Nucleon Structure with clover-Wilson Fermions

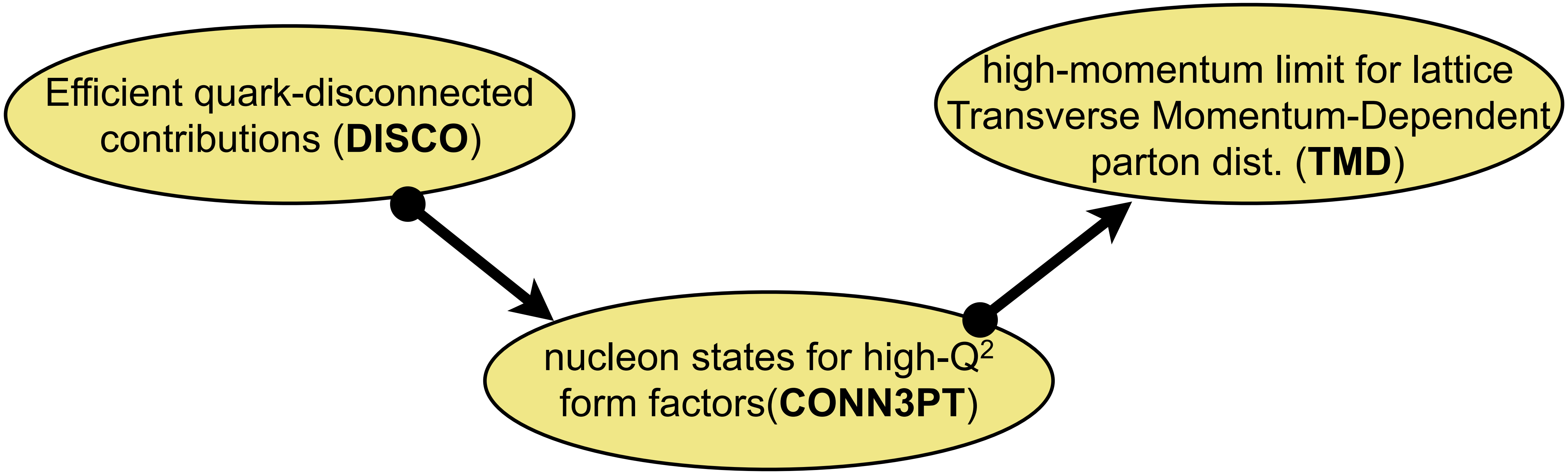
LHP proposal
M.Engelhardt (PI), A.Gambhir, J.Green,
J.Negele, A.Pochinsky, S.Syritsyn(co-PI),

USQCD All-Hands Meeting, Jefferson Lab
Apr 28-30, 2017

Nucleon Structure with Isotropic Wilson Lattices

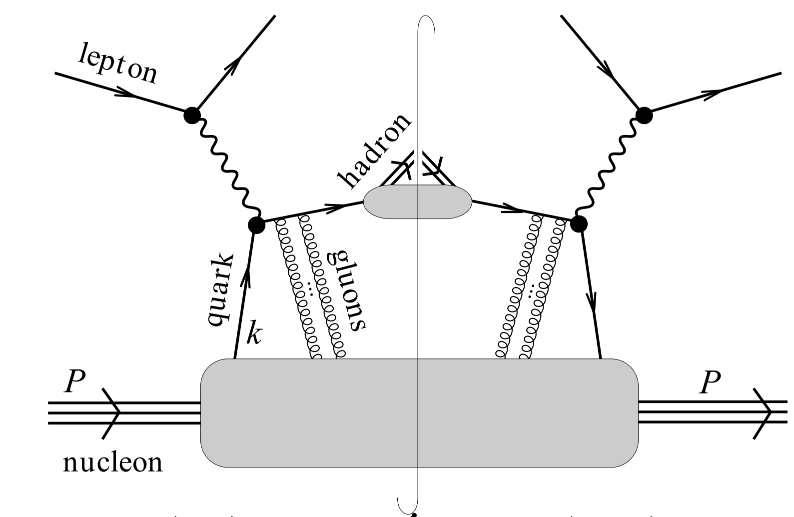
Goal : Study Flavor-Dependent Nucleon Structure at High Momentum with Stat.signal Improvement and Inclusion of Disconnected Quarks

- **DISCO**: disconnected diagrams with Hierarchical Probing and Deflation [A.Gambhir, K.Orginos] with all lattice coordinate/momenta
- **CONN3PT** : Nucleon form factors with high momentum transfer with boosted nucleon operators
- **TMD** : Transverse-momentum dependent PDFs with boosted high-momentum initial/final states

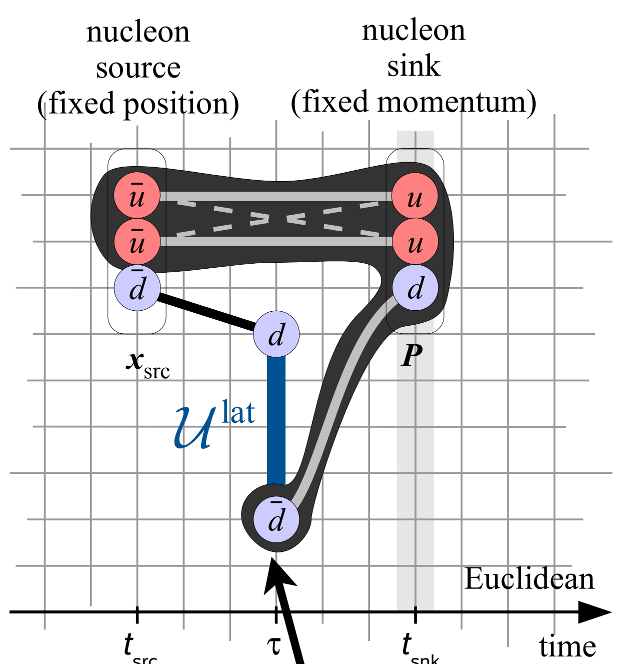


TMD Program

SIDIS



$$l + N(P) \longrightarrow l' + N(P_h) + X$$

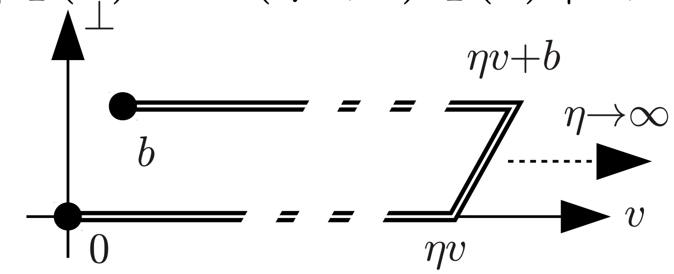


operator localized
at Euclidean time τ

Non-local lattice operator

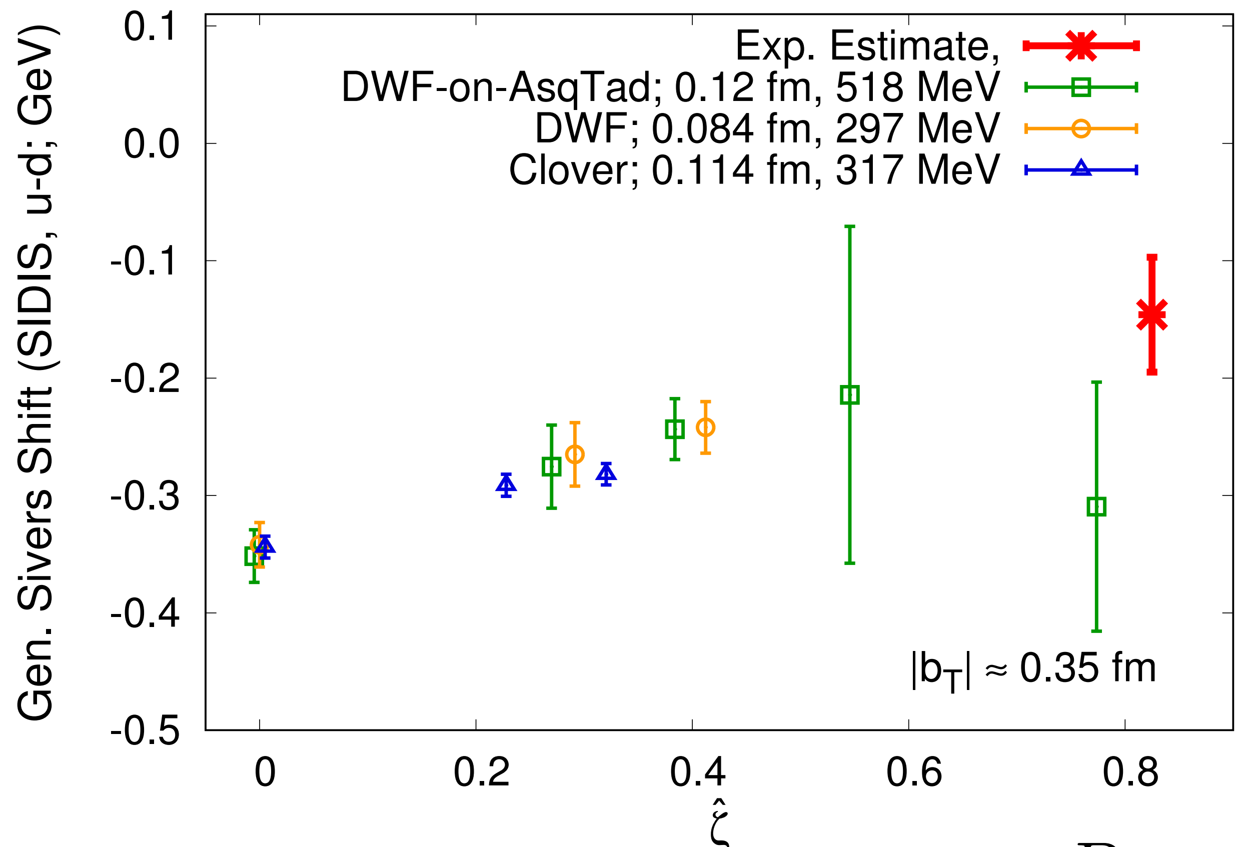
$$\Phi(b, P, S, \hat{\zeta}, \mu) = \frac{1}{2} \langle P, S | \, \bar{q}(0) \, \Gamma \, \mathcal{U}(\eta v, b) \, q(b) \, | P, S \rangle$$

with spacelike link path $\mathcal{U}$ =



probes $k_\perp$-moments
("shifts") of TMDs

$$\sim \int dx \int d^2\vec{k}_\perp \, k_i \, f(x, \vec{k}_\perp)$$

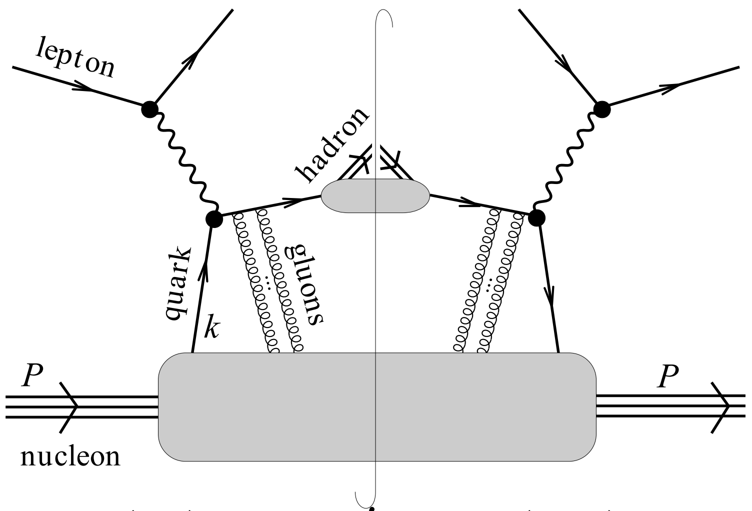


LC limit of spacelike staple
Collins-Soper parameter

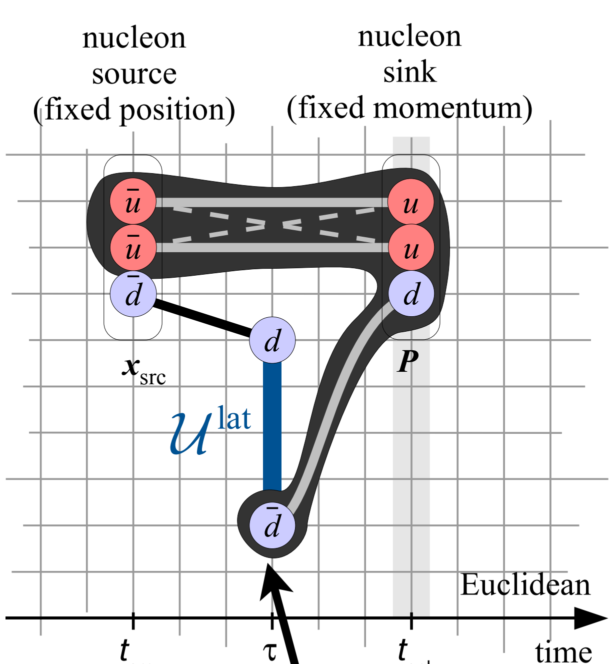
$$\hat{\zeta} = \frac{P \cdot v}{m_N \, |v|} \to \infty$$

TMD Program

SIDIS



$$l + N(P) \longrightarrow l' + N(P_h) + X$$

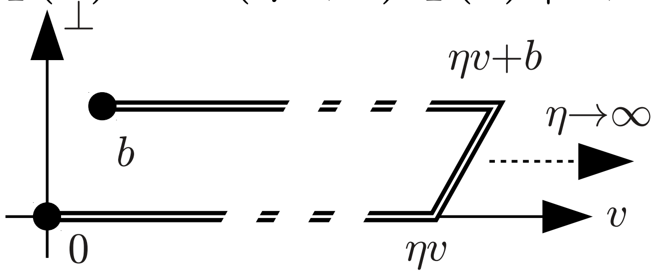


operator localized
at Euclidean time τ

Non-local lattice operator

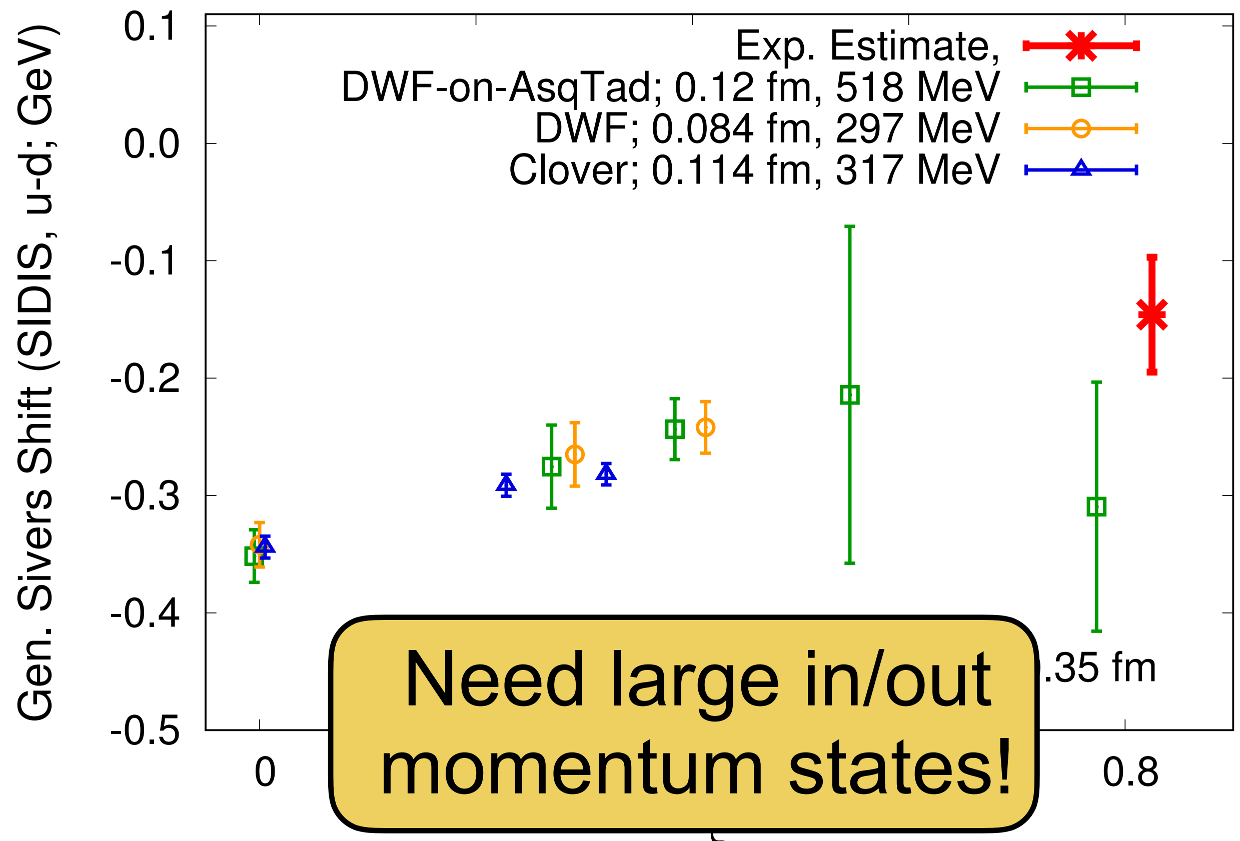
$$\Phi(b, P, S, \hat{\zeta}, \mu) = \frac{1}{2} \langle P, S | \, \bar{q}(0) \, \Gamma \, \mathcal{U}(\eta v, b) \, q(b) \, | P, S \rangle$$

with spacelike link path $\mathcal{U}$ =



probes $k_\perp$-moments
("shifts") of TMDs

$$\sim \int dx \int d^2 \vec{k}_\perp \, k_i \, f(x, \vec{k}_\perp)$$

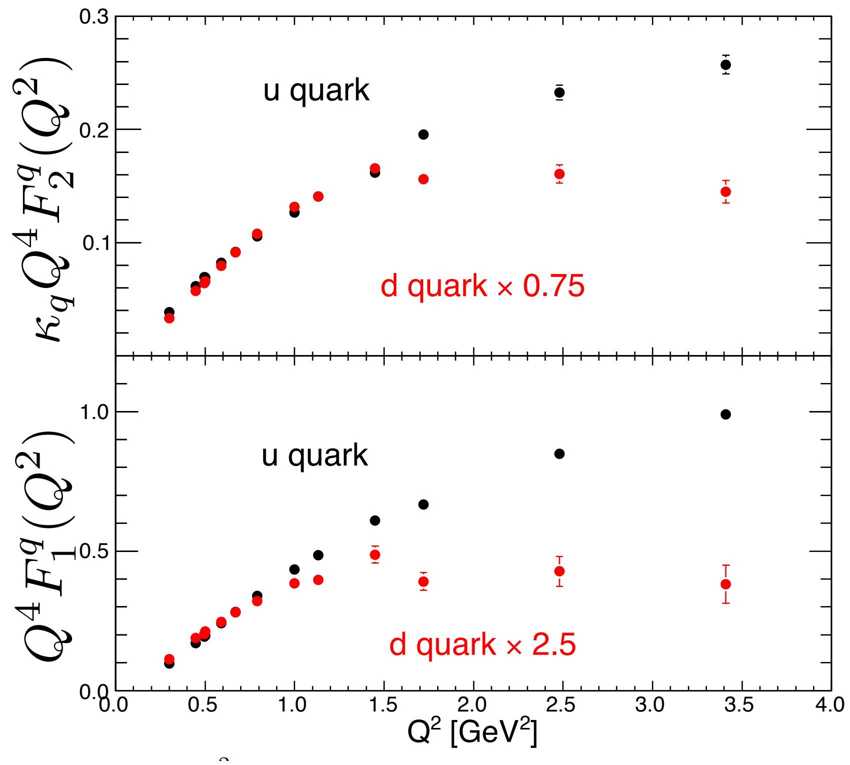


LC limit of spacelike staple
Collins-Soper parameter

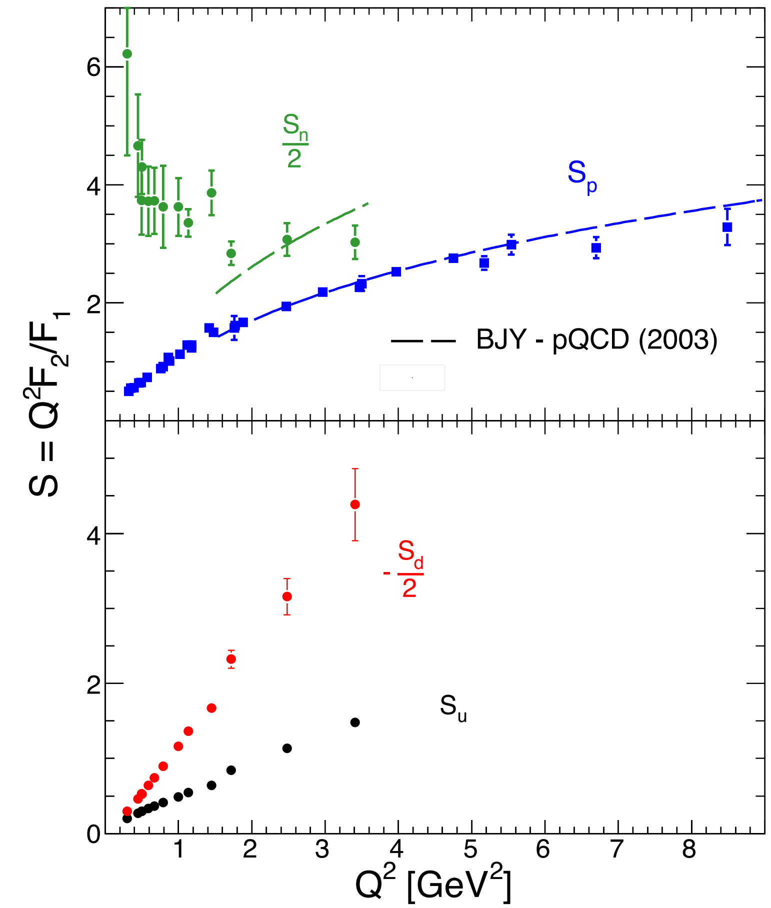
$$\hat{\zeta} = \frac{P \cdot v}{m_N \, |v|} \to \infty$$

High-Q^2 Nucleon Form Factors in Experiments

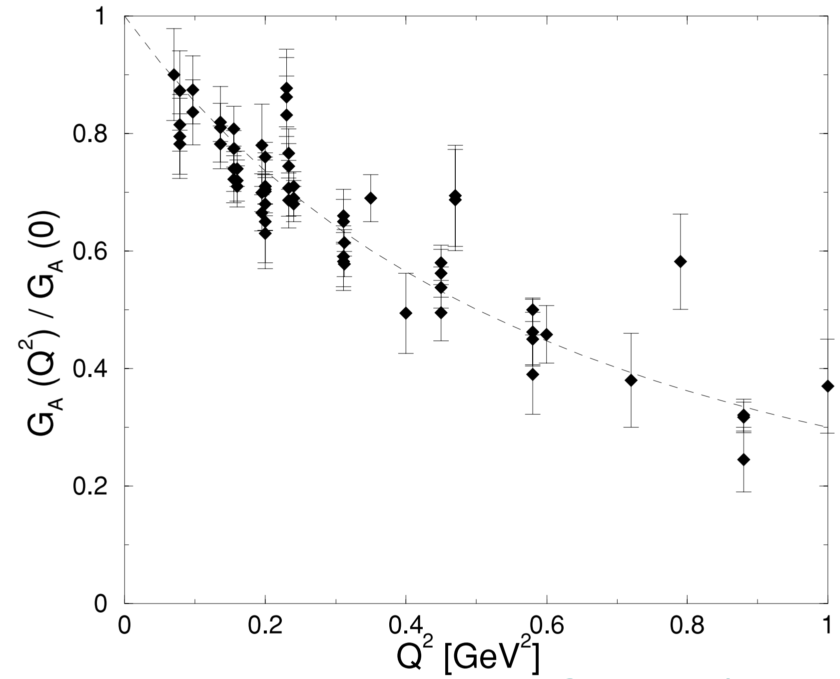
- Form Factors at high momentum:
  JLab@12GeV :
  up to 18 GeV^2;
  $Q^2 \to \infty$ scaling;
  flavor separation



u & d contributions to $F_{1,2}$ form factors [G.D.Cates et al., PRL106:252003]



- $G_A(Q^2)$ are measured in ν-scattering, π-production;
  implications for neutrino flux norm. in IceCube, DUNE
- Axial radius $\langle r_A^2 \rangle = 12 / m_A^2$: model dependence
  varying nuclear / G_A shape models: m_A=0.9 ... 1.4 GeV
- Strange quark $G^s_{A,P}(Q^2)$: MiniBooNE



[V.Bernard et at, J.Phys.G28:R1(2002)]

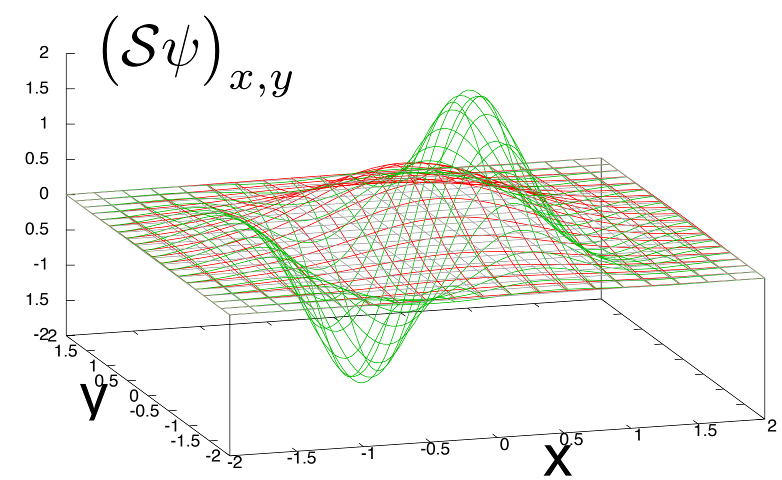
High-Momentum Nucleon States and Form Factors

- Nucleon operator on a lattice with Gaussian-"smeared" quarks does not couple well to moving hadron

$$N_{\rm lat}(x) = (\mathcal{S}\,u)^a_x \left[(\mathcal{S}\,d)^b_x\, C\gamma_5\, (\mathcal{S}\,u)^c_x\right] \epsilon^{abc}$$

$$\mathcal{S}_{\text{at-rest}} = \exp[-\frac{w^2}{4}(i\vec\nabla)^2] \sim exp(-\frac{w^2\vec k^2_{\rm lat}}{4})$$

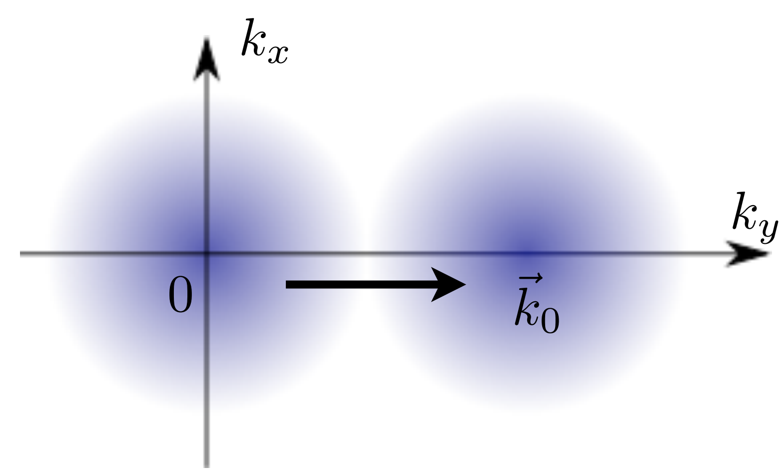
reduced overlap with boosted WF



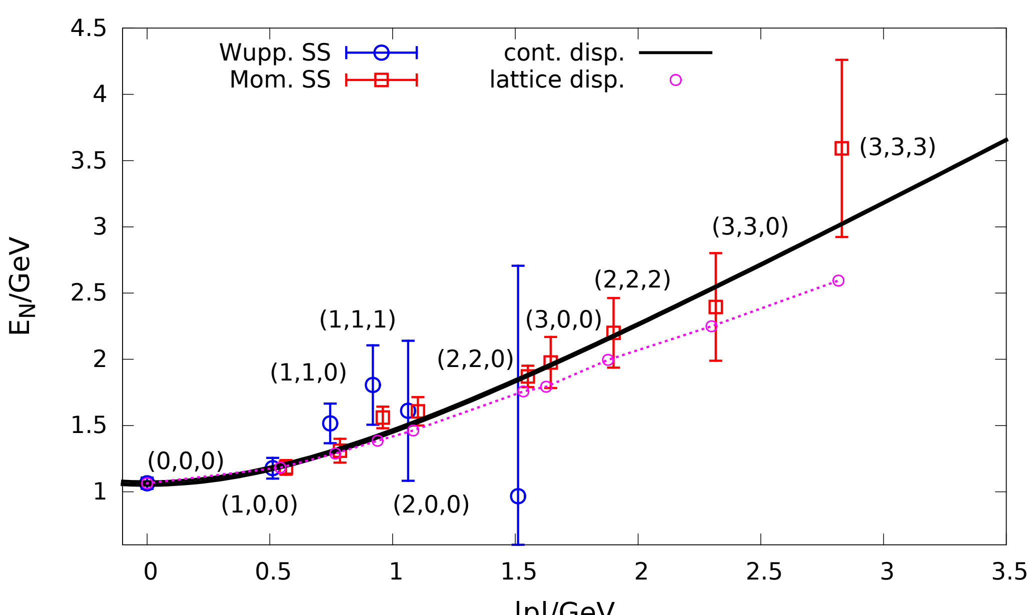
- Optimize smearing for boosted nucleon states [orig. B.Musch]

$$\mathcal{S}_{\rm boosted} = \exp[-\frac{w^2}{4}(i\vec\nabla - \vec k_0)^2]$$

$$\sim exp(-\frac{w^2(\vec k_{\rm lat} - \vec k_0)^2}{4})$$



RQCD results for spectrum [G. Bali et al, arXiv:1602.05525]



This Proposal (**CONN3PT**): continued study of nucleon structure with boosted sources

m_π =*320,190* MeV with a=0.114, a=0.081 fm

In Breit frame:

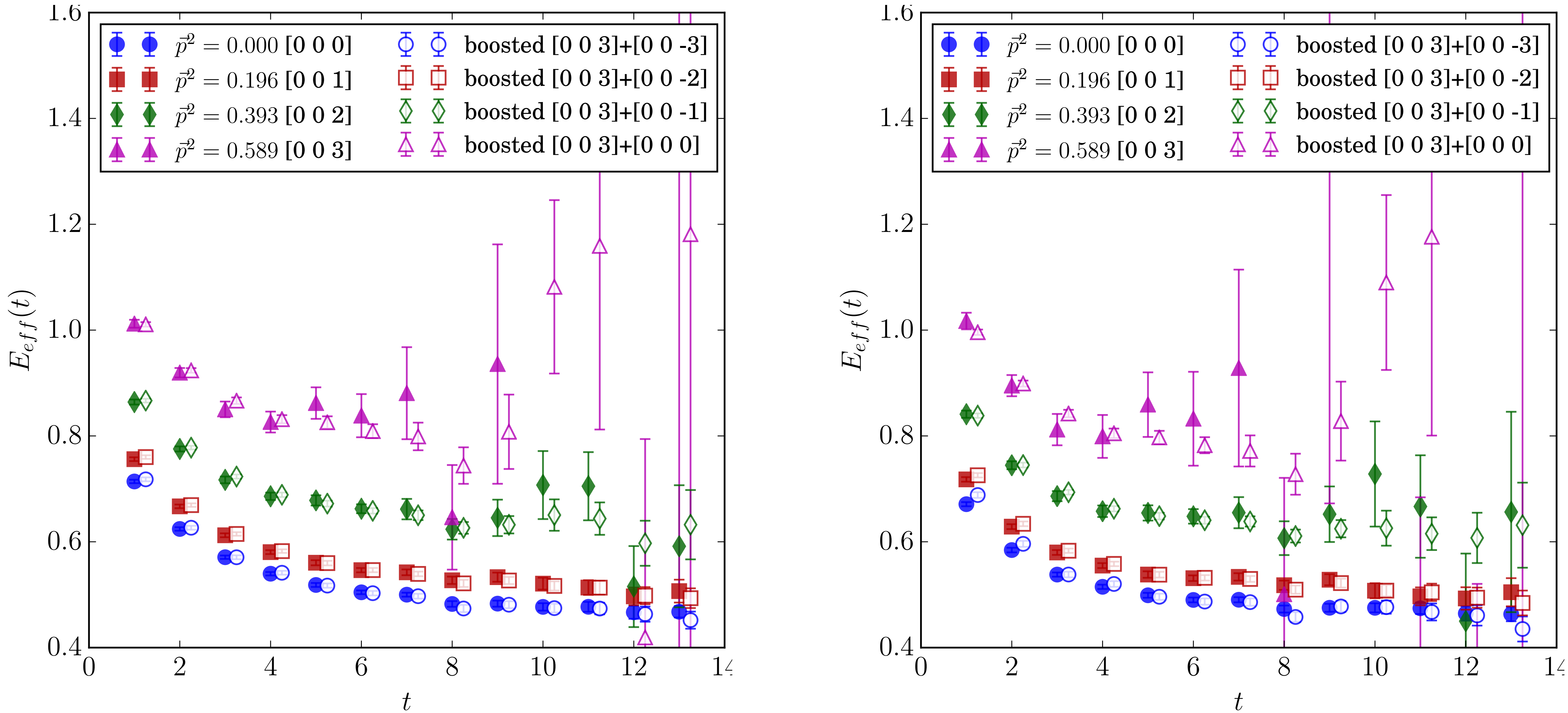
- periodic BC $\quad Q^2_{\rm opt} = (6\,\vec k_{\rm min})^2 = 4.2\ldots 8.2{\rm GeV}^2$
- antiperiodic (twisting) $\quad Q^2_{\rm opt} = (6\,\vec k_{\rm min})^2 = 1.1\ldots 2.1{\rm GeV}^2$

\+ Include disconnected diagrams (**DISCO**)

Motivation : JLab @12 GeV will measure proton, neutron form factors up to Q^2 = 12..18 GeV^2

Signal Gain : Traditional vs. Boosted Smearing

Nucleon Effective Energy: m_π = 320 MeV, a=0.081 fm, 32^3x64

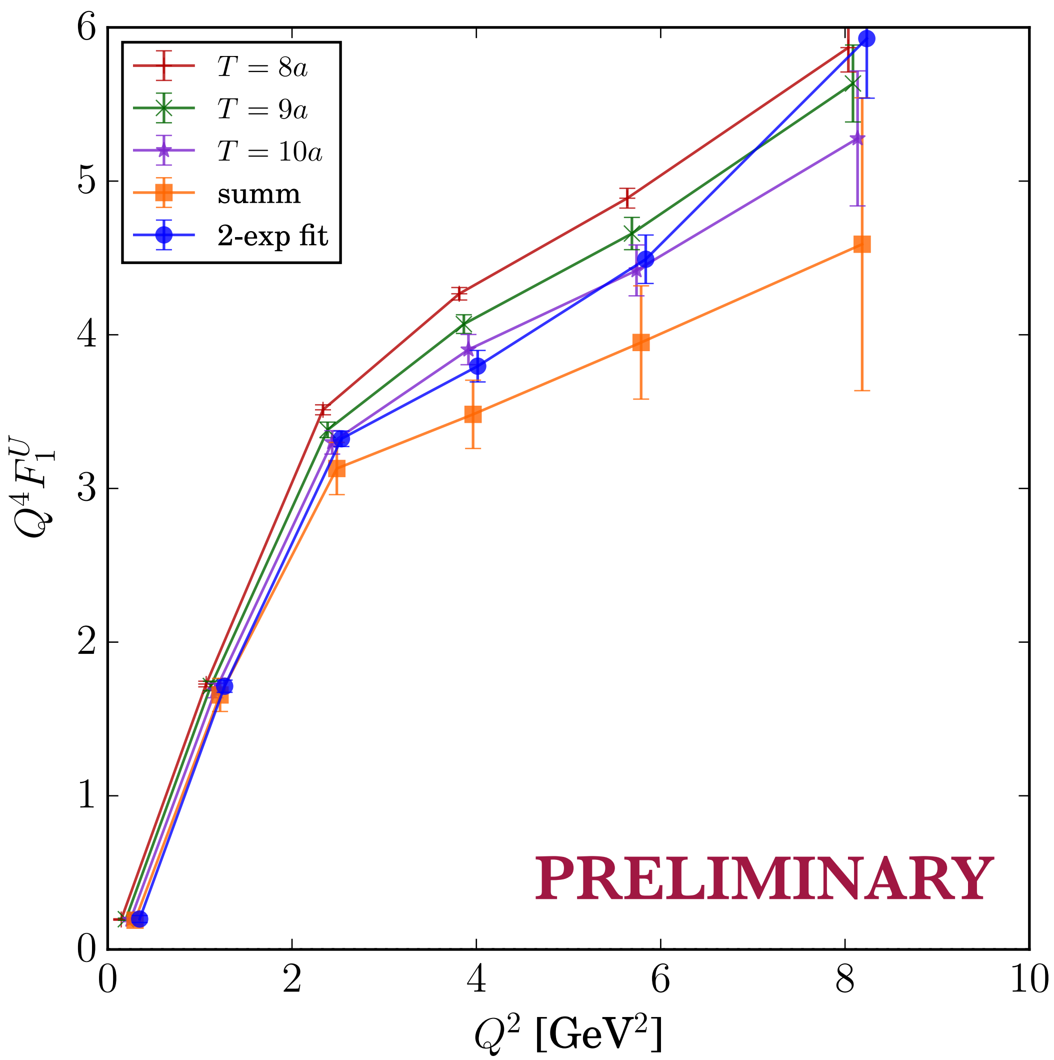


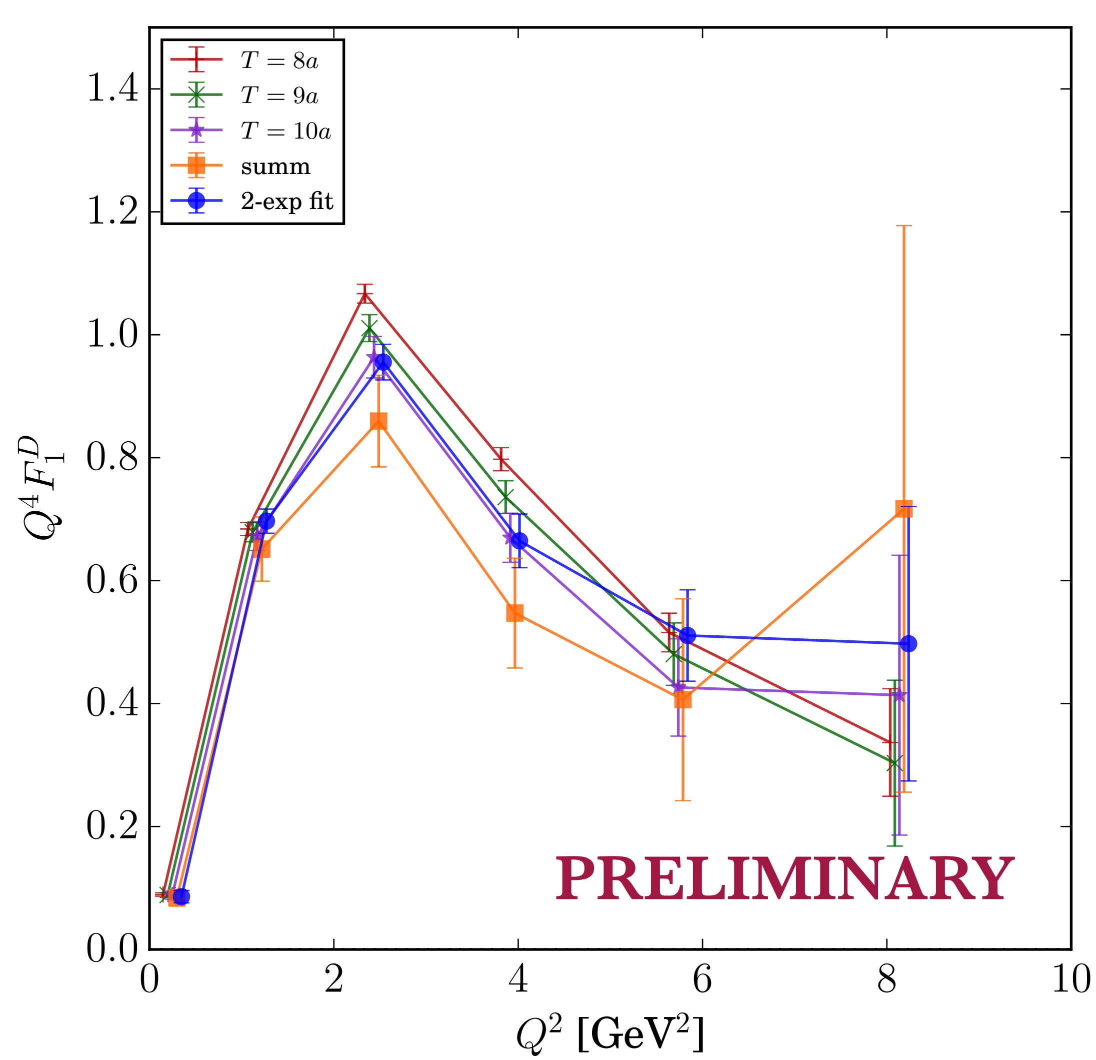
w=4.96 (N=40) ⟵ Gaussian smearing ⟶ *w=6.56 (N=70)*

- each quark is boosted with the same k=[0 0 1]
- *w=5.55 (N=45)* chosen for preliminary structure study

[SNS, Lattice 2016]

Q^2 Dependence of F_1^u and F_1^d

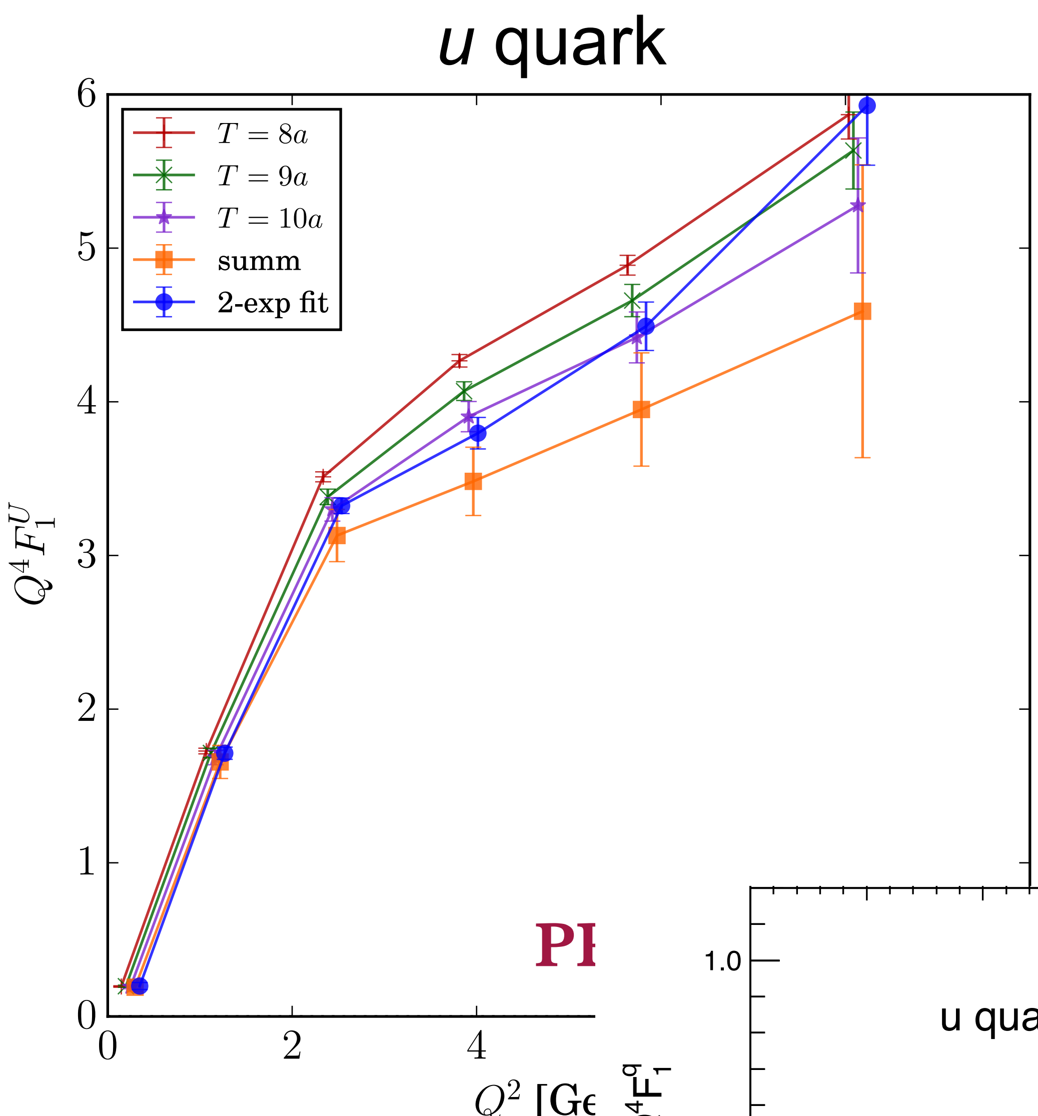


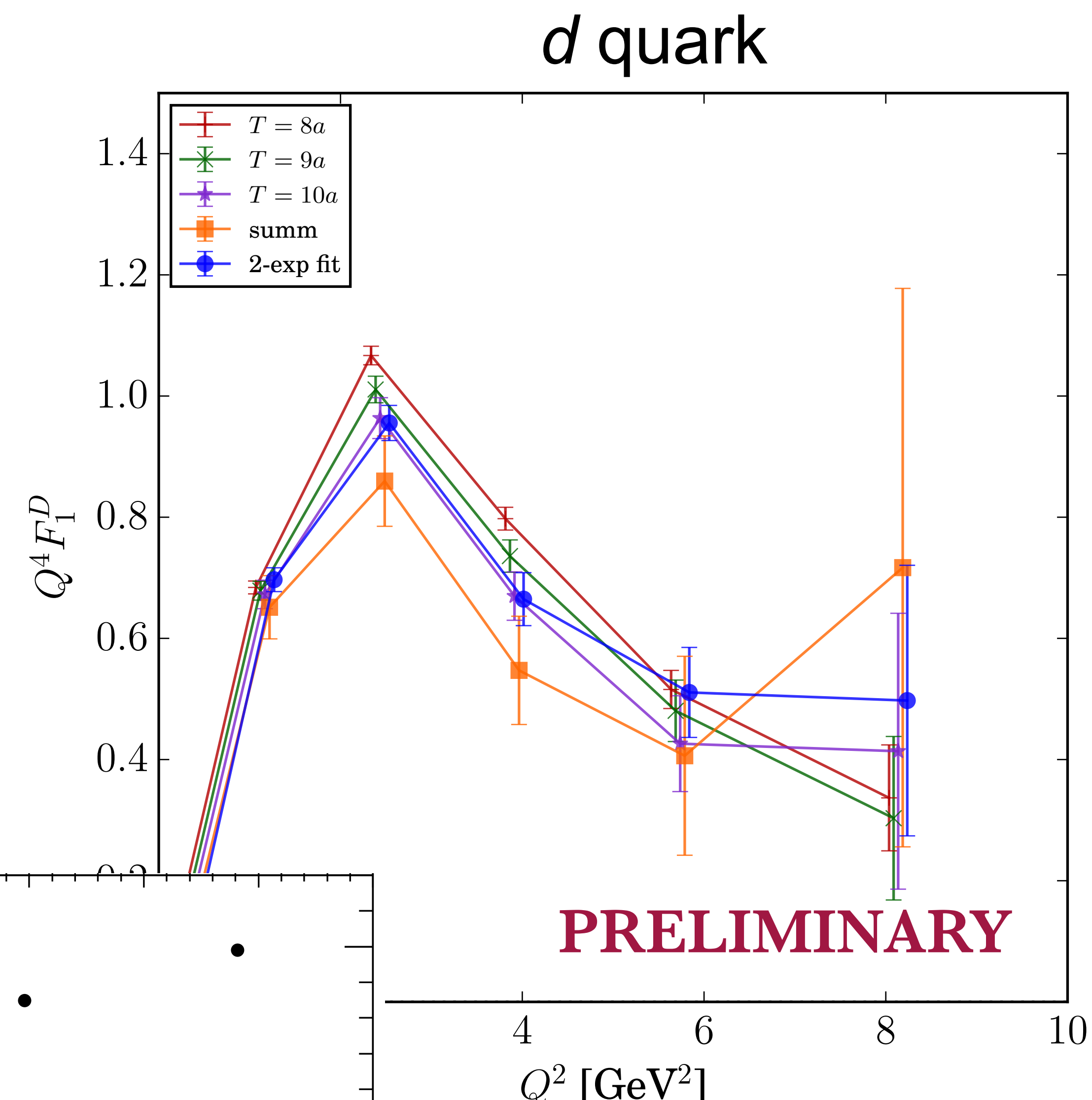


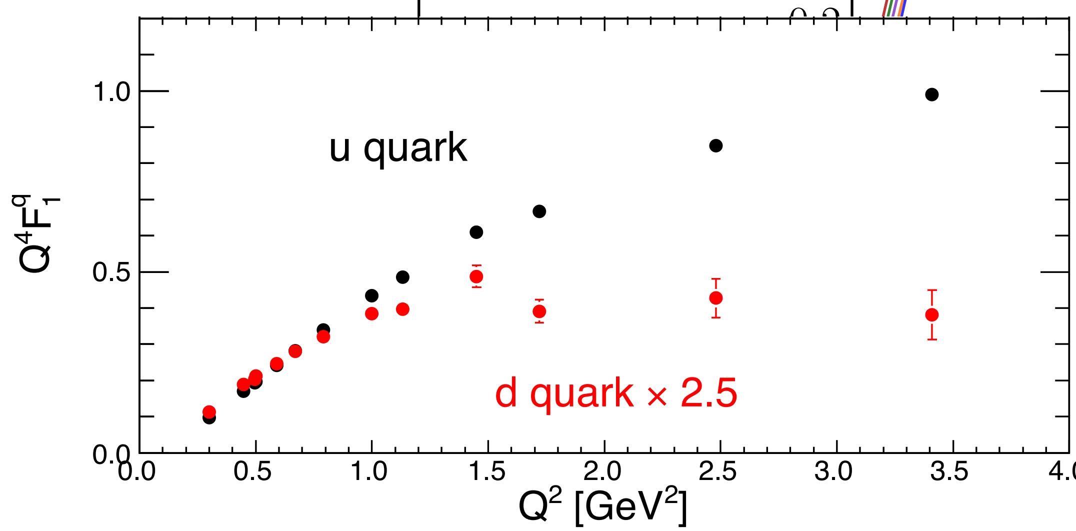
- expect $F_1(Q^2) \sim Q^4$ scaling [Lepage, Brodsky (1979)]
- Both form factors overshoot experiment (x3-4)

[SNS, Lattice 2016]

Q^2 Dependence of F_1^u and F_1^d





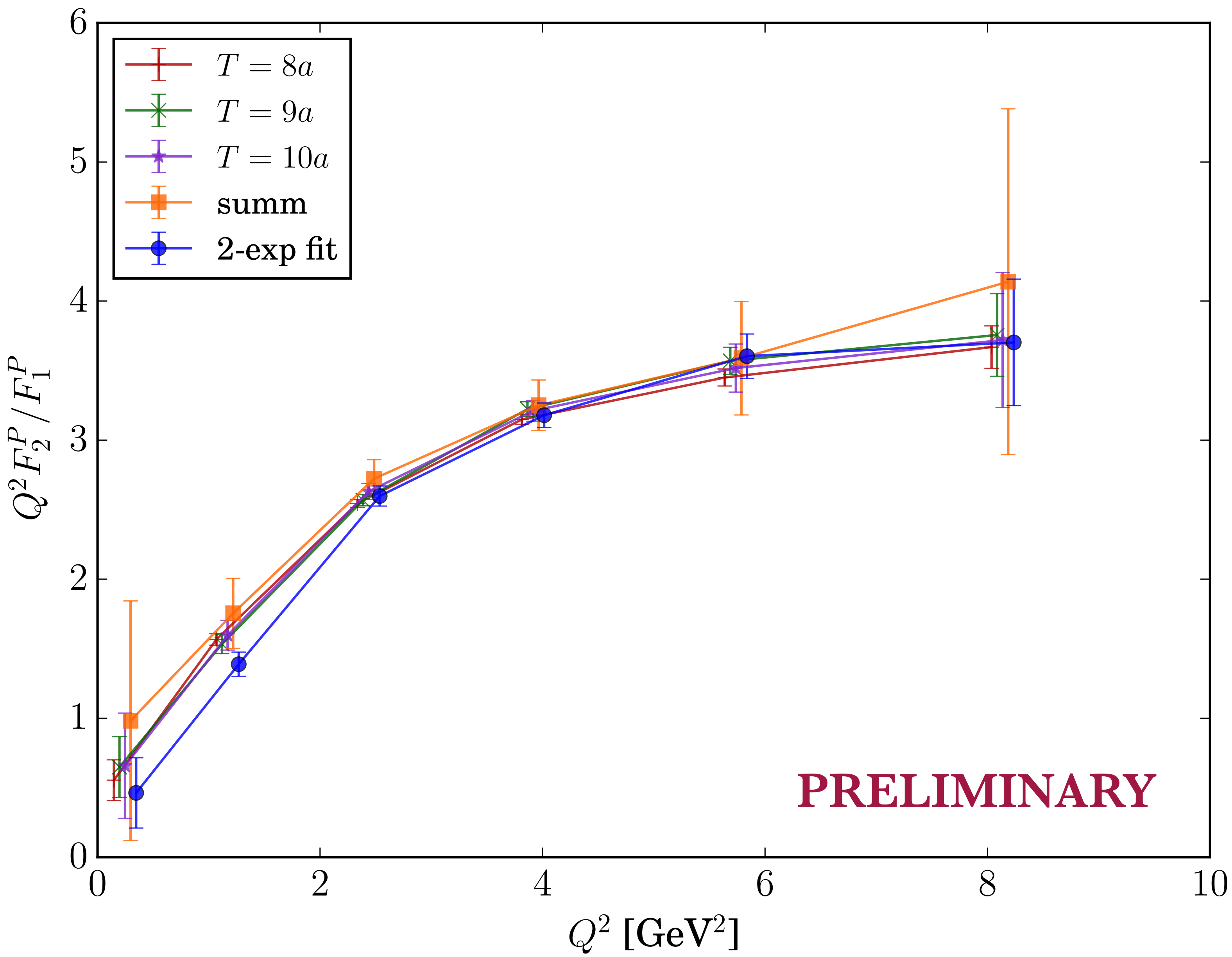


[G.D.Cates, C.W.de Jager, S.Riordan, B.Wojtsekhovski, PRL106:252003, arXiv:1103.1808]

- expect $F_1(Q^2) \sim Q^4$ scaling [Lepage, Brodsky (1979)]
- Both form factors overshoot experiment (x3-4)

[SNS, Lattice 2016]

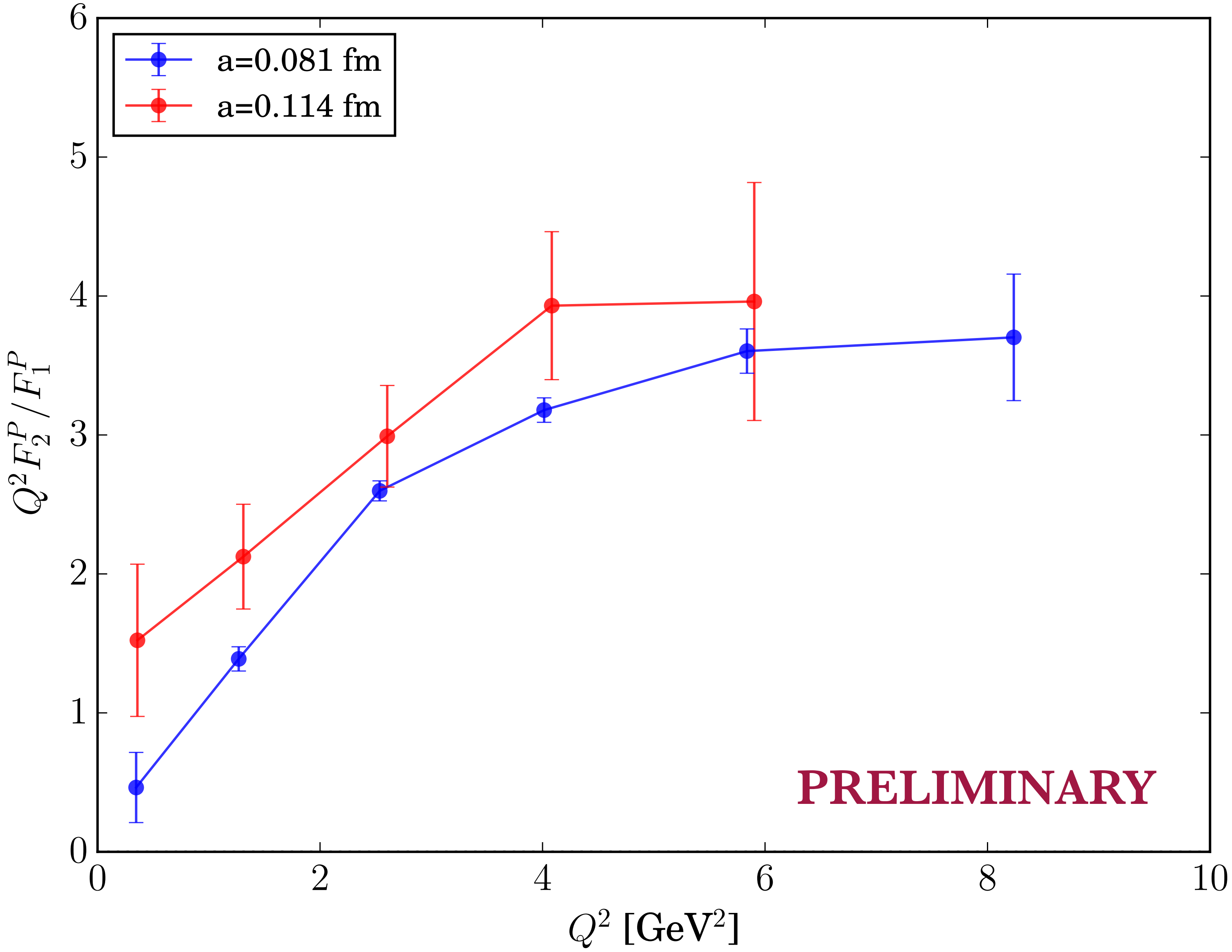
Q^2F_2/F_1 for Proton

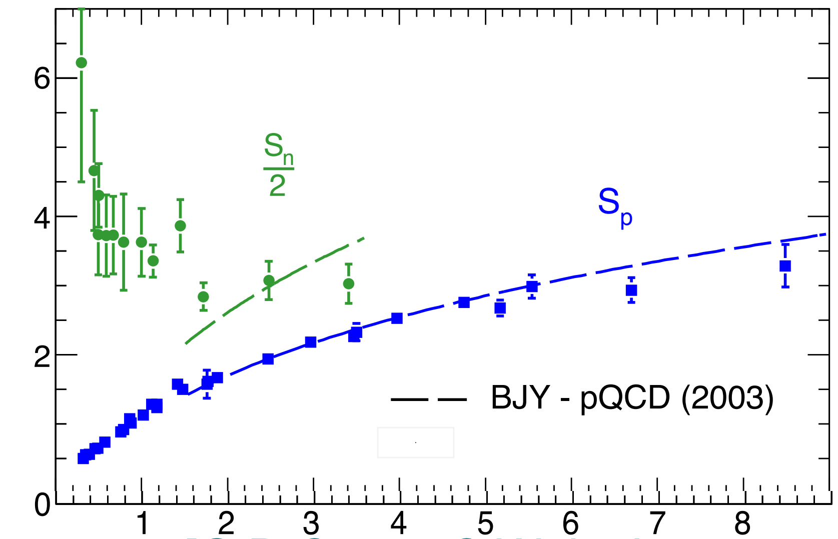


- expect $Q^2 F_1(Q^2)/F_2(Q^2) \sim \log[Q^2/\Lambda^2]$ scaling
  [Belitsky, Ji, Yuan (2003)]
- Qualitative behavior of F1u, F1d agrees with phenomenology

[SNS, Lattice 2016]

Q2F2/F1, Comparison to pQCD scaling



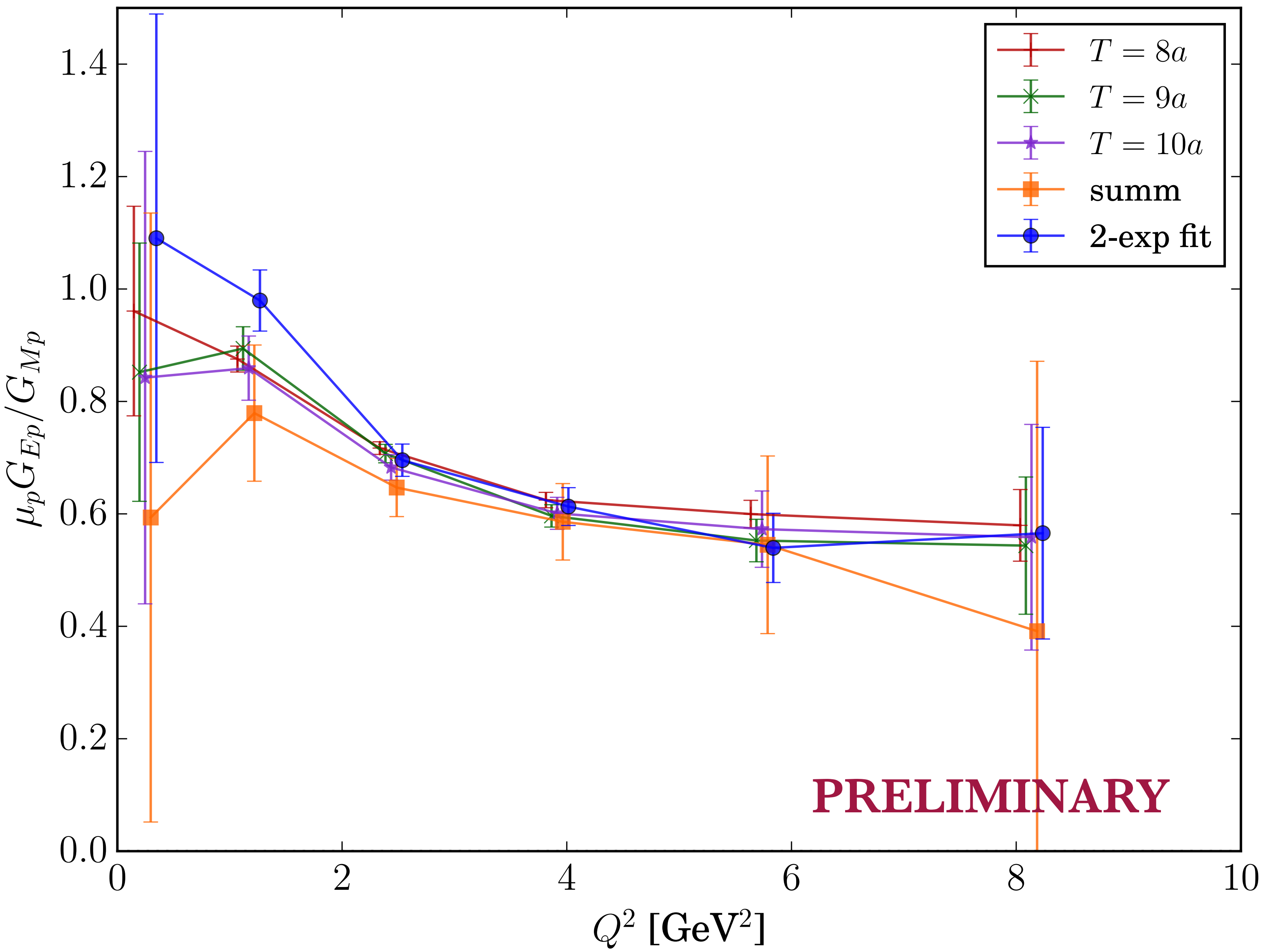


[G.D.Cates, C.W.de Jager, S.Riordan, B.Wojtsekhovski, PRL106:252003, arXiv:1103.1808]

- expect $Q^2 F_1(Q^2)/F_2(Q^2) \sim \log[Q^2/\Lambda^2]$ scaling
  [Belitsky, Ji, Yuan (2003)]
- Qualitative behavior of F1u, F1d agrees with phenomenology

[SNS, Lattice 2016]

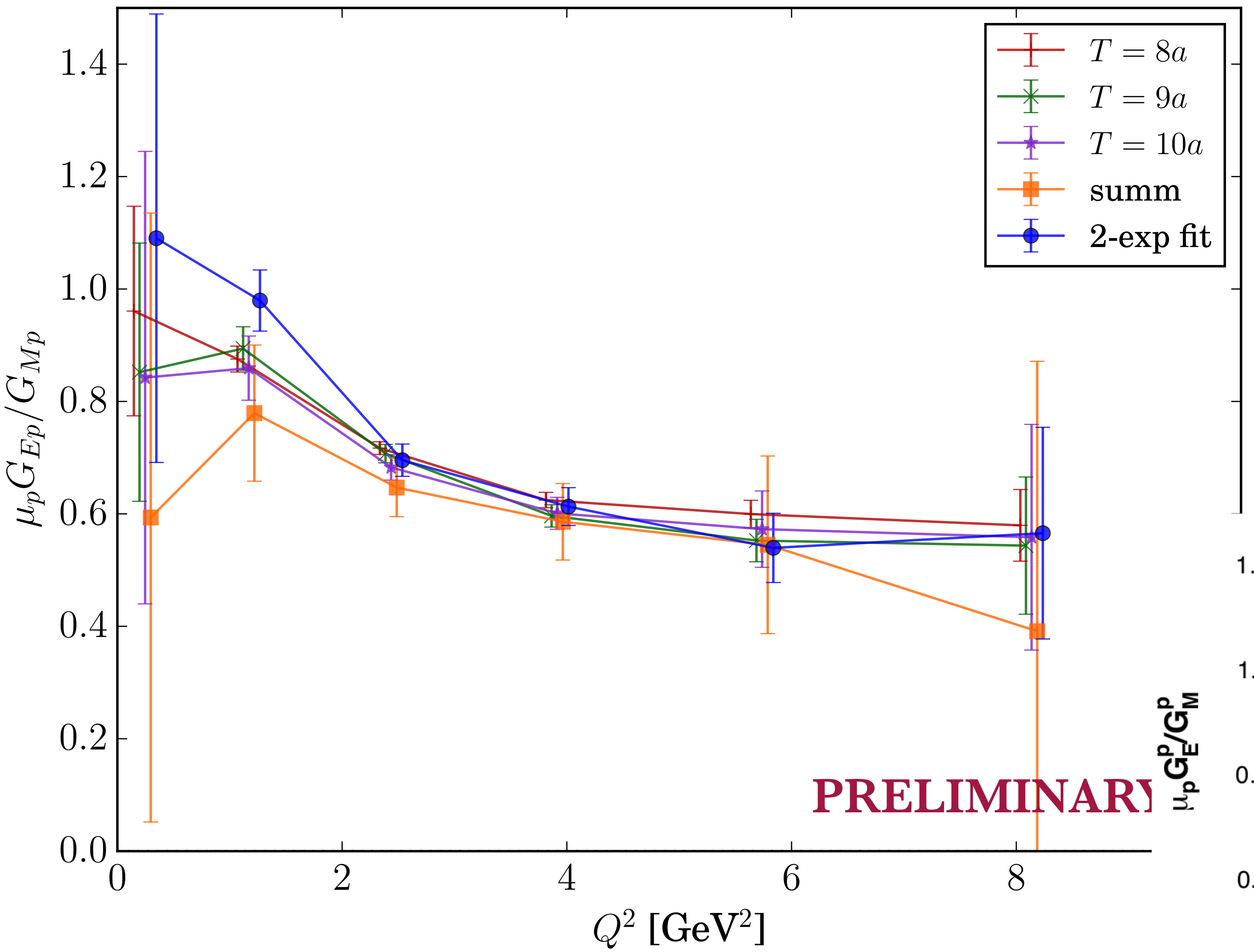
G_{Ep}/G_{Mp} for Proton

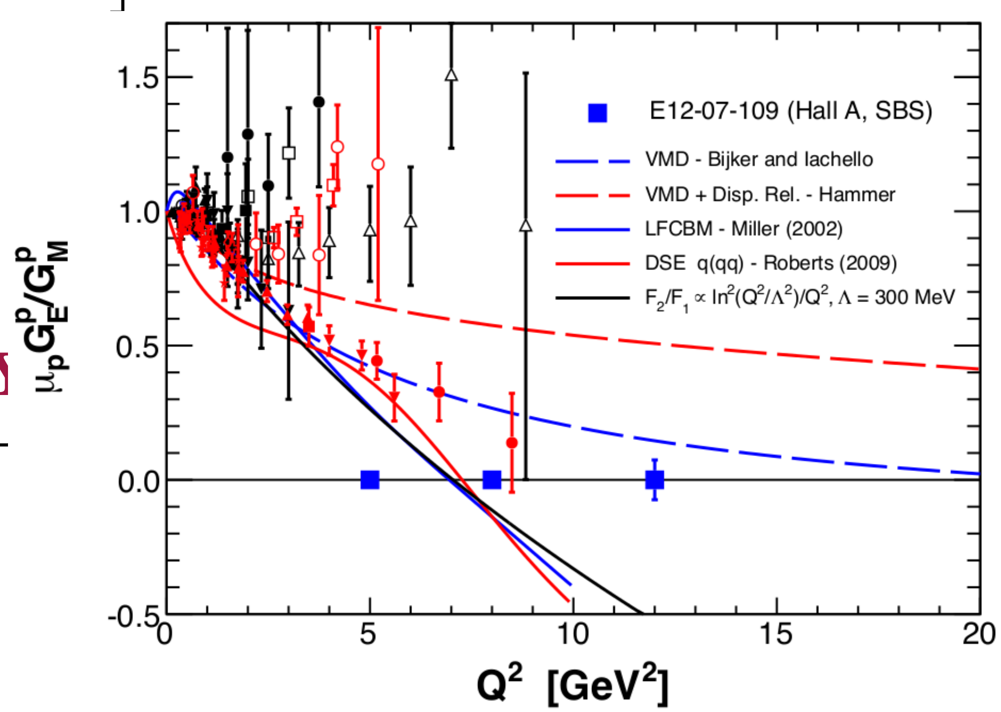


- Need to evaluate disconnected diagrams and operator improvement term

[SNS, Lattice 2016]

GEp/GMp

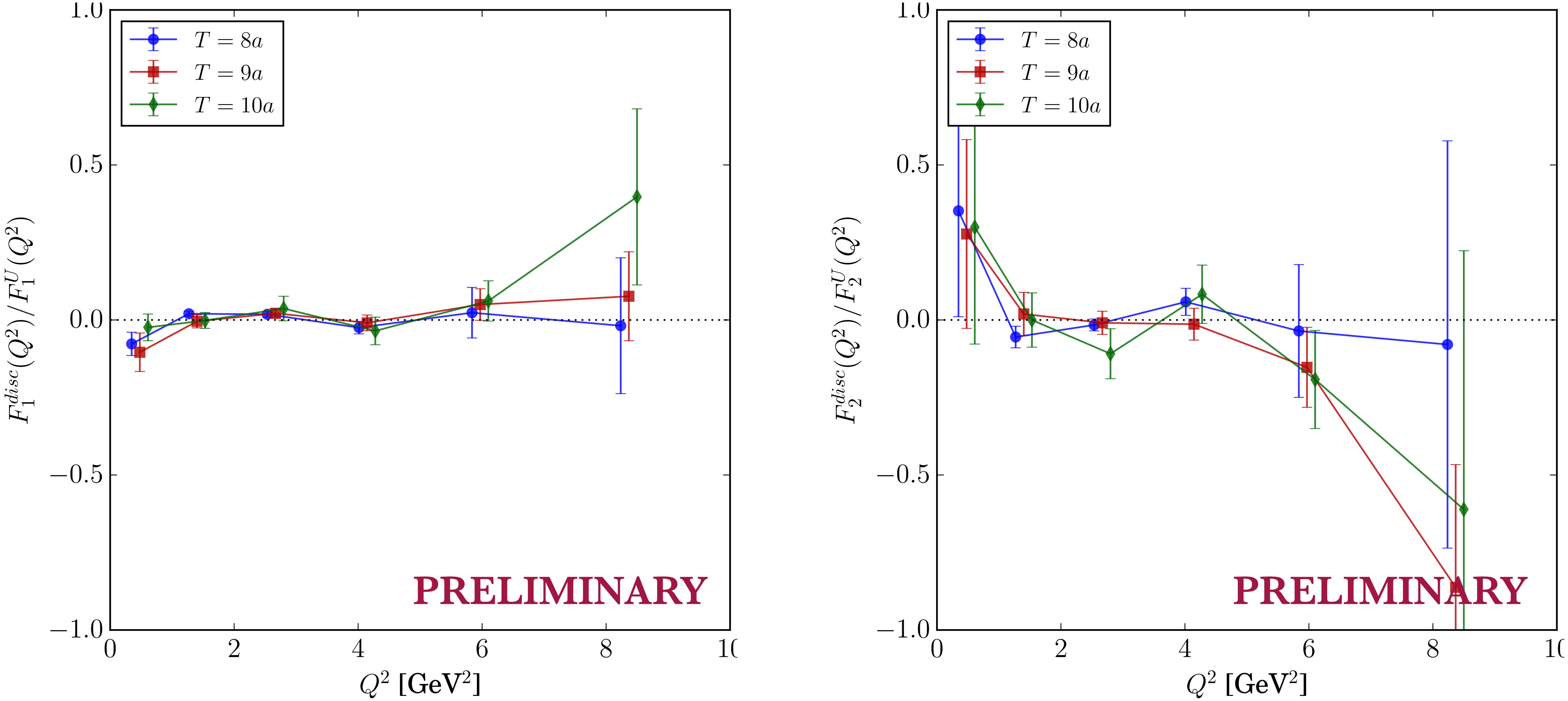




- Need to evaluate disconnected diagrams and operator improvement term
- Experiments hint at GEp/GMp 0-intersection at Q^2=8 GeV^2 (cancellation between F_1 and $(Q^2/4M^2)F_2$)

[SNS, Lattice 2016]

Disconnected Contributions to F_1, F_2



- Ratio of disconnected to connected(U) contributions
- Preliminary analysis (plateau averages), a=0.081 fm ensemble

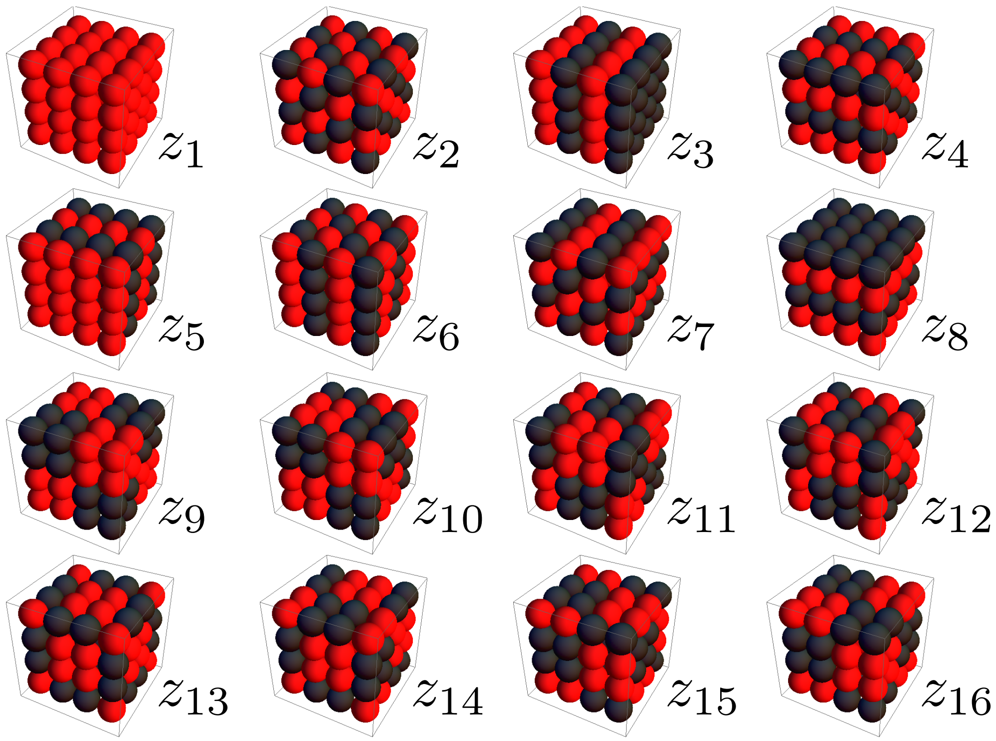
[SNS, Lattice 2016]

Efficient Calculation of Disconnected Diagrams

Hierarchical probing [K.Orginos, A.Stathopoulos, '13] :
In sum over 2^{dk+1} vectors (d=3),
dist(x,y) ≤ 2^k terms cancel exactly:

$$1 \le \sum_a |x_a - y_a| \le 2^k \; : \quad \frac{1}{N}\sum_i^N z_i(x) z_i(y)^\dagger \equiv 0$$

$$z_i \longrightarrow z_i \odot \xi \,, \quad \xi(x) = \text{ random } Z_2\text{-vector}$$



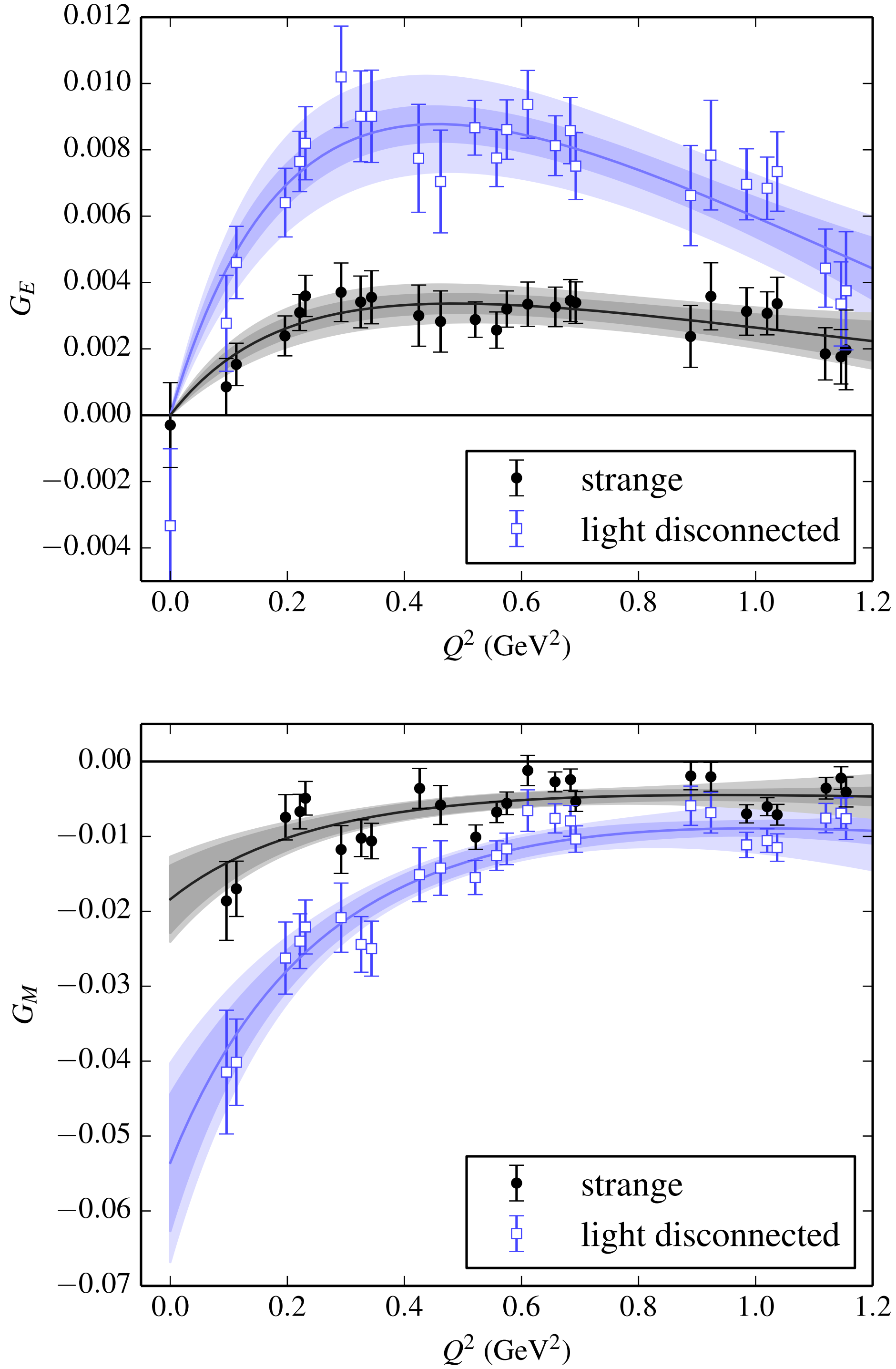
- reduce variance by treating low modes of $(\not{D}^\dagger \not{D})$ exactly [K.Orginos, A.Gambhir]

- Wide range of momenta is required for (1) form factors; (2) RI-MOM renormalization;
  ⇒ save all momenta / coordinates

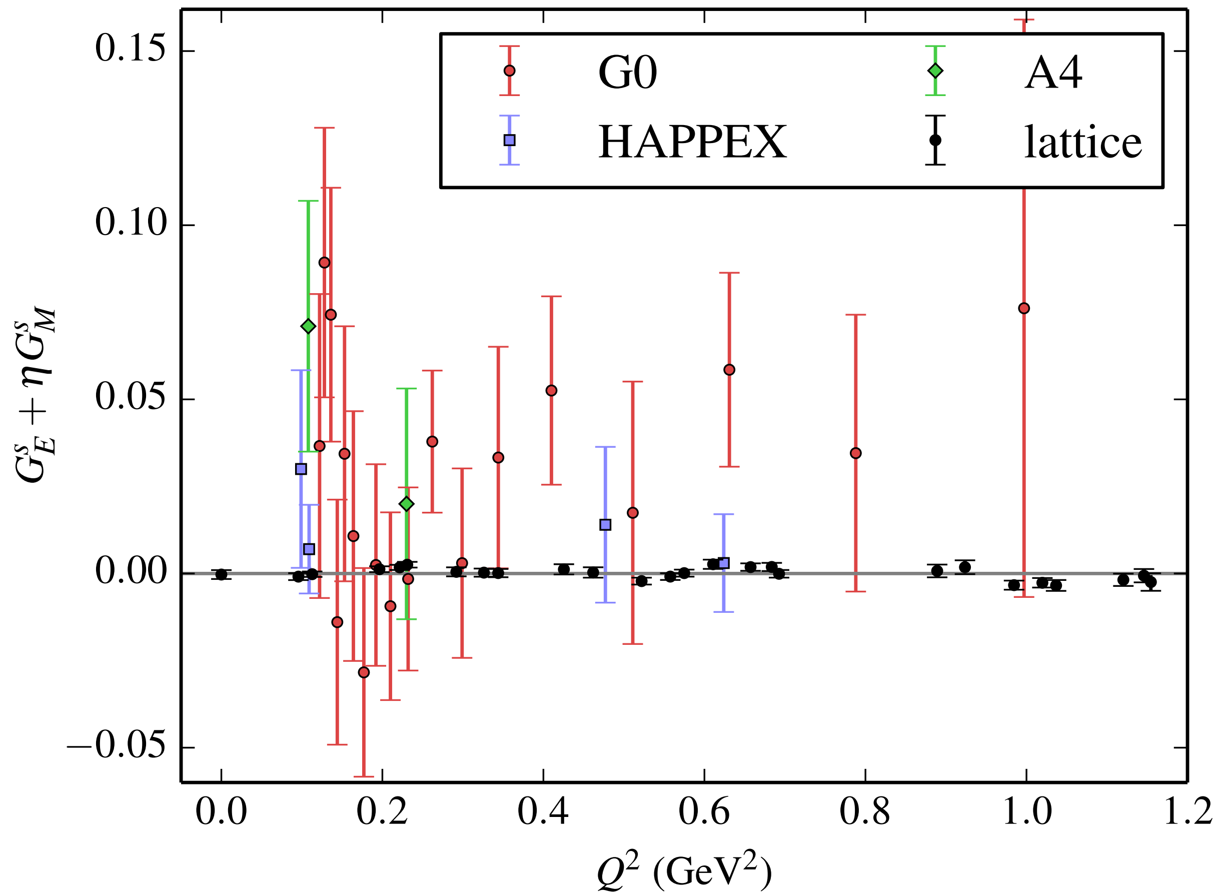
- Highly reusable data : hadron structure, π-π scattering
  ⇒ must be preserved&shared similarly to gauge configurations

Disconnected Vector Form Factors

JLab isotropic Clover *(m_π = 317 MeV)*
[J. Green, S. Meinel; PRD92:031501]



Comparison to
HAPPEX, G0, A4 data
[PRL108:102001(2012)]

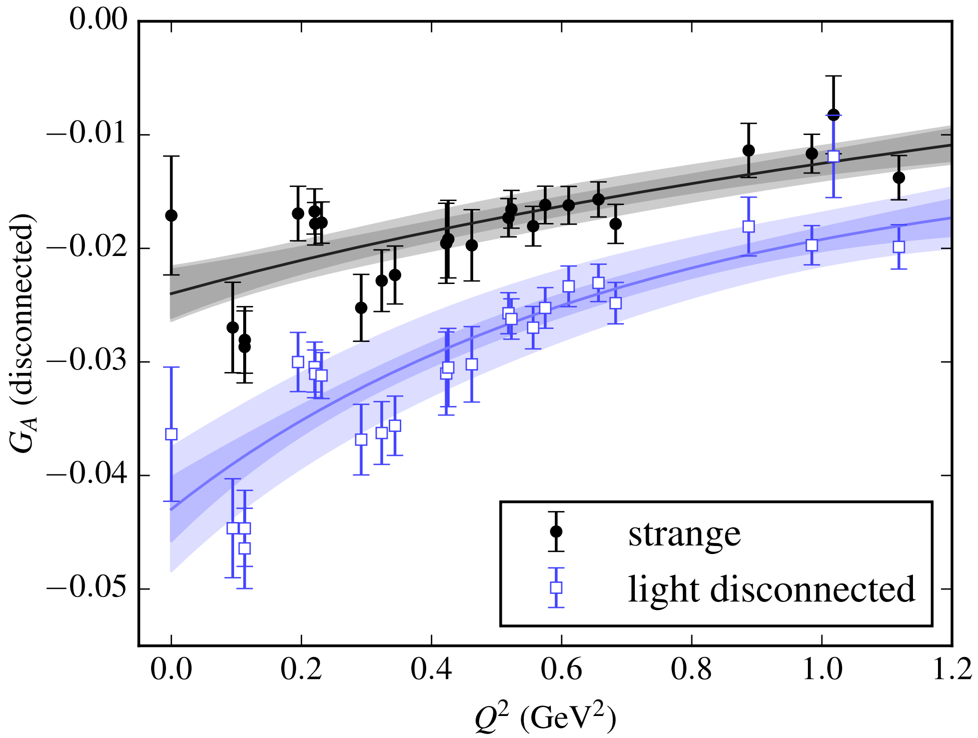


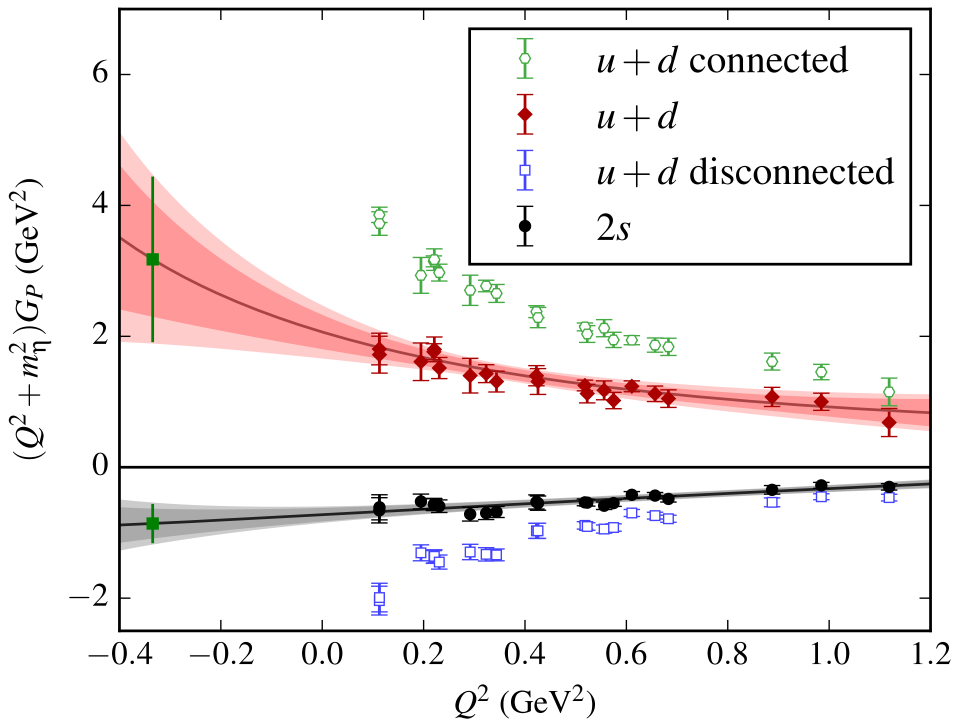
elastic *e–p* scattering asymmetry
~ Strange quark contrb.

(Lattice "kinematic factor" $\eta = \dfrac{Q^2}{0.94\mathrm{GeV}^2}$)

Disconnected Axial Form Factors

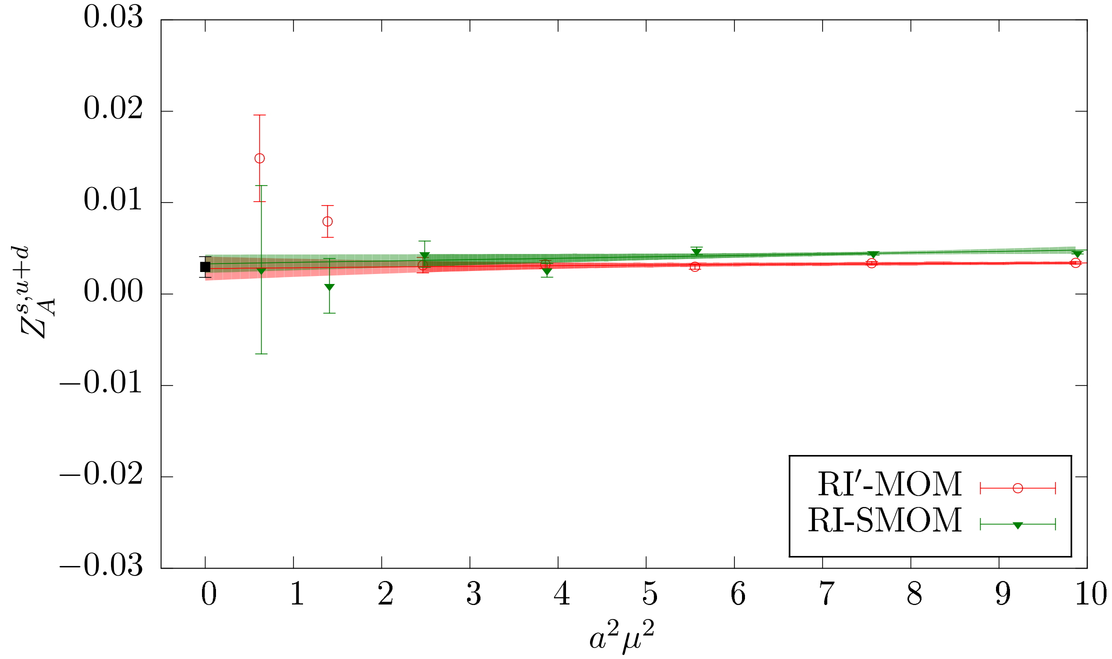
JLab isotropic Clover *(m_π = 317 MeV)*
[J.Green et at, arxiv:1703.06703]

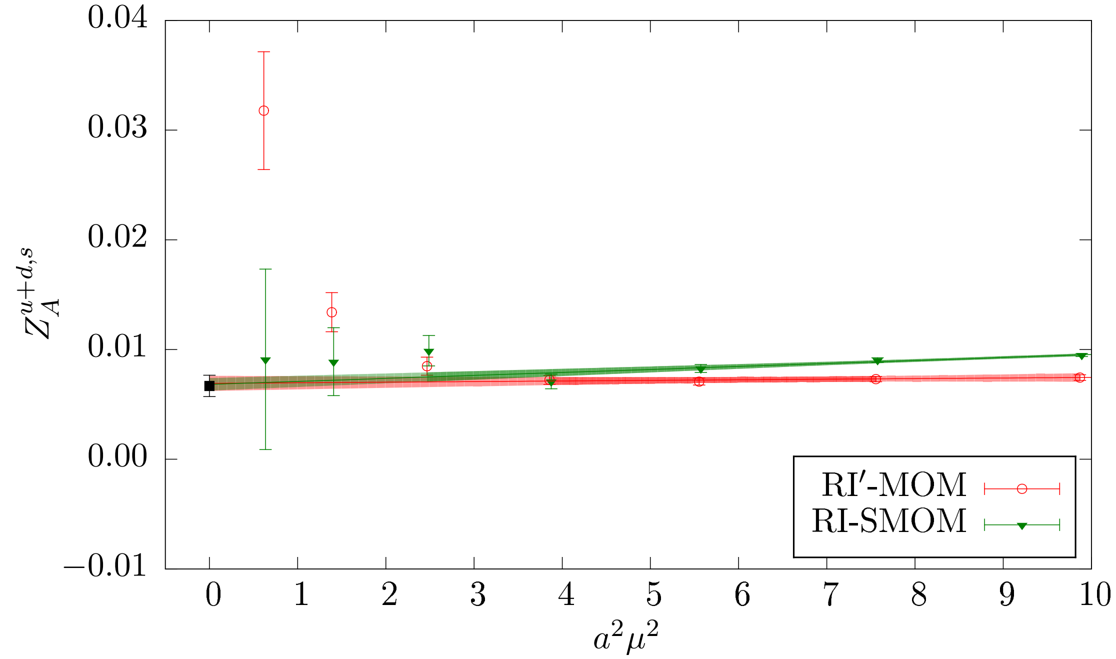




Up/down/strange axial current mixing

RI-SMOM with quark loops





Next: extend to quark / gluon energy-momentum mixing

Total Request

- [**DISCO**] disconnected quark loops with HP and deflation, up to one link insertions, all momenta, preserve & share similarly to gauge configurations
- [**CONN3PT**] form factors at high momentum transfer with control of exc.states
- [**TMD**] TMD and PDF contractions for high-momentum nucleon in- & out-states

	C13 : 32^3x96 $m\pi$=320 MeV a=0.114 fm	D5 : 32^3x64 $m\pi$=320 MeV a=0.080 fm	D6 : 48^3x96 $m\pi$=170 MeV a=0.090 fm	D7 : 64^3x128 $m\pi$=170 MeV a=0.090 fm	REQUEST [Jpsich]
DISCO	200c * 512 v.	200c * 512 v.	200c * 512 v.	150c * 512v.	33.5M [GPU]
CONN3PT	25,600 samp.	19,200 samp.	25,600 samp.	24,000 samp.	32.6M [GPU]
TMD(contr.)		14,400 samp.			12.0M [CPU]
TOTAL					**77.3M**

And now for something completely different...

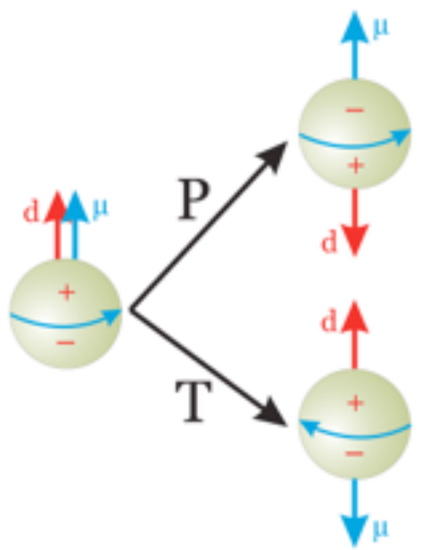
[Monthy Python]

Nucleon EDMs, form factors, and proton decay amplitudes using domain wall fermions

RBC+LHP proposal
Yasumichi Aoki, Tom Blum, Taku Izubuchi, Chulwoo Jung, Christoph Lehner, Hiroshi Ohki, Eigo Shintani, Amarjit Soni, Sergey Syritsyn (PI)

USQCD All-Hands Meeting, Jefferson Lab
Apr 28-30, 2017

Electric Dipole Moments of Nucleons



Motivations to search for new CP-odd interactions
- Evidence for SM Extensions
- Baryogenesis Requirement
- Strong CP problem (θ_{QCD})

$$\vec{d}_N = d_N \frac{\vec{S}}{S}$$

$$\mathcal{H} = -\vec{d}_N \cdot \vec{E}$$

$$\langle N|V_\mu(q)|N\rangle = \bar{u}_N \Big[\gamma_\mu F_1(q^2) + i\frac{[\gamma_\mu,\gamma_\nu]}{2} q_\nu \frac{F_2(q^2)}{2m_N} + (2im_N\gamma_5 q_\mu - \gamma_\mu\gamma_5 q^2)\frac{F_A(q^2)}{m_N^2} + \frac{[\gamma_\mu,\gamma_\nu]}{2} q_\nu \gamma_5 \frac{F_3(q^2)}{2m_N}\Big] u_N$$

$\not{P}$ anapole form factor — $\not{P}$, $\not{CP}$ EDM form factor

Role of Lattice QCD : connect quark/gluon-level (effective) operators to hadron/nuclei matrix elements and interactions

$$\mathcal{L}_{eff} = \sum_n \frac{c_n}{\Lambda^{d_n-4}} \mathcal{O}_n^{(d_n)}$$

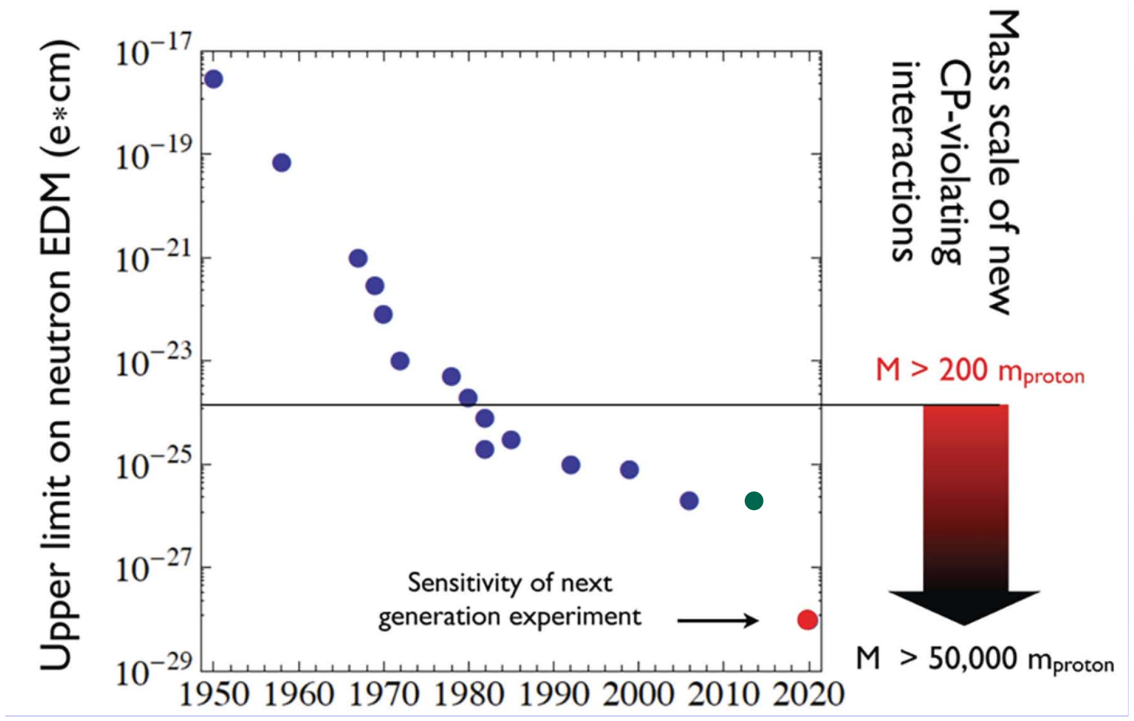
$$\begin{cases} \mathcal{L}^{(4)} = \theta \frac{g^2}{32\pi^2} G\tilde{G} & \text{(QCD theta-angle)} \\ \mathcal{L}^{(\text{"6"})} = \sum_q \left[d_q\, \bar{q}(F\cdot\sigma)\gamma_5 q + \tilde{d}_q \bar{q}(G\cdot\sigma)\gamma_5 q\right] & \text{Quark (chromo-)EDM} \\ \ldots & \text{(3-gluon, 4-quark, etc)} \end{cases}$$

$\Rightarrow$ $d_{n,p}$, $F_3^{n,p}(Q^2)$

Experimental Outlook: Neutron EDM

	10^{-28} *e* cm
CURRENT LIMIT	<300
Spallation Source @ORNL	**< 5**
Ultracold Neutrons @LANL	~30
PSI EDM	<50 (I), **<5 (II)**
ILL PNPI	<10
Munich FRMII	**< 5**
RCMP TRIUMF	<50 (I), **<5 (II)**
JPARC	**< 5**
Standard Model (CKM)	< *0.001*

[B.Filippone's talk, KITP 2016]



nEDM sensitivity :

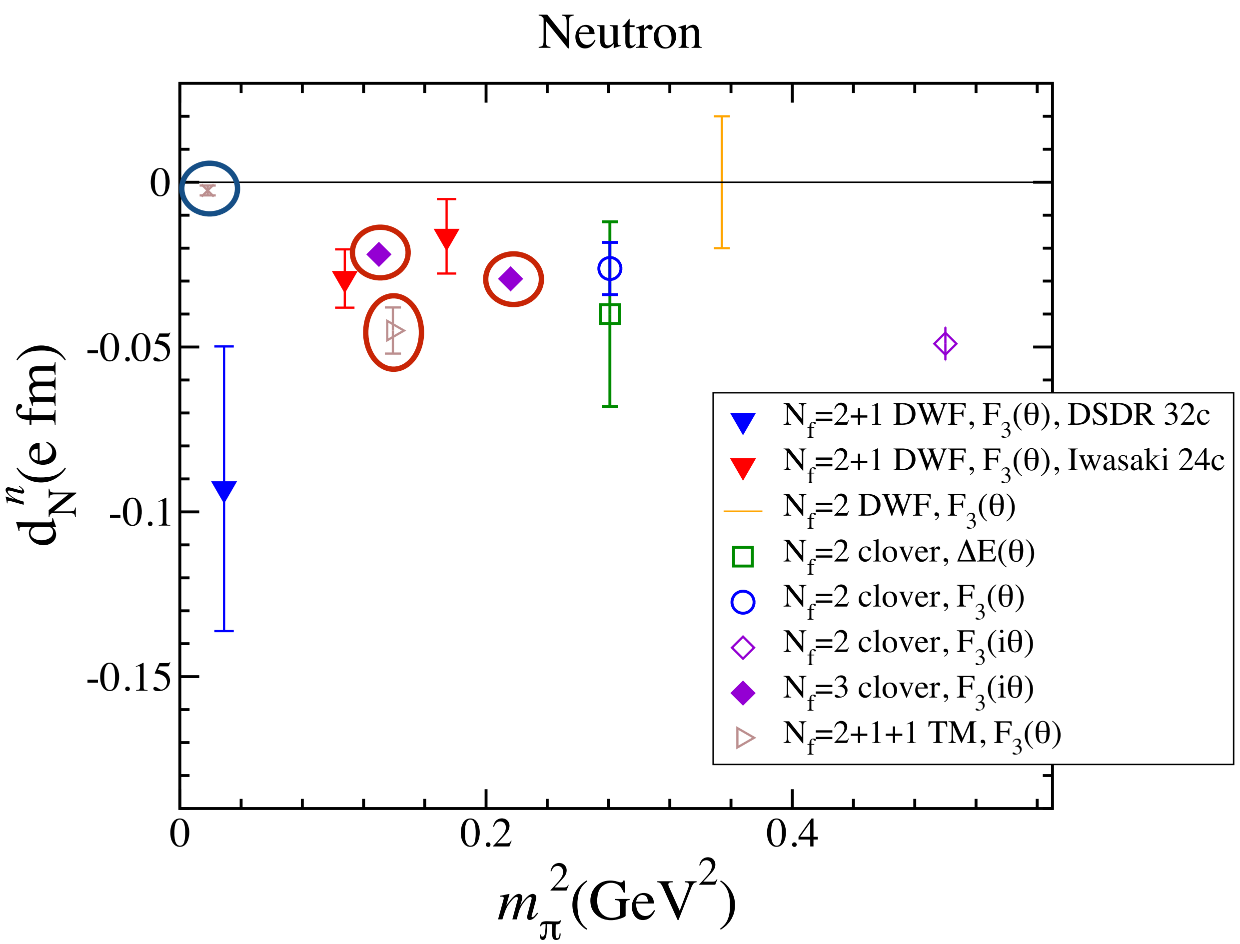
- 1–2 years : next best limit
- 3–4 years : x10 improvement
- 7–9 years : x100 improvement

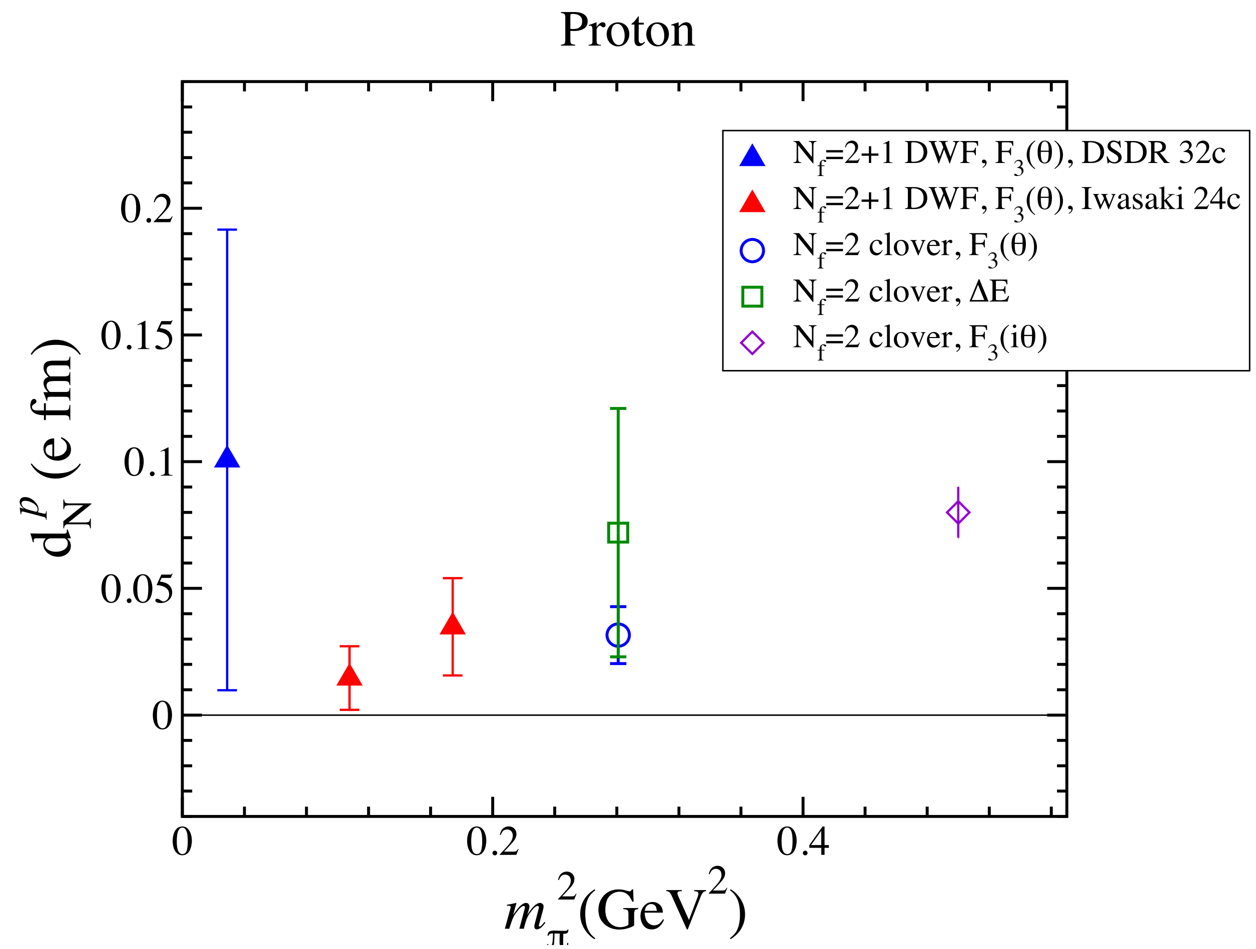
Experimental Outlook: Nuclei, Protons, etc

- ^{225}Ra : rigid octupole deformation (parity partner at 55 keV)
  + strong enhancement of P,T-odd πNN coupling in NN potential
  – *connection to CPv parameters (theta, cEDM, ...) depends on ChPT and nuclear models*

- Protons and light nuclei (d, t, h) in storage rings :
  + potential for stat. sensitivity $|d_p| \lesssim 10^{-29}\ e\cdot\mathrm{cm}$
  ++ potential to disentangle different sources of CPv
  – not clear if sys. uncertainties may be controlled

θ_{QCD}-induced Nucleon EDM

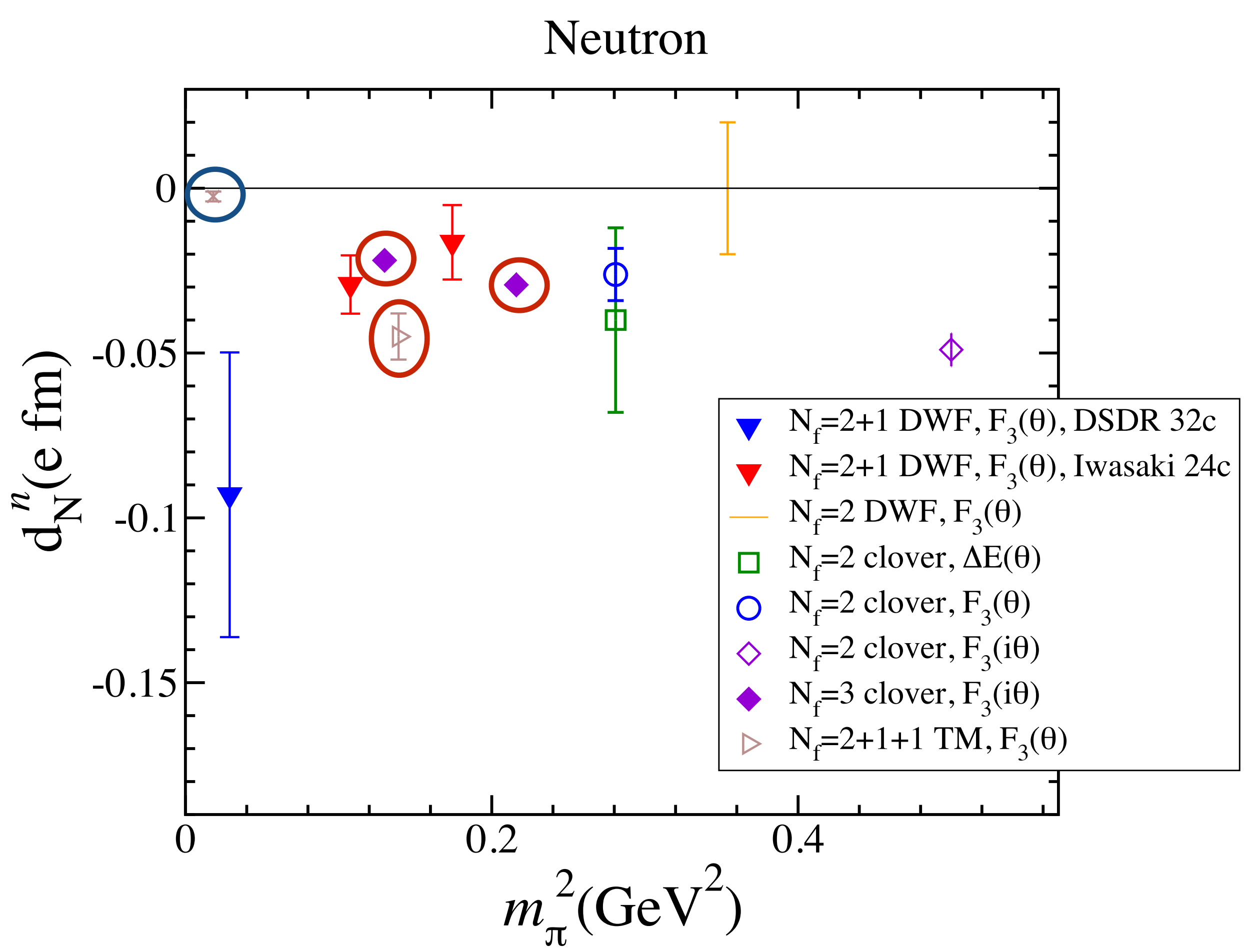


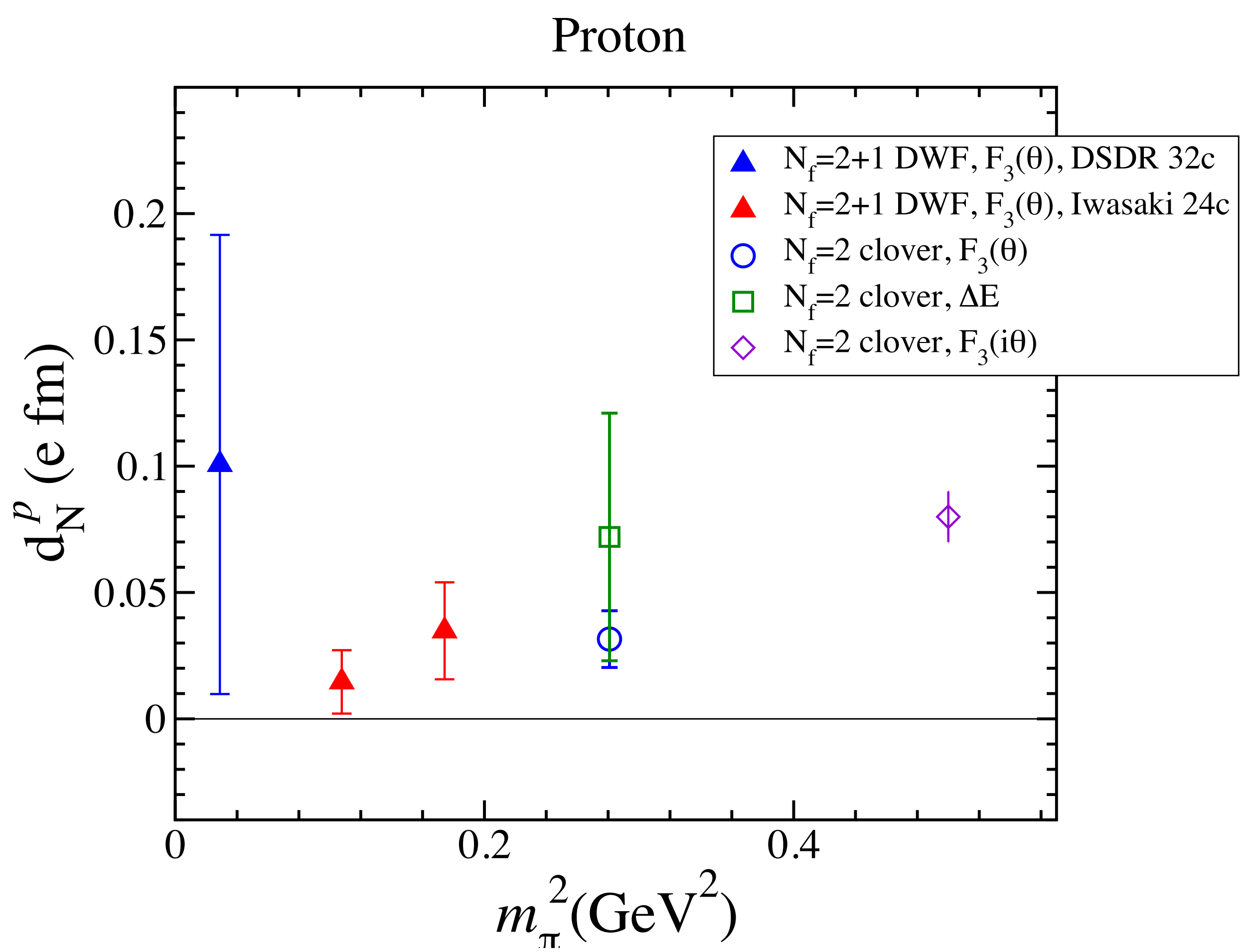


[E.Shintani, T.Blum, T.Izubuchi, A.Soni, PRD93, 094503(2015)]

- Phenomenology: $|d_n| \simeq \theta_{QCD}$ ×(few 10^{-3} e fm) $\Longrightarrow |\theta_{QCD}| \lesssim 1.5\times10^{-10}$
- Lattice : $|d_n| \simeq \theta_{QCD}$×(few 10^{-2} e fm) $\Longrightarrow$ tighter constraint on θ_{QCD} ?

θ_{QCD}-induced Nucleon EDM





[E.Shintani, T.Blum, T.Izubuchi, A.Soni, PRD93, 094503(2015)]

- Phenomenology: $|d_n| \simeq \theta_{QCD} \times$(few 10^{-3} e fm) $\Longrightarrow$ $|\theta_{QCD}| \lesssim 1.5\times10^{-10}$
- Lattice : $|d_n| \simeq \theta_{QCD}\times$(few 10^{-2} e fm) $\Longrightarrow$ tighter constraint on θ_{QCD}?

Unfortunately, there is a problem:
unaccounted-for mixing between electric and magnetic moments

Nucleon "Parity Mixing" on a Lattice

Lattice nucleon operator $N = u\left[u^T C\gamma_5 d\right]$

Ground state in CPv vacuum $\langle \text{vac}|N|p,\sigma\rangle_{\not{CP}} = e^{i\alpha\gamma_5} u_{p,\sigma} = \tilde{u}_{p,\sigma}$

Solutions to
$(\not{\partial} + m_N e^{-2i\alpha\gamma_5})\tilde{u}_p = 0$

Nucleon propagator $\langle N(t)\bar{N}(0)\rangle_{\not{CP}} = e^{-E_N t} e^{i\alpha\gamma_5} \frac{-i\not{p}_{\mathcal{E}} + m_N}{2E_N} e^{i\alpha\gamma_5}$

$$\sim \frac{-i\not{p}_{\mathcal{E}} + m_N e^{2i\alpha\gamma_5}}{2E_N} = \sum_\sigma \tilde{u}_{p,\sigma}\bar{\tilde{u}}_{p,\sigma}$$

The mixing phase α has to be calculated and removed by field redefinition
Similar issues may appear in EFT (ChPT) calculations

Nucleon "Parity Mixing" : EDM and aMDM

Nucleon-current correlator spin structure in the **original** works [S.Aoki et al (2005), ...]

$$\langle N_{p'}\, J^\mu\, \bar{N}_p \rangle_{\not{CP}} \overset{?}{\sim} \Big(\sum_{\sigma'} \tilde{u}_{\sigma'} \bar{\tilde{u}}_{\sigma'}\Big)_{p'} \Gamma^\mu_{\mathcal{E}} \Big(\sum_{\sigma} \tilde{u}_{\sigma} \bar{\tilde{u}}_{\sigma}\Big)_{p}$$

Vector current vertex in Euclid

$$\Gamma^\mu_{\mathcal{E}} = F_1 \gamma^\mu + (F_2 + iF_3\gamma_5) \frac{\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}$$

P,T-odd (electric dipole f.f.)

Correct spin structure [SNS, S.Aoki, et al (2017)]

$$\langle N_{p'}\, J^\mu\, \bar{N}_p \rangle_{\not{CP}} \sim \sum_{\sigma',\sigma} \tilde{u}_{p',\sigma'} \left(\bar{u}_{p',\sigma'} \Gamma^\mu_{\mathcal{E}} u_{p,\sigma}\right) \bar{\tilde{u}}_{p,\sigma}$$

Solutions to $(\not{\partial} + m_N) u_p = 0$ $\Leftrightarrow$ at rest: parity proj. $(1-\gamma_4)u = 0$

Chiral rotation results in "rotation" in the $F_{2,3}$ plane

$$e^{i\alpha\gamma_5} \Gamma^\mu e^{i\alpha\gamma_5} \leftrightarrow \Gamma^\mu$$

$$e^{2i\alpha}(\text{“}F_2\text{”} + i\text{“}F_3\text{”}) = (F_2 + iF_3)_{\text{true}}$$

... and spurious contributions to anomalous mag.moment $F_2(0)$ electric dipole moment $F_3(0)$

$$\begin{cases} \text{“}F_2\text{”} & = [\cos(2\alpha)F_2 + \sin(2\alpha)F_3]_{\text{true}} \\ \text{“}F_3\text{”} & = [\cos(2\alpha)F_3 - \sin(2\alpha)F_2]_{\text{true}} \end{cases}$$

With CPv interaction as a perturbation over QCD vacuum

$$\text{“}F_3\text{”} \approx [F_3]_{\text{true}} - 2\alpha[F_2]_{\text{true}}$$

$$\text{“}d_{n,p}\text{”} \approx [d_{n,p}]_{\text{true}} - 2\alpha \frac{\kappa_{n,p}}{2m_N}$$

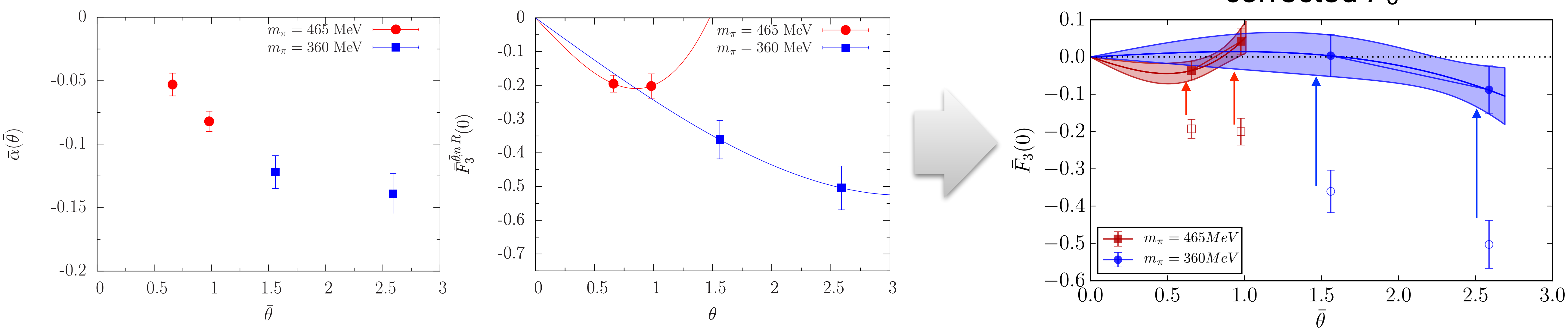
Recent Lattice Results on θ_{QCD}-induced nEDM

Correction is simple: $[F_3]_{\text{true}} = \text{“}F_3\text{”} + 2\alpha F_2$

- [F. Guo *et al* (QCDSF), PRL115:062001 (2015)]

  dynamical calculations with finite imag. θ^I angle

corrected F_3



- [C.Alexandrou *et al* (ETMC), PRD93:074503 (2016)]

  d_n=–0.045(06) *e* fm (~7.5σ) → +0.008(6) *e* fm (1.3σ)

  \+ zero result confirmed by the authors

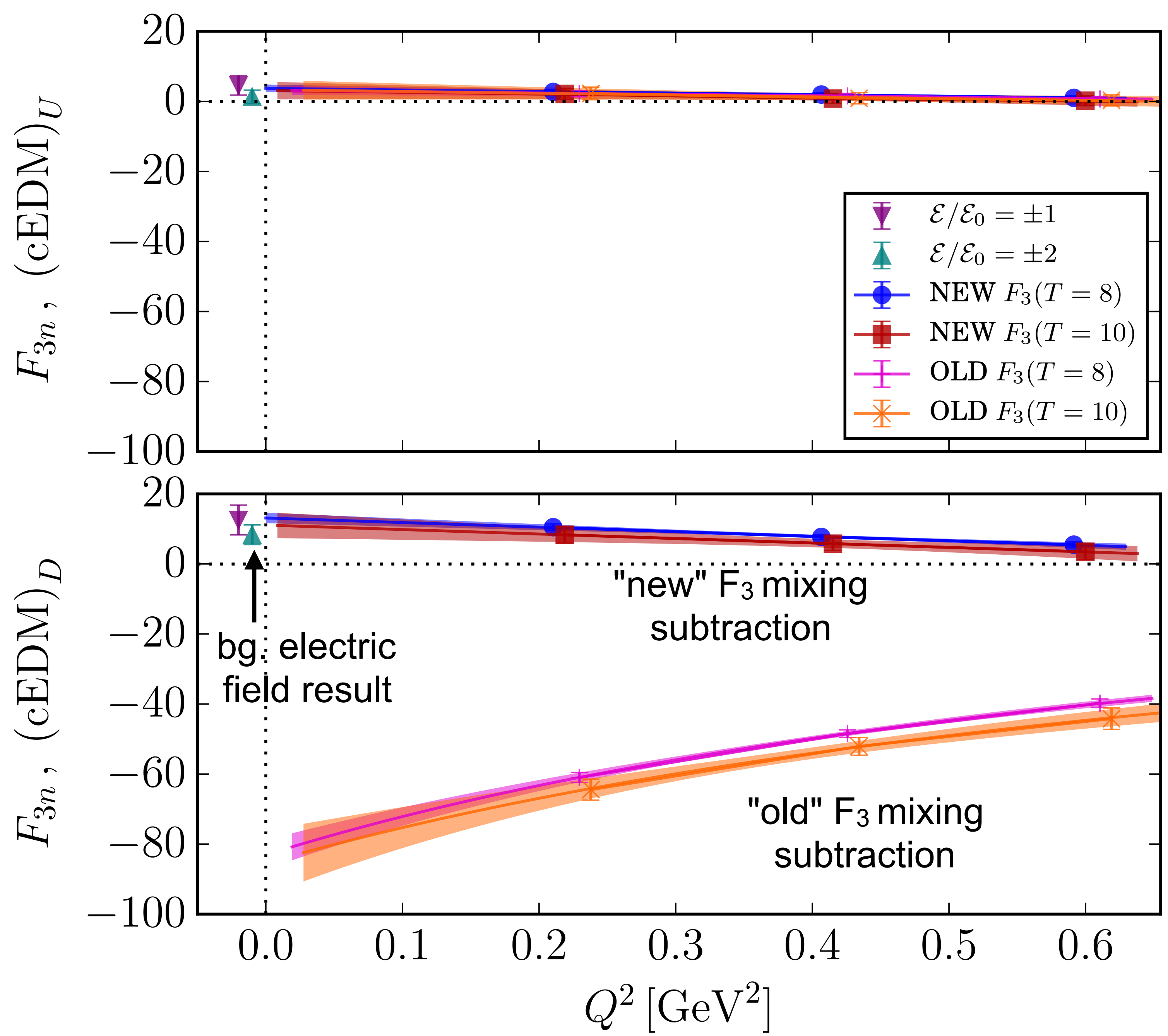
- [E.Shintani *et al,* D78:014503 (2008)],

  uniform Minkowski-real bg. electric field: **not affected** by the spinor "parity mixing"

  d_n=–0.040(28) *e* fm (~1.4σ) at m_π ≈530 MeV; *Precision is insufficient for comparison*

After removing spurious contributions, no significant lattice signal for θ_{QCD}-induced nEDM !
However, the conflicts with phenomenology value and m_q scaling disappears

Energy Shift vs. Form Factors (Neutron)



Mixing $\alpha_U \approx 0$

No F_2 contribution to F_3

$$\text{“}F_{3n}^U\text{”} \approx [F_{3n}^U]_{\mathrm{true}}$$

Mixing $\alpha_D \approx 30(0.2)$

Large F_2 contribution to "F_3"

$$\text{“}F_{3n}^D\text{”} = [F_{3n}^D]_{\mathrm{true}} - 2\alpha_D F_{2n}$$

[S.Aoki, SNS, *et al* (2017) arXiv:1701.07792]

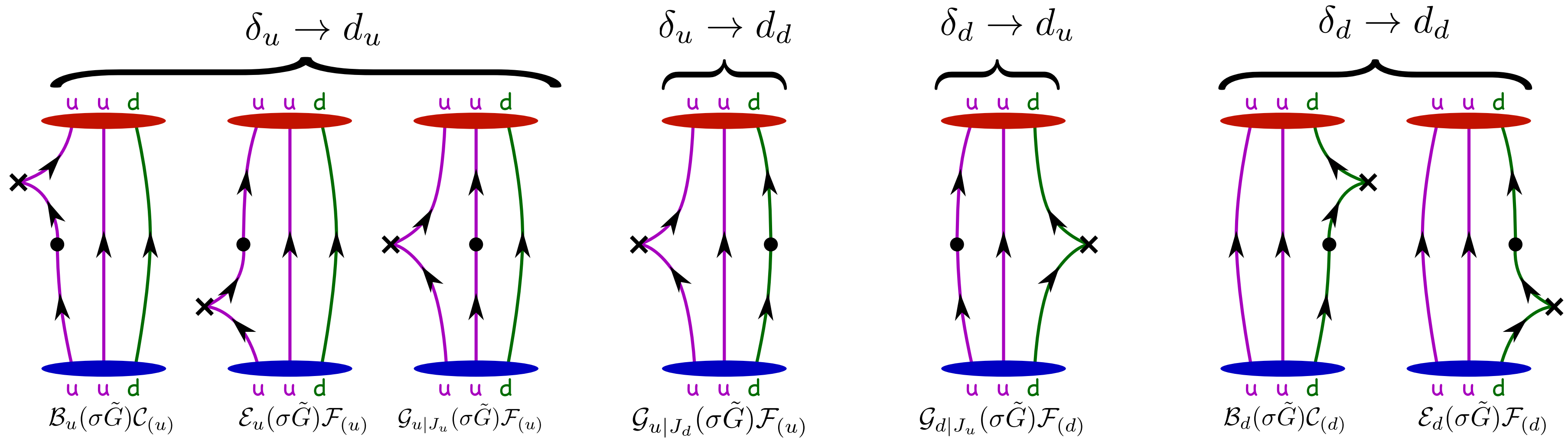
*Agreement between the **new** F_3 formula and the energy shift method*

Quark chromo-EDM: Insertions of dim-5 Operators

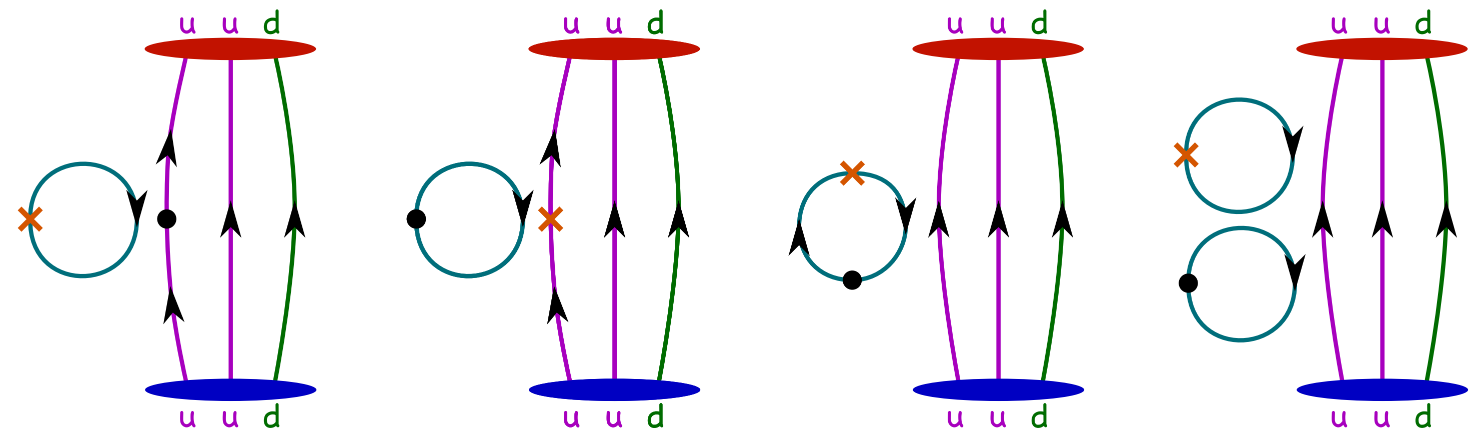
$$\mathcal{L}^{(5)} = \sum_q \tilde{d}_q\, \bar{q}(G\cdot\sigma)\gamma_5 q \quad \longrightarrow \quad \begin{cases} \langle N(y)\,\bar{N}(0)\,\int d^4x\, \bar{q}(G\cdot\sigma)\gamma_5 q\rangle \\ \langle N(y)\,[\bar{\psi}\gamma^\mu\psi]_z\,\bar{N}(0)\,\int d^4x\, \bar{q}(G\cdot\sigma)\gamma_5 q\rangle \end{cases}$$

First calculations : [T.Bhattacharya et al(LANL, LATTICE'15,'16]

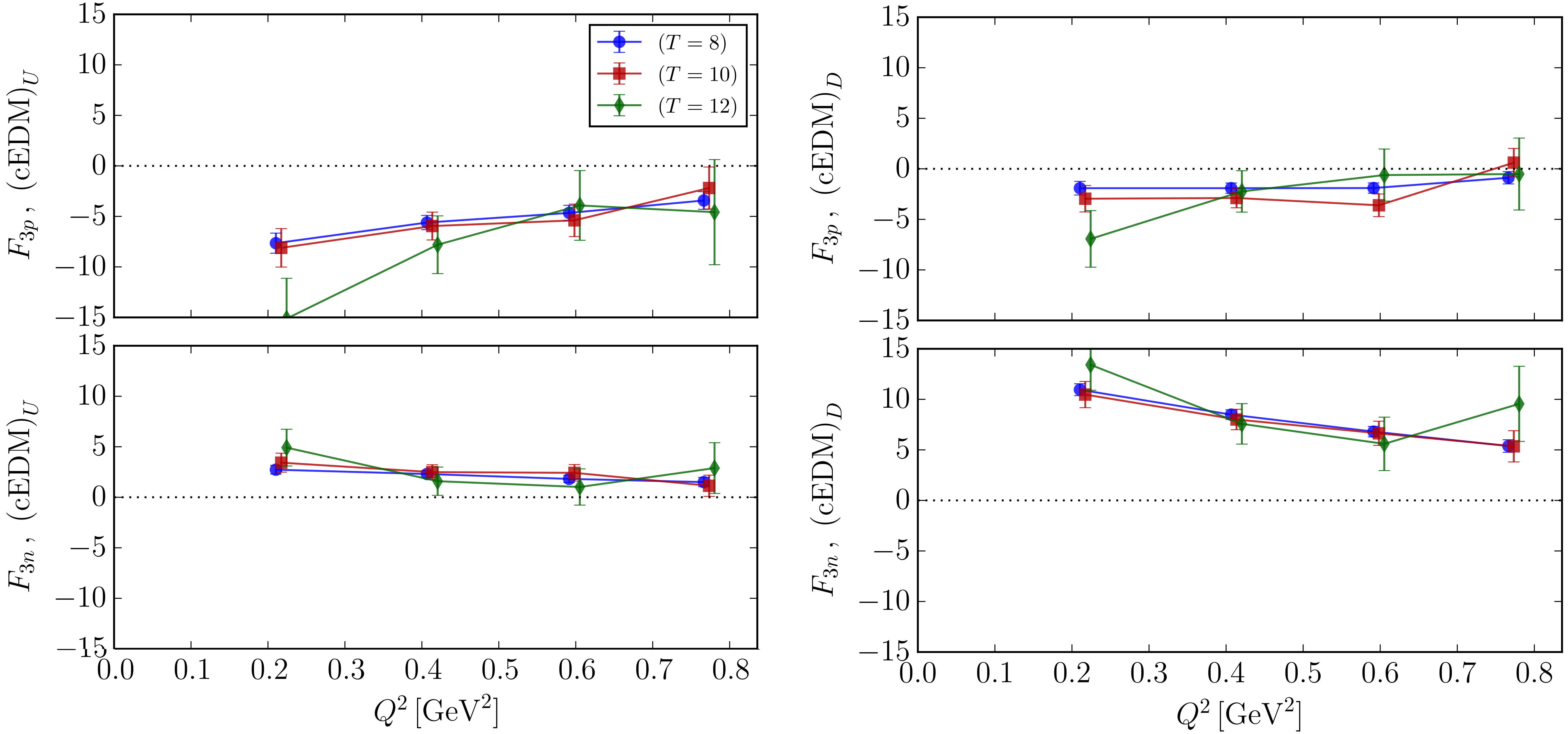
- So far: Only quark-connected insertions



- Future (hopefully): Single- and double-disconnected diagrams (contribute to isosinglet cEDM, mix with θ-term)



Current Results on cEDM-induced nEDM



- connected-only
- bare cEDM operators on a lattice (no renormalization/mixing subtraction)
- statistics = 10,500 samples on 24^3x64 m_π = 330 MeV DW ensemble

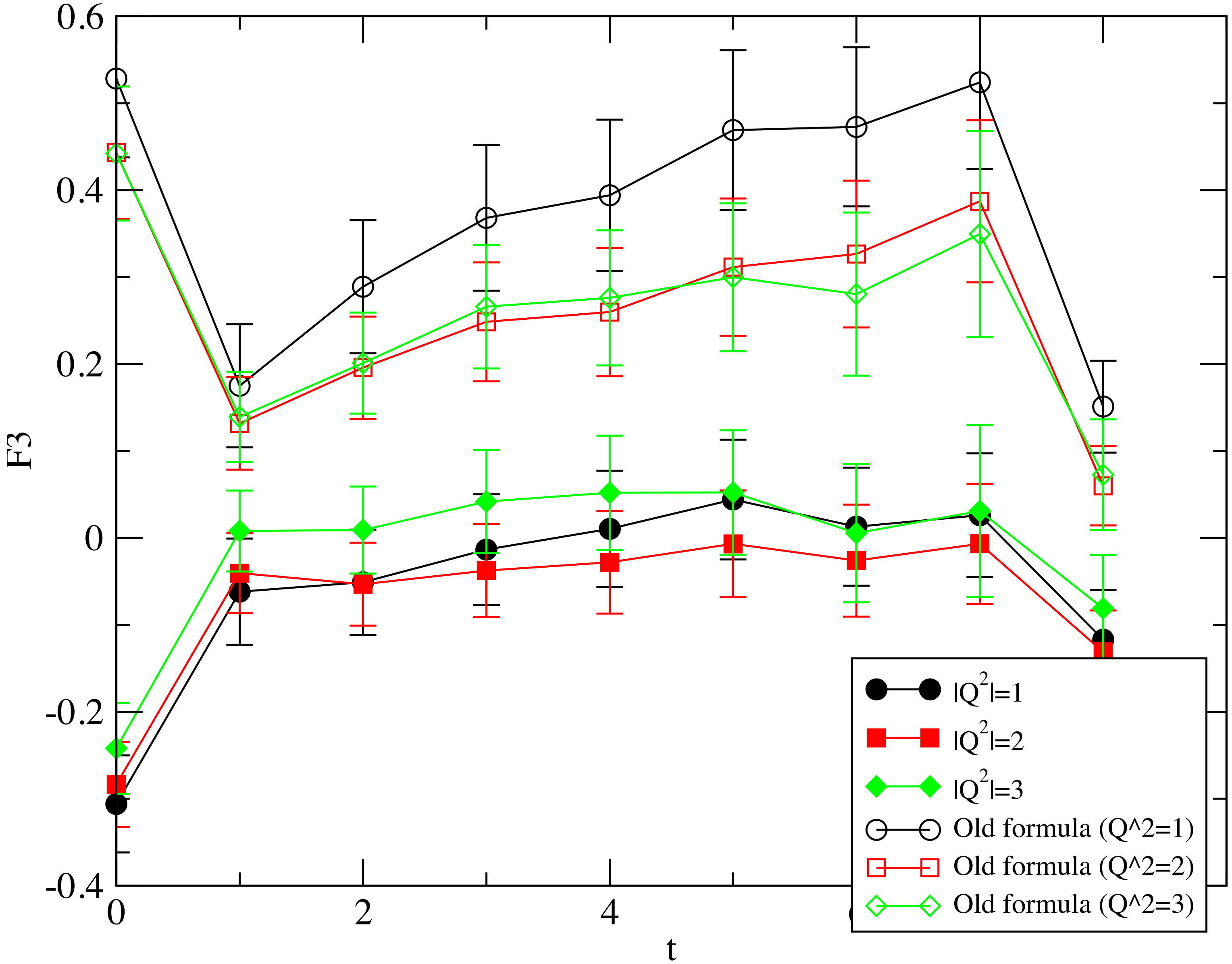
Request

- [**EDM**] high-statistics calculation of theta- and cEDM-induced p,nEDM
- [**FormFac**] exploration of required stat. for EDM at the physical point
- [**Pdecay**] opportunistic calculation to reuse (relatively expensive) chirally symmetric light-quark propagators

	DSDR 32^3x64 $m\pi$=250 MeV a=0.142 fm	DSDR 32^3x64 $m\pi$=170 MeV a=0.142 fm	DSDR 32^3x64 $m\pi$=140 MeV a=0.200 fm	ID 64^3x128 $m\pi$=140 MeV a=0.090 fm	REQUEST [Jpsich]
EDM	9,600 samp.	9,600 samp.	6,400 samp.		43.3M [CPU]
FORMFAC				6,400 samp.	16.1M [CPU]
PDECAY				6,400 samp.	6.1M [CPU]
TOTAL					**70.1M**

- At present: continue exploration on 24^3x64 330 MeV
- Reuse eigenvectors from the HVP/HLbL project

Current theta-EDM estimate



- 24^3x64 $m\pi$=330 MeV, 3,200 samples

$$|F_3| \lesssim 0.1 \quad \Rightarrow \quad |d_n|/\theta \lesssim 0.01\, e \cdot \mathrm{fm}$$

Nucleon "Parity Mixing" : EDM and aMDM

Correct identification of $F_{2,3}$ in nucleon ME based on parity of the vector current matrix element: $F_{1,2}$ P,T-even, F_3 P,T-odd [S.Aoki, SNS, *et al* (2017) arXiv:1701.07792]

$$\langle N_{p'}|\bar{q}\gamma^\mu q|N_p\rangle_{\not{CP}} = \bar{u}_{p'}\left[F_1\gamma^\mu + (F_2 + iF_3\gamma_5)\frac{i\sigma^{\mu\nu}(p'-p)_\nu}{2m_N}\right]u_p$$

with $u_{\vec{p},\sigma} \to u_{-\vec{p},\sigma} = \gamma_4\, u_{\vec{p},\sigma} \quad \Leftrightarrow \quad (i\not{p} + m)u_{p,\sigma} = 0$

- poles of the Dirac equation with CPv nucleon mass in bg. electric & magnetic fields

$$\mathcal{L}_N = \bar{N}\left[i\not{\partial} - me^{-2i\alpha\gamma_5} - Q\gamma_\mu A^\mu - (\tilde{\kappa} + i\tilde{\zeta}\gamma_5)\frac{1}{2}F_{\mu\nu}\frac{\sigma^{\mu\nu}}{2m_N}\right]N$$

$$E_N(\vec{p}=0) - m_N = -\frac{\kappa}{2m_N}\vec{\Sigma}\cdot\vec{H} - \frac{\zeta}{2m_N}\vec{\Sigma}\cdot\vec{E} + O(\kappa^2, \zeta^2)$$

where $\kappa + i\zeta = e^{2i\alpha\gamma_5}(\tilde{\kappa} + i\tilde{\zeta})$

Numerical test: compare EDFF to mass shift in uniform bg. electric field

Full flux through the "side" of the periodic box $= q\Phi = 2\pi \cdot n$

Constant Electric field has to be quantized, $\mathcal{E}_{\min} = \frac{1}{|q_d|}\frac{2\pi}{L_x L_t}$