Lattice Calculation of PDFs

Two Challenges….

- Euclidean lattice precludes the calculation of light-cone correlation functions
  - So… …Use *Operator-Product-Expansion* to formulate in terms of *Mellin Moments* with respect to *Bjorken x*.

$$q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P \mid \bar{\psi}(\xi^-)\gamma^+ e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)} \psi(0) \mid P \rangle$$

$$\downarrow$$

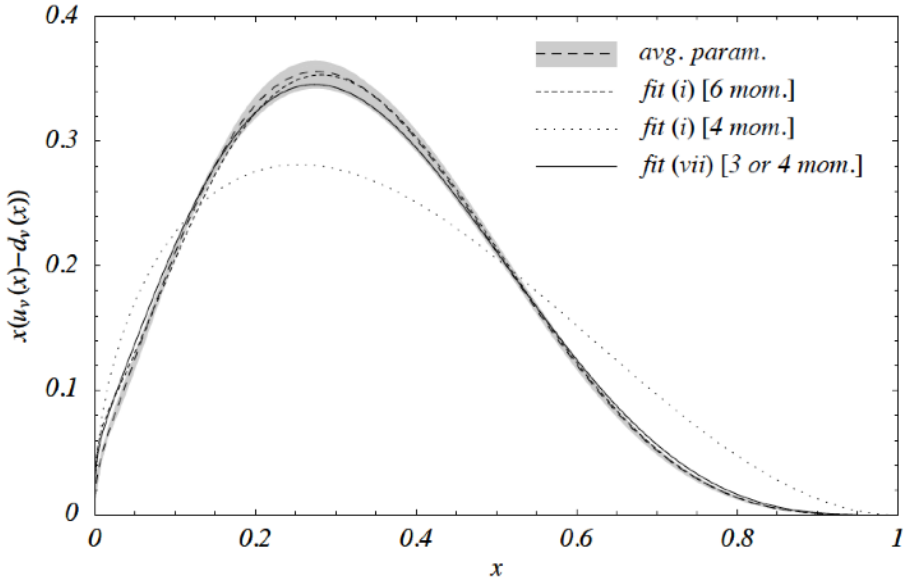
$$\langle P \mid \bar{\psi}\gamma_{\mu_1}(\gamma_5) D_{\mu_2} \ldots D_{\mu_n} \psi \mid P \rangle \rightarrow P_{\mu_1} \ldots P_{\mu_n} a^{(n)}$$

  - *Generalized Parton Distributions (off-forward): GPDs*
  - *Quark Distribution Amplitudes in exclusive processes: PDAs*
  - *(Transverse-Momentum-Dependent Distributions): TMDs*

- Discretisation, and hence reduced symmetry of the lattice, introduces power-divergent mixing for N >3 moment.

Higher Moments of Parton Distributions

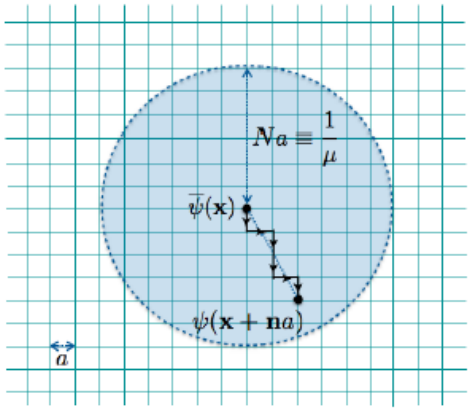
$$x(u_v(x) - d_v(x)) = a x^b (1-x)^c (1 + \epsilon\sqrt{x} + \gamma x)$$



IsoVector Distribution

Need to constrain parameters from phenomenology.

Detmold, Melnitchouk, Thomas
Eur.Phys.J.direct C3:1-15,2001



Use **improved, extended operators** to reduce power-divergent mixing. c.f. restoration of rotational symmetry for interpolating operators in spectroscopy

Davoudi and Savage, PRD86, 054505 (2012)

"Higher Moments of Parton Distribution Functions", Z. Davoudi et al, exploratory quenched calculation at fine lattice spacing, 800 MeV pion.

Quasi Distributions

- A solution, LaMET (Large Momentum Effective Theory) was proposed by X.Ji

X. Ji, Phys. Rev. Lett. 110 (2013) 262002

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig\int_0^z dz' A^z(z')}\psi(0) \mid P > + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2))$$

- Quasi distributions approach light-cone distributions in limit of large P^z

$$q(x,\mu^2,P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

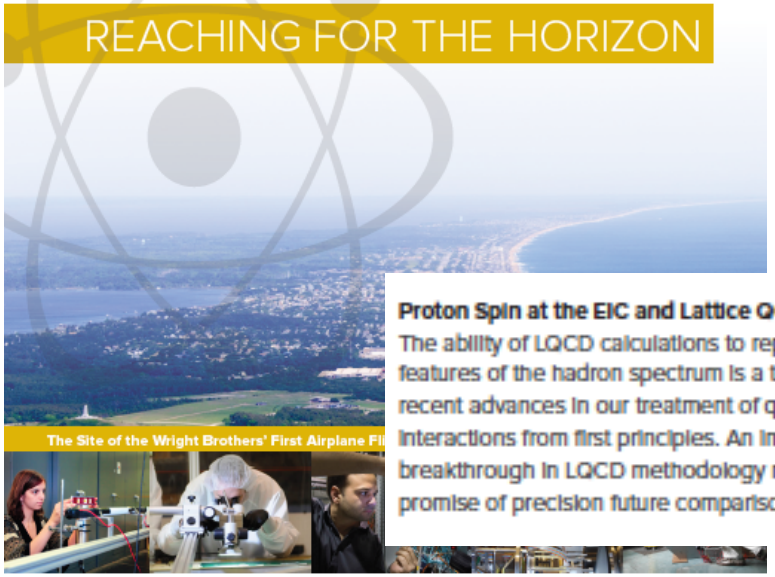
Y-Q Ma and J-W Qiu, arXiv:1404.6860

- Matching and evolution of quasi- and light-cone distributions

Carlson, Freid, arXiv:1702.05775
Isikawa et al., arXiv:1609.02018
Monahan and Orginos, arXiv:1612.01584
Radyushkin (Evolution of quasi-distributions, pion QDA,..)
Briceno, Hansen, Monahan, arXiv:1703.06072 (Euclidean Signature)

- Direct lattice calculation of hadronic tensor

K.F. Liu and S.J.Dong, PRL72, 1790 (1994); arXiv:1703.04690



The 2015
LONG RANGE PLAN
for NUCLEAR SCIENCE

The Proton as a Laboratory for QCD

How are the gluons and sea quarks, and their intrinsic spins, distributed in space and momentum inside the nucleon? What is the role of sea quark and gluon orbital motion in building the nucleon spin?

Proton Spin at the EIC and Lattice QCD

The ability of LQCD calculations to reproduce many features of the hadron spectrum is a testimony to striking recent advances in our treatment of quark and gluon interactions from first principles. An important recent breakthrough in LQCD methodology now provides the promise of precision future comparisons of theory with such detailed EIC measurements of proton spin structure as the images and distributions in Figures 2.16 and 2.17. Such comparisons will not only bring deep insight into the origin of the spin but will also shed light on the role the abundant gluons play in generating the proton's mass and confining quarks and gluons inside the proton.

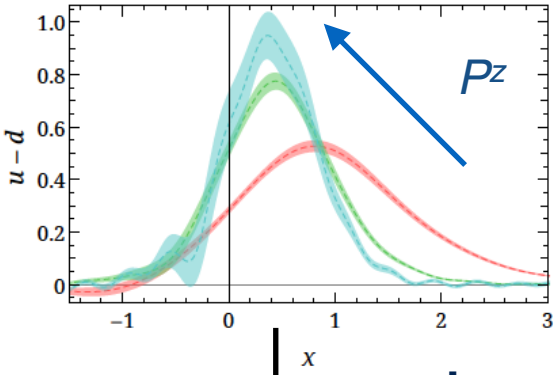
Proposals

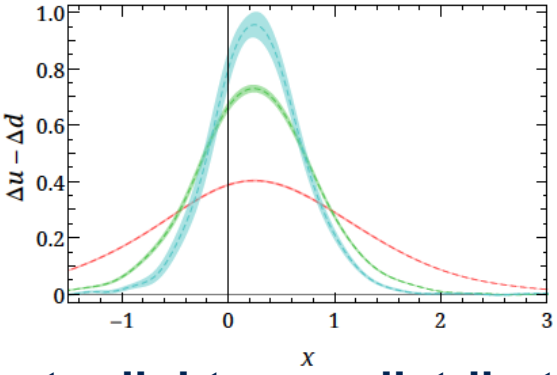
Proposal/	PI	Action	Cluster	GPU	KNL
Spin and Three-Dim. Structure of Nucleon	Lin	Clover on HISQ	61.3M		
Pion Properties from Lattice QCD	Orginos	Isotropic Clover			56.6M (169.8M)
Pion Parton Distribution Function on Fine Lattice	Jin	HYP clover on HISQ	12.17M	??	
Higher Moments of Parton Dust.	Davoudi	Quenched Clover	8.6M		

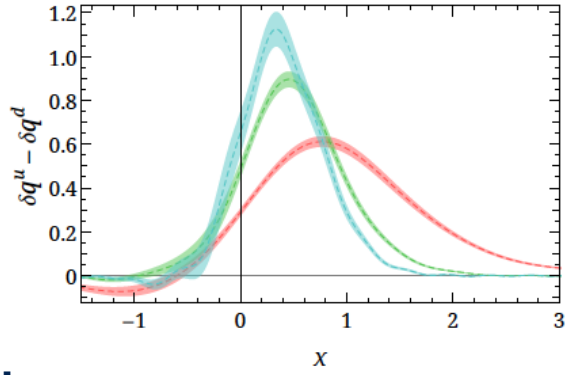
Highlights - I

H-W Lin, arXiv:1612.09366

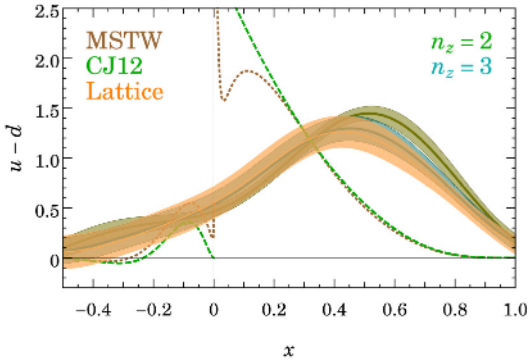
Iso-vector quasi distributions

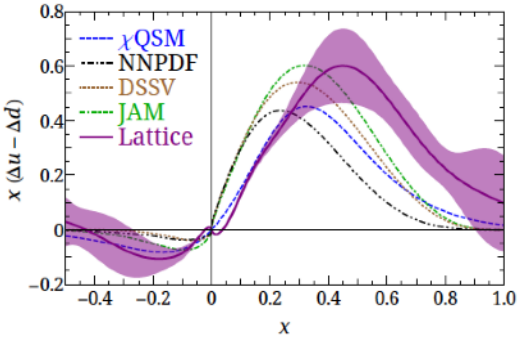


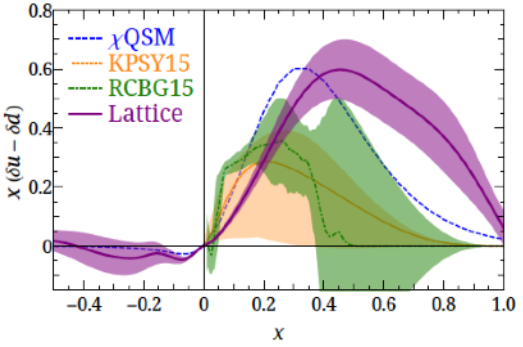




Iso-vector light-cone distributions

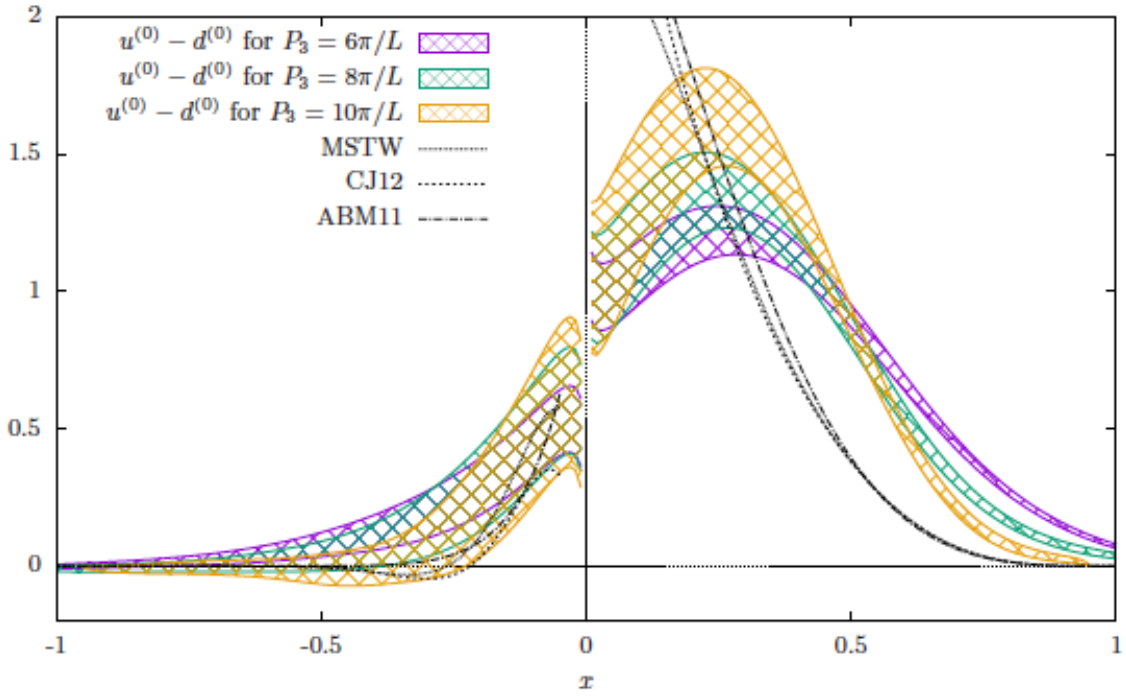






Highlights - II

Alexandrou et al., arXiv:1610.03689

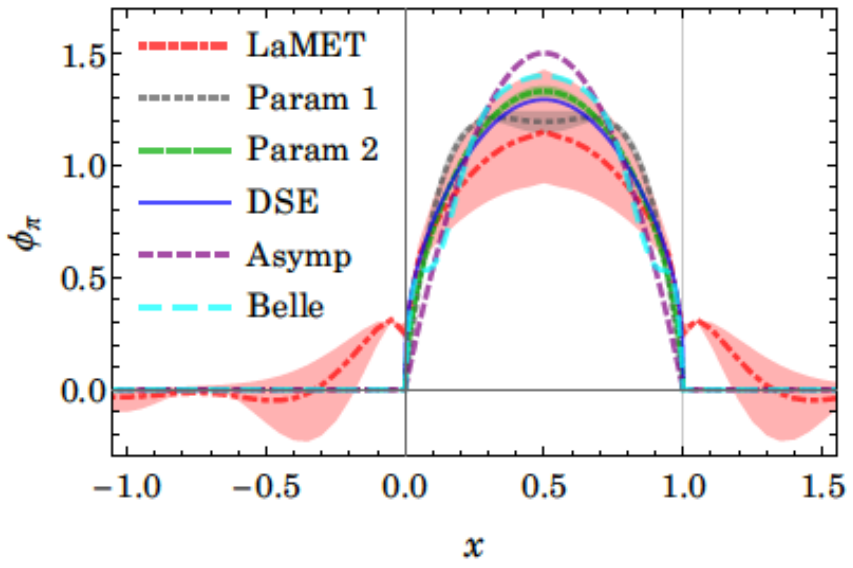


Unrenormalized PDFs
- **Twisted-Mass Fermions**
- **High Statistics**
- **Momentum-Smearing for high momenta**

Pion Distribution Amplitude

- Same operators as in polarized structure functions
- …BUT two-point function A. Radyushkin, Phys.Rev. D95 (2017) no.5, 056020
- Governs EM form factors at high Q^2

$$\phi_\pi(x) = \frac{i}{f_\pi}\int \frac{d\xi}{2\pi} e^{i(x-1)\xi\lambda\cdot P}\langle \pi(P) \mid \bar{\psi}(0)\lambda\cdot\gamma\gamma_5\Gamma(0,\xi\lambda\psi(\xi\lambda) \mid 0>$$



Zhang et al., arXiv:1702.00008

Observations

- Two of the proposals focus on properties of the pion
  - Can attain smaller values of M/P
  - Computationally less demanding
  - Renormalization should be independent of external states
- Different methods for performing matching, e.g. gradient-flow in proposal of Orginos
- Questions:
  - What is the largest value of P attainable? Use of boosted smearing, distillation.
  - What is the range of x accessible? Does it depend on P, Volume, etc?
  - How do computations impact experiment: RHIC-spin, JLab, EIC
- Road map for computations?