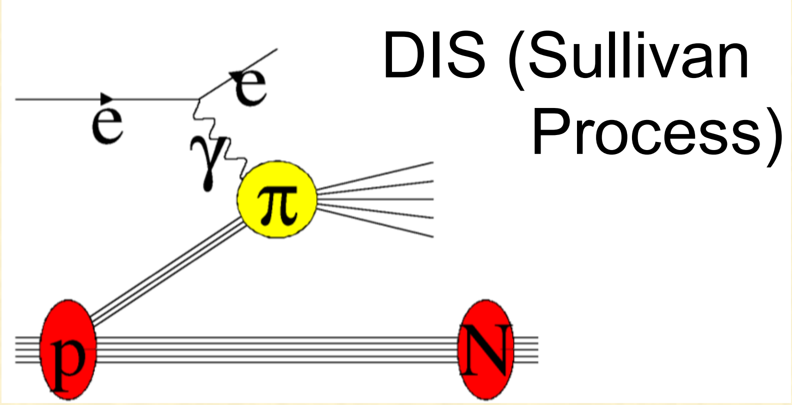
R. Briceno, B. Chakraborty, R. Edwards, A. Gambhir, B. Joo, J. Karpie, A. Kusno, C. Monahan, **K. Orginos**, D. Richards, S. Zafeiropoulos

WILLIAM & MARY
CHARTERED 1693

Jefferson Lab
Thomas Jefferson National Accelerator Facility

MOTIVATION

- Understand from first principles the structure of the pion
- Unlike the nucleon, pion parton distribution function are not well determined experimentally
- Experimental effort at JLab 12GeV to determine pion PDFs.
  - Sullivan process
  - Pion Drell-Yan
- Future EIC experiments



[T. Horn DIS2017]

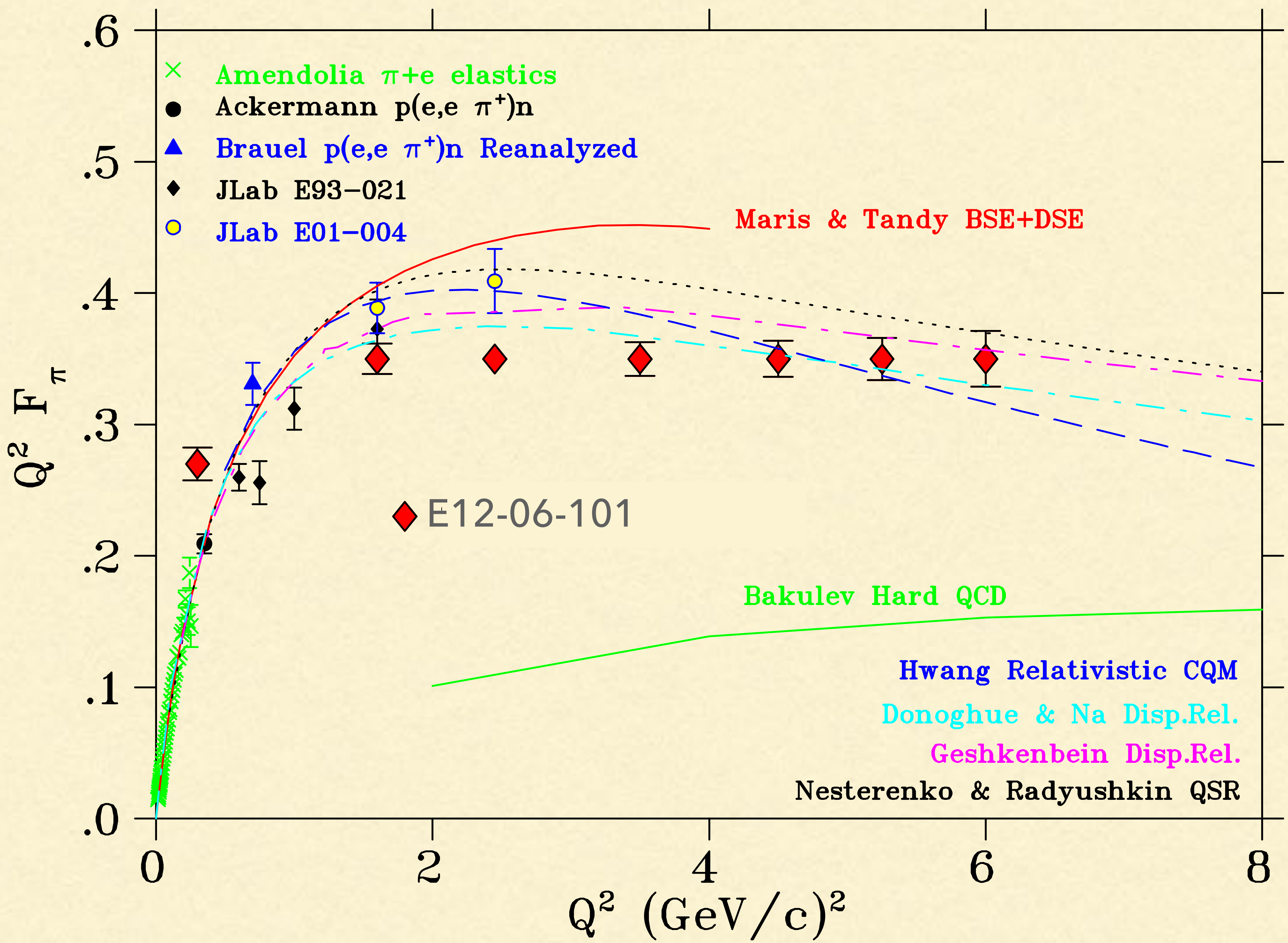
MOTIVATION

- JAM global fit analysis (@JLAB)
  - Will benefit from theoretical input from Lattice QCD
  - Exploring the impact on the global fits first couple of moments will have
  - Interested in the large x region (x>.2)
- Pion is the cloud in the nucleon: Understand pion structure leads to insight to nucleon structure.
  - Light quark asymmetries

MOTIVATION

- Pion distribution amplitude
- Offers insight to the interplay of high an low energy scales in a hadron
- High Q^2 factorization (ex. EM form factor)

PION FORM FACTOR



MOTIVATION

- Pion is the lightest hadron
- Pion is relatively easy for lattice calculations
- Statistical noise much smaller than the nucleon
- Offers an interesting playground to test new ideas for hadron structure calculations

QUASI-PARTON DISTRIBUTIONS

- Goal: Compute properties of hadrons from first principles
  - Parton distribution functions (PDFs)
- Lattice QCD calculations is a first principles method
  - For many years calculations focused on Mellin moments
  - Can be obtained from local matrix elements of the proton in Euclidean space
    - Breaking of rotational symmetry –> power divergences
    - only first few moments can be computed
- Recently direct calculations of PDFs in Lattice QCD are proposed
- First lattice Calculations already available

X. Ji, Phys.Rev.Lett. 110, (2013)

Y.-Q. Ma J.-W. Qiu (2014) 1404.6860

H.-W. Lin, J.-W. Chen, S. D. Cohen, and X. Ji, Phys.Rev. D91, 054510 (2015)

C. Alexandrou, et al, Phys. Rev. D92, 014502 (2015)

QUASI-PARTON DISTRIBUTIONS

- Defined as non-local (space), equal time matrix elements in Euclidean space
- Equal time: rotation to Minkowski space is trivial
- PDFs are obtained in the limit of infinite proton momentum
- Matching to the infinite momentum limit can be obtained through perturbative calculations

X. Xiong, X. Ji, J. H. Zhang, Y. Zhao, Phys. Rev. D 90, no. 1, 014051 (2014)
T. Ishikawa et al. arXiv:1609.02018 (2016)

QPDFS: DEFINITION

Light-cone PDFs:

$$f^{(0)}(\xi) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega^-}{4\pi} e^{-i\xi P^+ \omega^-} \left\langle P \middle| T\,\overline{\psi}(0,\omega^-,\mathbf{0}_\mathrm{T}) W(\omega^-,0)\gamma^+ \frac{\lambda^a}{2}\psi(0) \middle| P \right\rangle_\mathrm{C} .$$

$$W(\omega^-,0) = \mathcal{P}\exp\left[-ig_0 \int_0^{\omega^-} \mathrm{d}y^- A_\alpha^+(0,y^-,\mathbf{0}_\mathrm{T}) T_\alpha\right]$$

$$\langle P'|P\rangle = (2\pi)^3 2P^+ \delta\left(P^+ - P'^+\right)\delta^{(2)}\left(\mathbf{P}_\mathrm{T} - \mathbf{P}'_\mathrm{T}\right)$$

Moments:

$$a_0^{(n)} = \int_0^1 \mathrm{d}\xi\,\xi^{n-1}\left[f^{(0)}(\xi) + (-1)^n \overline{f}^{(0)}(\xi)\right] = \int_{-1}^1 \mathrm{d}\xi\,\xi^{n-1} f(\xi)$$

Local matrix elements:

$$\left\langle P|\mathcal{O}_0^{\{\mu_1\ldots\mu_n\}}|P\right\rangle = 2a_0^{(n)}\left(P^{\mu_1}\cdots P^{\mu_n} - \text{traces}\right)$$

$$\mathcal{O}_0^{\{\mu_1\cdots\mu_n\}} = i^{n-1}\overline{\psi}(0)\gamma^{\{\mu_1}D^{\mu_2}\cdots D^{\mu_n\}}\frac{\lambda^a}{2}\psi(0) - \text{traces}$$

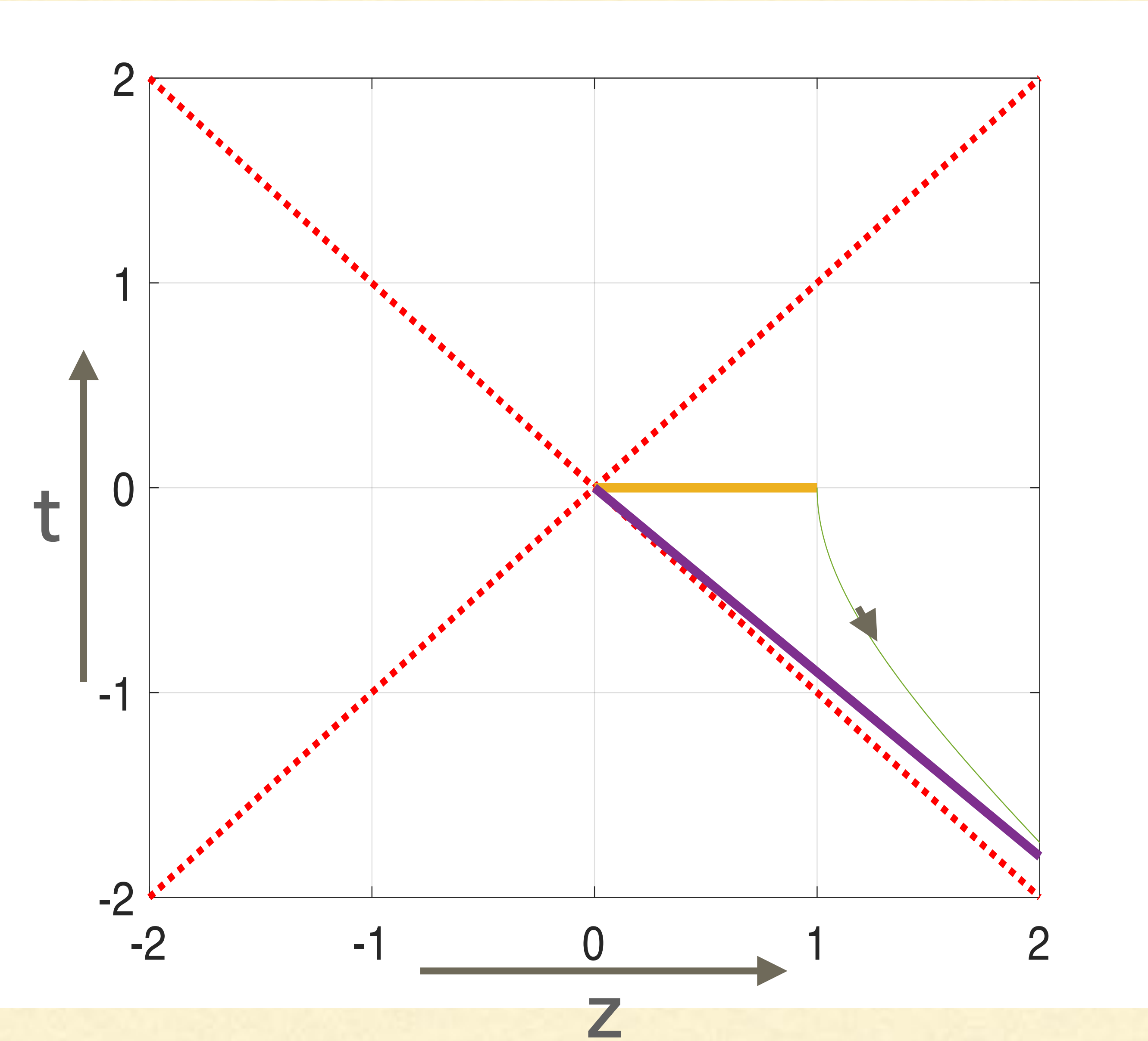
QPDFS: DEFINITION

$$h^0\left(z,P_z\right)=\frac{1}{2P_z}\left\langle P_z\left|\overline{\psi}(z)\mathbf{W}(0,z;\tau)\gamma_z\frac{\lambda^a}{2}\psi(0)\right|P_z\right\rangle_{\mathrm{C}}$$

$$\mathbf{W}(z,0)=\mathcal{P}\exp\left[-ig_0\int_0^z \mathrm{d}z'\,A^3_\alpha(z'\mathbf{v})\mathbf{T}_\alpha\right],\quad \mathbf{v}=(0,0,1,0)$$

$$q^{(0)}\left(\xi,P_z\right)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}z\,e^{i\xi zP_z}h^{(0)}\left(z,P_z\right)$$

QPDFS: MAIN IDEA



$$\lim_{P_z \to \infty} q^{(0)}(x, P_z) = f(x)$$

X. Ji, Phys.Rev.Lett. 110, (2013)

Euclidean space time local matrix element is equal to the same matrix element in Minkowski space

$$q\left(x,P_z\right)=\int_{-1}^{1}\frac{d\xi}{\xi}\,\widetilde{Z}\left(\frac{x}{\xi},\frac{\mu}{P_z}\right)f(\xi,\mu)+\mathcal{O}(\Lambda_{\rm QCD}/P_z,M_N/P_z)$$

The matching kernel can be computed in perturbation theory

X. Xiong, X. Ji, J. H. Zhang, Y. Zhao, Phys. Rev. D 90, no. 1, 014051 (2014)
T. Ishikawa et al. arXiv:1609.02018 (2016)

- Practical calculations require a regulator (Lattice)
- Continuum limit has to be taken
  - renormalization
- Momentum has to be large compared to hadronic scales to suppress higher twist effects
- Practical issue with LQCD calculations at large momentum … signal to noise ratio

GRADIENT FLOW SMEARING

Is a way to obtain a finite matrix element in the continuum that can then be used to obtain the light cone PDFs

For fermonic correlation functions an additional wave function renormalization remains.

Ringed fermions is a way to remove this renormalization in simple way.

Ringed smeared fermions

$$\mathring{\chi}(\tau, x) = \sqrt{\frac{-2\dim(R)N_{\mathrm{f}}}{(4\pi)^2\tau^2\left\langle\bar{\chi}(\tau,x)\overleftrightarrow{\not{D}}\chi(\tau,x)\right\rangle}}\,\chi(\tau,x),$$

$$\mathring{\bar{\chi}}(\tau, x) = \sqrt{\frac{-2\dim(R)N_{\mathrm{f}}}{(4\pi)^2\tau^2\left\langle\bar{\chi}(\tau,x)\overleftrightarrow{\not{D}}\chi(\tau,x)\right\rangle}}\,\bar{\chi}(\tau,x)$$

Ringed fermion correlation functions require no additional renormalization

. H. Makino and H. Suzuki, PTEP 2014, 063B02 (2014), 1403.4772.

. K. Hieda and H. Suzuki (2016), 1606.04193

SMEARED QUASI-PDFS

$$h^{(s)}\left(\frac{z}{\sqrt{\tau}}, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}M_{\mathrm{N}}\right) = \frac{1}{2P_z}\left\langle P_z \left| \overline{\chi}(z;\tau)\mathcal{W}(0,z;\tau)\gamma_z \frac{\lambda^a}{2}\chi(0;\tau) \right| P_z \right\rangle_{\mathrm{C}}$$

τ is the flow time

χ is the ringed smeared quark field

$\mathcal{W}$ is the smeared gauge link

$$q^{(s)}\left(\xi, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}M_{\mathrm{N}}\right) = \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{2\pi} e^{i\xi z P_z} P_z\, h^{(s)}(\sqrt{\tau}z, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}M_{\mathrm{N}}),$$

At fixed flow time the quasi-PDF is finite in the continuum limit

Using the previous definitions we have

$$\left(\frac{i}{P_z}\frac{\partial}{\partial z}\right)^{n-1} h^{(s)}\left(\frac{z}{\sqrt{\tau}}, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\rm QCD}, \sqrt{\tau}M_{\rm N}\right) = \int_{-\infty}^{\infty} \mathrm{d}\xi\, \xi^{n-1} e^{-i\xi z P_z} q^{(s)}\left(\xi, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\rm QCD}, \sqrt{\tau}M_{\rm N}\right)$$

By introducing the moments

$$b_n^{(s)}\left(\sqrt{\tau}P_z, \frac{\Lambda_{\rm QCD}}{P_z}, \frac{M_{\rm N}}{P_z}\right) = \int_{-\infty}^{\infty} \mathrm{d}\xi\, \xi^{n-1} q^{(s)}\left(\xi, \sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\rm QCD}, \sqrt{\tau}M_{\rm N}\right)$$

Taking the limit of z going to 0 we obtain:

$$b_n^{(s)}\left(\sqrt{\tau}P_z, \frac{\Lambda_{\rm QCD}}{P_z}, \frac{M_{\rm N}}{P_z}\right) = \frac{c_n^{(s)}(\sqrt{\tau}P_z)}{2P_z^n}\left\langle P_z\left|\left[\overline{\chi}(z;\tau)\gamma_z(i\overleftarrow{D}_z)^{(n-1)}\frac{\lambda^a}{2}\chi(0;\tau)\right]_{z=0}\right|P_z\right\rangle_{\rm C}.$$

i.e. the moments of the quasi-PDF are related to local matrix elements of the smeared fields

These matrix elements are not twist-2. Higher twist effects enter as corrections that scale as powers of

$$\frac{\Lambda_{\rm QCD}}{P_z}\,,\quad \frac{M_N}{P_z}$$

[H.-W. Lin, et. al Phys.Rev. D91, 054510 (2015)]

after removing M_N/P_z effects

$$b_n^{(s)}\left(\sqrt{\tau}P_z, \sqrt{\tau}\Lambda_{\rm QCD}\right) = c^{(s)}(\sqrt{\tau}P_z)b_n^{(s,{\rm twist-2})}\left(\sqrt{\tau}\Lambda_{\rm QCD}\right) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2}\right)$$

Small flow time expansion:

Luscher ['10,'13]

$$b_n^{(s,\mathrm{twist}-2)}\big(\sqrt{\tau}\Lambda_{\mathrm{QCD}}\big) = \widetilde{C}_n^{(0)}(\sqrt{\tau}\mu)a^{(n)}(\mu) + \mathcal{O}(\sqrt{\tau}\Lambda_{\mathrm{QCD}})$$

$a^{(n)}(\mu)$ are the moments of the PDFs

The quasi-PDF moments then are:

$$b_n^{(s)}\big(\sqrt{\tau}\Lambda_{\mathrm{QCD}}\big) = C_n^{(0)}(\sqrt{\tau}\mu, \sqrt{\tau}P_z)a^{(n)}(\mu) + \mathcal{O}\left(\sqrt{\tau}\Lambda_{\mathrm{QCD}}, \frac{\Lambda_{\mathrm{QCD}}^2}{P_z^2}\right)$$

$$\Lambda_{\mathrm{QCD}}, M_N \ll P_z \ll \tau^{-1/2}$$

Introducing a kernel function such that:

$$C_n^{(0)}(\sqrt{\tau}\mu, \sqrt{\tau}P_z) = \int_{-\infty}^{\infty} dx\, x^{n-1} \widetilde{Z}(x, \sqrt{\tau}\mu, \sqrt{\tau}P_z)$$

We can undo the Mellin transform:

$$q^{(s)}\left(x, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}P_z\right) = \int_{-1}^{1} \frac{d\xi}{\xi} \widetilde{Z}\left(\frac{x}{\xi}, \sqrt{\tau}\mu, \sqrt{\tau}P_z\right) f(\xi, \mu) + \mathcal{O}(\sqrt{\tau}\Lambda_{\mathrm{QCD}})$$

Therefore smeared quasi-PDFs are related to PDFs if

$$\Lambda_{\mathrm{QCD}}, M_N \ll P_z \ll \tau^{-1/2}.$$

PION DISTRIBUTION AMPLITUDE

Q-DA

$$\tilde{\phi}(x, P_z) = \frac{i}{f_\pi} \int \frac{dz}{2\pi} e^{-i(x-1)P_z z} \langle \pi(P) | \bar{\psi}(0) \gamma^z \gamma_5 \Gamma(0, z) \psi(z) | 0 \rangle$$

Factorization:

$$\tilde{\phi}(x, \Lambda, P_z) = \int_0^1 dy \, Z_\phi(x, y, \Lambda, \mu, P_z) \phi(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{P_z^2}, \frac{m_\pi^2}{P_z^2} \right)$$

X. Ji, Phys.Rev.Lett. 110, (2013)

Zhang et. al. ‘17

COMPUTATION METHOD

- Use distillation
- All 3-projects can be done with large overlap of resource requirements
  - Additional contraction cost and operator insertion for 3pt functions
  - Common feature: Large momentum for the pion

DISTILATION

Gaussian smearing:

$$S = e^{\tau \nabla^2(U)} \qquad \text{or:} \qquad S = e^{\tau \nabla^2(U)} = \sum_\lambda e^{-\tau\lambda} |\lambda\rangle\langle\lambda|$$

Distillation smearing smearing:

$$S_{dist} = \sum_\lambda^{\lambda_{max}} |\lambda\rangle\langle\lambda|$$

Result: Factorization of correlation functions

DISTILATION

$$C_{2pt}(t) = \mathrm{Tr}\left(M(t)P(t,0)M(0)^\dagger P(0,t)\right)$$ Two point function

$$C_{3pt}(t) = \mathrm{Tr}\left(M(t)G(t,0)M(0)^\dagger P(0,t)\right)$$ Three point function

$$M(t)^{s,s'}_{\lambda,\lambda'} = \langle\lambda|\,\Gamma^{s,s'}\,|\lambda'\rangle$$ Meson Operator

$$P(t,0)^{s,s'}_{\lambda,\lambda'} = \langle\lambda|\left[D^{-1}\right]^{s,s'}|\lambda'\rangle$$ Perambulator

$$G(t,0)^{s,s'}_{\lambda,\lambda'} = \langle\lambda|\left[D^{-1}\mathcal{O}D^{-1}\right]^{s,s'}|\lambda'\rangle$$ Generalized Perambulator

DISTILATION

★

$$M(t)^{s,s'}_{\lambda,\lambda'} = \langle\lambda|\,\Gamma^{s,s'}\,|\lambda'\rangle$$

Meson Operator

$$P(t,0)^{s,s'}_{\lambda,\lambda'} = \langle\lambda|\left[D^{-1}\right]^{s,s'}|\lambda'\rangle$$

Perambulator

$$G(t,0)^{s,s'}_{\lambda,\lambda'} = \langle\lambda|\left[D^{-1}\mathcal{O}D^{-1}\right]^{s,s'}|\lambda'\rangle$$

Generalized Perambulator

General building blocks

Computed and stored to be used in many projects

Contractions done in an general fashion
Red Star Software (R. Edwards)

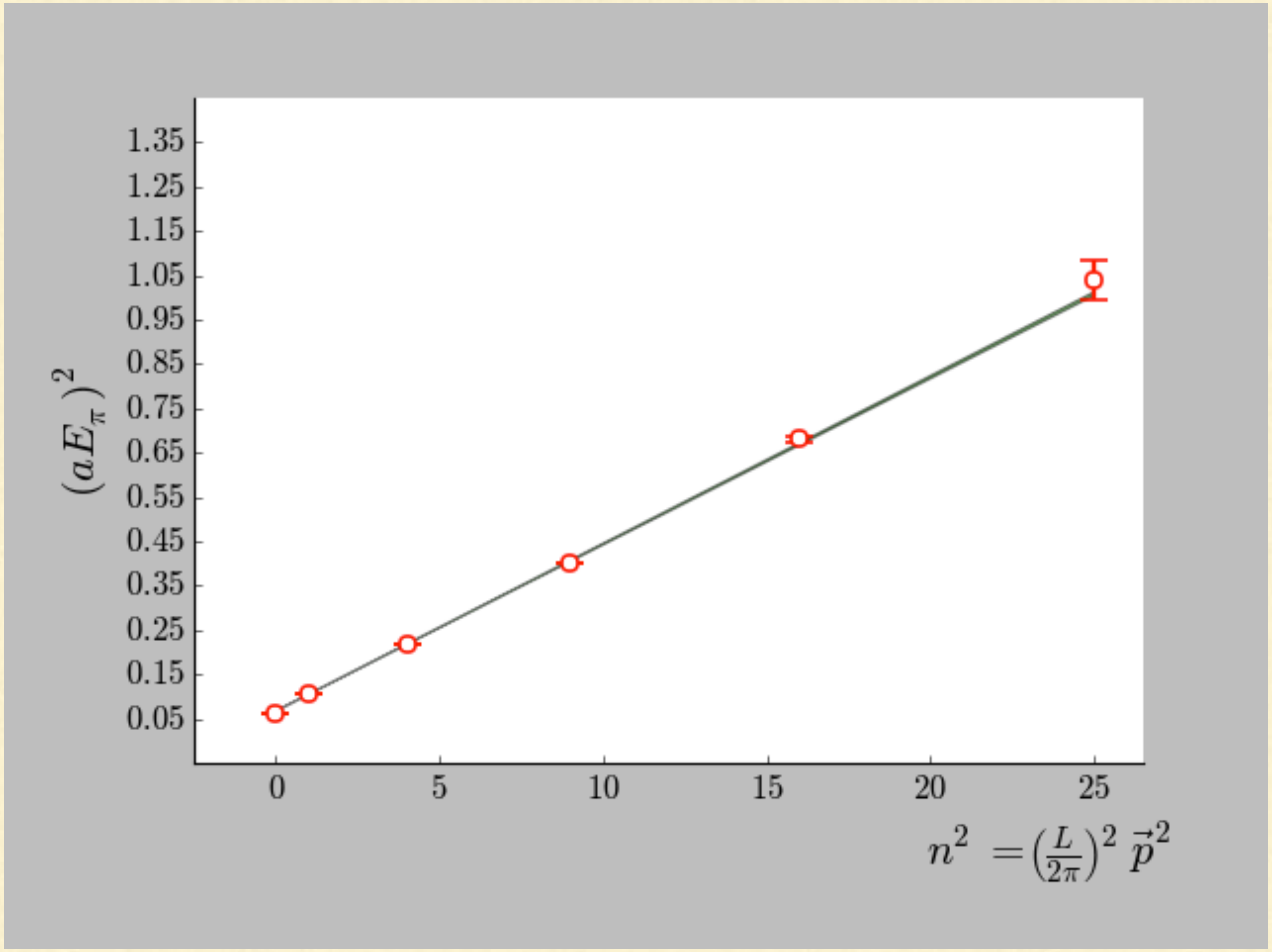
PRELIMINARY TESTING

Quenched calculation under way

Test codes (analysis and aspects of the methodology)

Using pre-existing building blocks high-Q^2 pion form factor calculation under way

Pion dispersion relation (quenched β=6.0) 32^3x64



Max P ~ 2 GeV

REQUEST

We request an allocation of 56.6M KNL core-hours (169.8M JPsi core-hours) on the KNL machine at JLab. We request disk and achival storage of 200 TByte and 200 TByte respectively, equivalent to 8M and 1.2M JPsi core-hours.

Ensemble: $64^3 \times 128$, $N_f = 2 \oplus 1$ $M_\pi \simeq 170$ MeV $a \simeq 0.091$fm (200 configs) $M_\pi L$=4.8

Projects

- Pion DA
- Pion PDF
- Pion Form Factor

Can you compare your proposed numerical method, distillation , to other approaches like AMA? What are the advantages of distillation? How does the costs compare?

Distillation and AMA are two different things. Distillation is a method for obtaining interpolating fields that allow exceptionally good control of excited states as the JLab spectroscopy program has demonstrated. AMA is a noise reduction method. Unfortunately the AMA assumes that the contraction cost is small relative to propagator cost therefore by substantially reducing the cost of the propagator calculation by solving with low precision for most of the propagators a major efficiency gain is obtained. In our case the cost of contractions is not negligible and as a result AMA in the form that it has been formulated is not beneficial for us. Methods for improving the overall efficiency of our computations are currently under way by members of our team.

You plan to analyze 200 configurations using significant computational resources. Is that sufficient to get physical predictions?

We think that 200 configurations are sufficient. Our improved interpolating fields with Distillation play an important role in this.

Would it make more sense to start with smaller volumes, perhaps larger pion mass, to gain experience with the method?

Perhaps this is a possibility, although not preferable, if resources are not available. Here we would like to stress that the generated data with Distillation (perambulators and meson elementals) are the same as those used in spectroscopy and will be useful for other on going projects at JLab for the years to come.

This proposal is much larger than many others with similar physics goals, even though the lattice spacing is not particularly fine. Also the method is new and exploratory. How do you justiffy such a large request?

We are not aware of the other proposals so we cannot comment on this point. The perambulators and generalized perambulators that we will generate, even though absolutely essential for our calculation, will be of value to subsequent investigations of hadron structure, a further advantage of the distillation approach.

Can you comment on other works/groups using the Ji method? What is their experience? How does your proposed work fit into the international and USQCD program?

We are aware of the work of Lin et. al. as well as the ETMC work. In both cases they renormalization issues are not addressed. Our gradient flow methods allows for the computation of a matrix element that requires no renormalization, therefore it can be extrapolated to the continuum. Perturbative calculations to obtain the matching kernel are underway (C. Monahan).

You request a significant fraction of the available KNL resources. Would your calculation be better fitted to an ALCC proposal that rewards high risk, high reward projects, leaving local USCQD resources to projects that do not require such large scale computing?

We do not have an ALCC allocation for this project.
Besides the renormalization issues, a primary issue is the range of x accessible with the P_z in your calculation. Do you have plans to study how large a P_z is accessible for a given lattice, and the values of x of the light-cone distributions that can be faithfully reproduced?

We are already studying this issue in an ongoing exploratory calculation of a high momentum transfer pion form factor.
Based on the results we get from this study we will consider ways to further improve our high momentum interpolating fields.
An advantage of the distillation approach is the ability to construct interpolating operators that satisfy the symmetries of the lattice, even for states in motion, thereby minimizing the number of states that can contribute as the momentum is increased, and the ability to use the variational approach at non-zero momentum. Finally, we emphasize that reaching the smallest values of Bjorken x is not the aim here. There is considerable interest in parton distributions at mid- and higher values of x, and indeed this is a key part of the JLab program.