Muon g-2 hadronic vacuum polarization from 2+1+1 flavors of sea quarks using the HISQ action

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Motivation

- Muon anomalous magnetic moment (g-2) provides sensitive probe of physics beyond the Standard Model:
  - Mediated by quantum-mechanical loops
  - Known to very high precision of 0.54ppm
- Measurement from BNL E821 disagrees with Standard-Model theory expectations by more than 3σ
- Muon g-2 Experiment being mounted at Fermilab to reduce the experimental error by a factor of four
  - Will begin running this year, and expect first results in Spring 2018!
- **Theory error must be reduced to a commensurate level to identify definitively whether any deviation observed between theory and experiment is due to new particles or forces**

▷ Our ongoing project uses *ab-initio* lattice-QCD to target the hadronic vacuum-polarization contribution, which is the largest source of theory error

Methodology

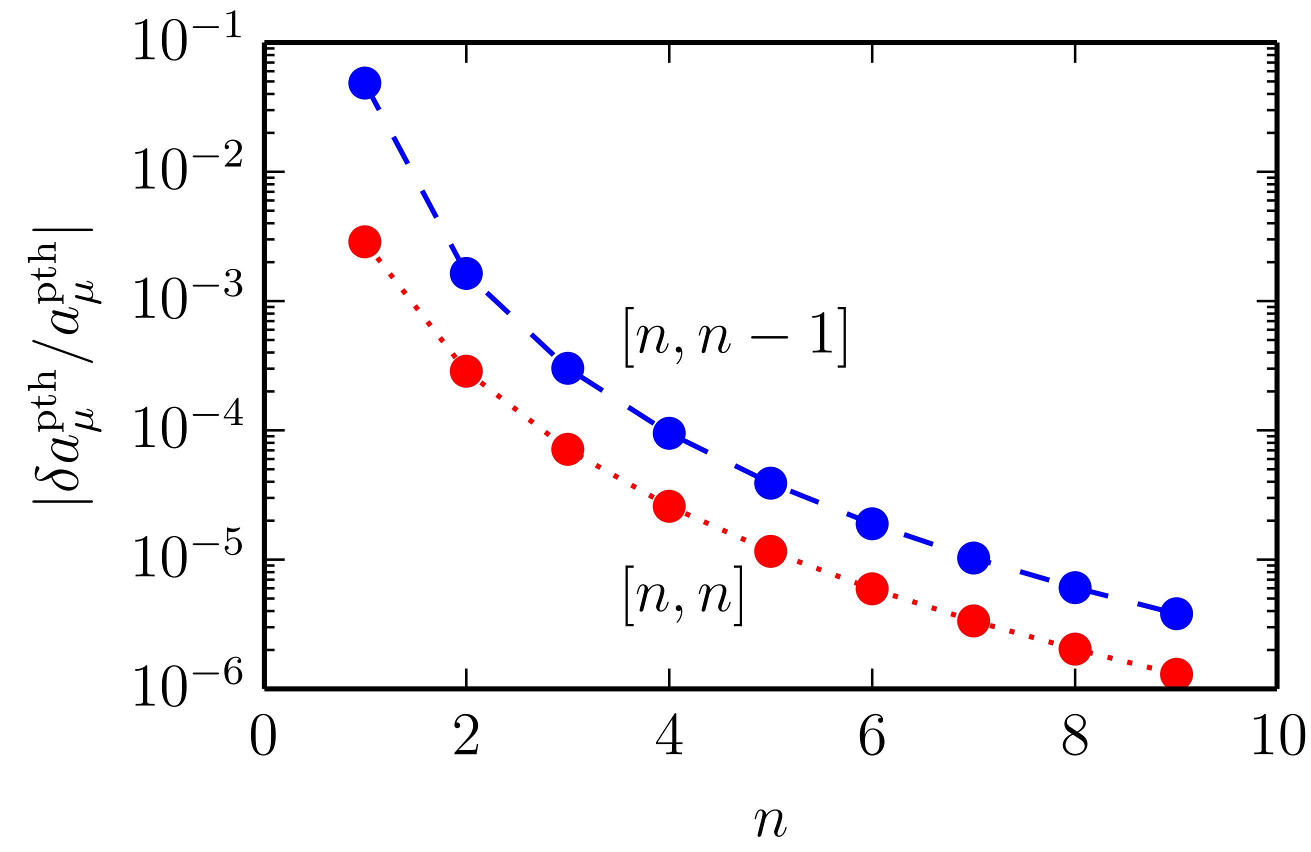
✦ Use "time moments" method introduced by HPQCD in PRD89 (2014) no.11, 114501

(1) **Calculate Taylor coefficients of vacuum polarization function $\Pi(q^2)$** from time moments of vector current-current correlators

(2) **Replace Taylor series for $\Pi(q^2)$ by its [n,n] and [n,n-1] Padé approximants** to obtain the correct high-q^2 behavior

- ❖ Exact result always between [n,n] and [n,n-1] Padé
- ❖ [2, 2] approximant sufficient to obtain ~0.5% precision

(3) **Plug $\Pi(q^2)$ into standard 1-loop QED integral** to obtain a_μ^{HVP}



Light-quark-connected contribution

- HQCD demonstrated method on (2+1+1)-flavor HISQ ensembles with physical light-quark masses

- **Obtain total uncertainty on light-quark-connected contribution $a_\mu^{\mathrm{HVP,LO}}$ (u/d) of ~2%** [arXiv:1601.03071]

	$a_\mu^{\mathrm{HVP,LO}}(u/d)$
QED corrections:	1.0%
Isospin breaking corrections:	1.0%
Staggered pions, finite volume:	0.7%
Valence m_ℓ extrapolation:	0.4%
Monte Carlo statistics:	0.4%
Padé approximants:	0.4%
$a^2 \to 0$ extrapolation:	0.3%
Z_V uncertainty:	0.4%
Correlator fits:	0.2%
Tuning sea-quark masses:	0.2%
Lattice spacing uncertainty:	$< 0.05\%$
Total:	1.8%

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  - Errors from the nonzero lattice spacing and finite spatial volume are next

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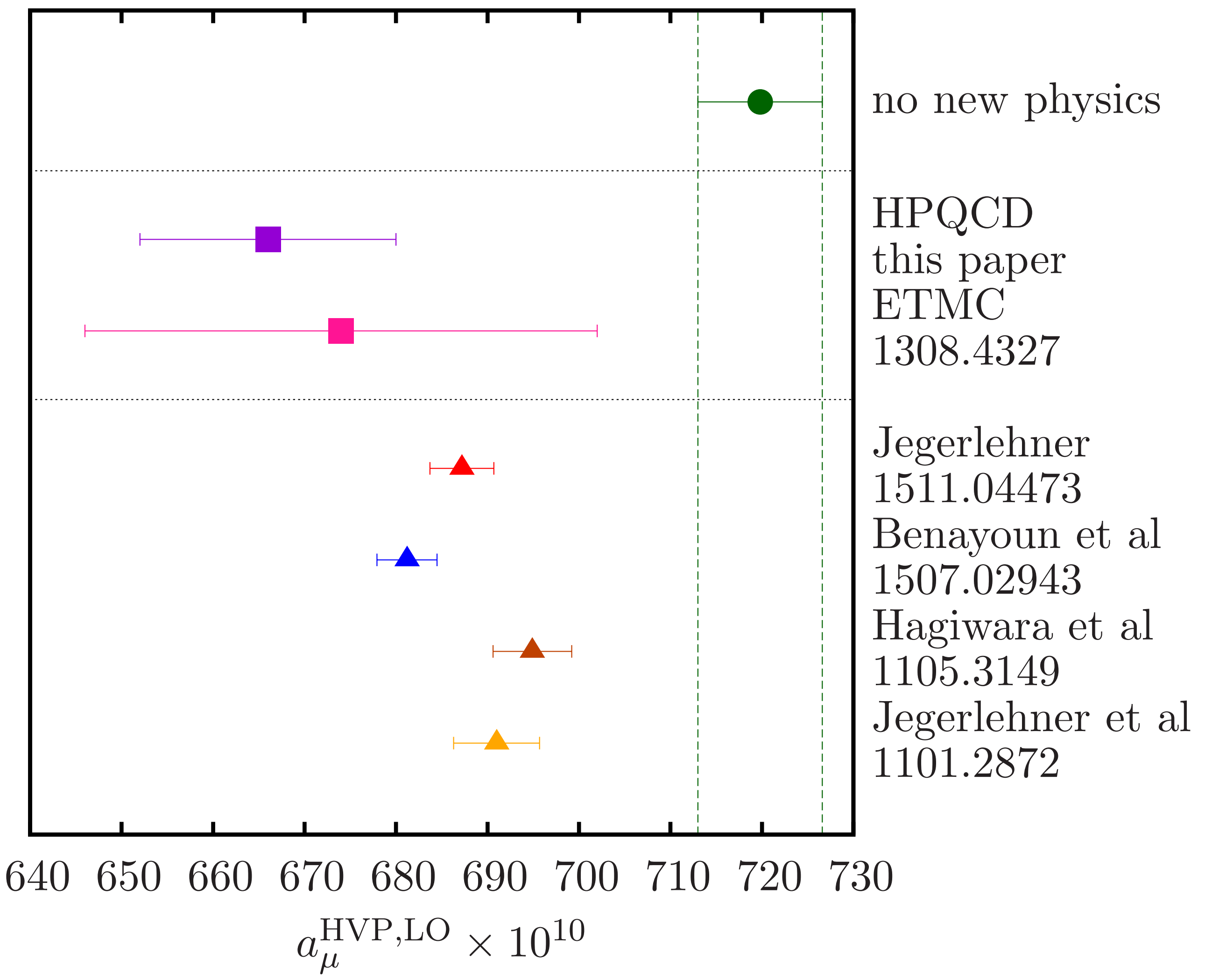
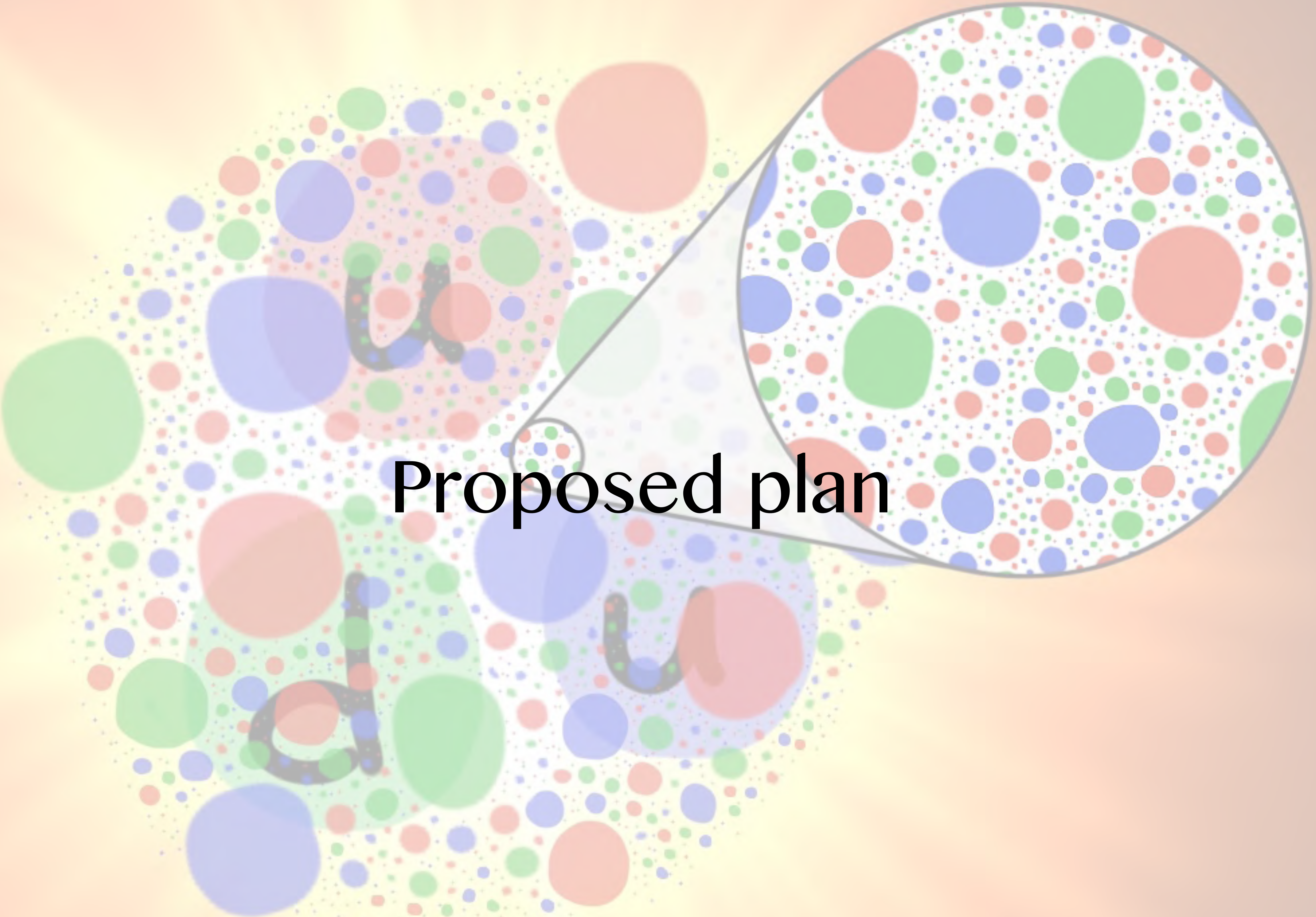
Total leading-order HVP contribution

- To obtain total leading-order HVP contribution to g-2, also include connected contributions from strange, charm, & bottom quarks [PRD 89, no. 11, 114501 (2014); PRD 91, no. 7, 074514 (2015)]
- HPQCD + Hadron Spectrum Collaboration unable to obtain a statistically-significant signal for the quark-disconnected contribution [PRD 93, no. 7, 074509 (2016)]
  - Calculation bounds error from omitted disconnected contributions to be comparable to that of the light-quark connected contribution



$$a_\mu^{\mathrm{HVP,LO}} \times 10^{10} = 666(11)_{u,d}(1)_{s,c,b}(9)_{\mathrm{disc.}}$$



Proposed plan

Overview

- Employ large set of MILC ensembles with four flavors of dynamical HISQ sea quarks with:
  - **Three lattice spacings** a~0.09—0.15 fm
  - **Multiple spatial volumes** at a~0.12 fm
  - **Physical light-quark masses**
- Multi-year strategy to improve the HPQCD result and meet target experimental precision includes both adding more data and refining the analysis
- Focus on reducing the leading sources of error from

(1) Omission of isospin-breaking and electromagnetism,

(2) Omission of the quark-disconnected contribution, and

(3) Finite spatial volume and staggered discretization effects

- Since joining efforts, have added substantially to the existing data set

▷ *Analysis is in progress, and will present preliminary result at Lattice 2017*

Discretization & finite-volume errors

- Combined "Staggered pions, finite volume" error (currently ~0.7%) accounts for both finite spatial lattice volume & taste-breaking discretization errors from staggered action
  - 1-pion-loop corrections to the moments due to these effects calculated in scalar QED and used to correct the moments in our analysis
  - Corrections largely from taste splittings between staggered pions in the sea, and become smaller and better controlled as the continuum limit is approached
- Addressing discretization errors by analyzing ensembles with yet finer lattice spacings
  - Computed vector-current correlators on ~1000 physical-mass configurations with a~0.09 fm
  - Using USQCD full-priority INCITE time and 1-year ALCC allocation on Mira, analyzed about 170 configurations on physical-mass ensemble with a~0.06 fm
- a~0.06 fm ensemble has very small taste splittings (root-mean-squared pion mass is ~140 MeV) and will substantially reduce "Staggered pions, finite volume" error

➡ Requesting USQCD zero-priority time on Mira to continue this running

QED & isospin breaking

- Dominant systematic uncertainties in HPQCD calculation are from the omission of isospin breaking and electromagnetism
- **Estimated based on QCD models and analysis of experimental data to each enter at the level of 1%** [Hagiwara et al., PRD 69, 093003 (2004), Wolfe & Maltman, PRD 83, 077301 (2011)]

➡ **To bring errors on $a_\mu^{HVP,LO}$ to below 1%, must directly include isospin-breaking and electromagnetism in our calculations**

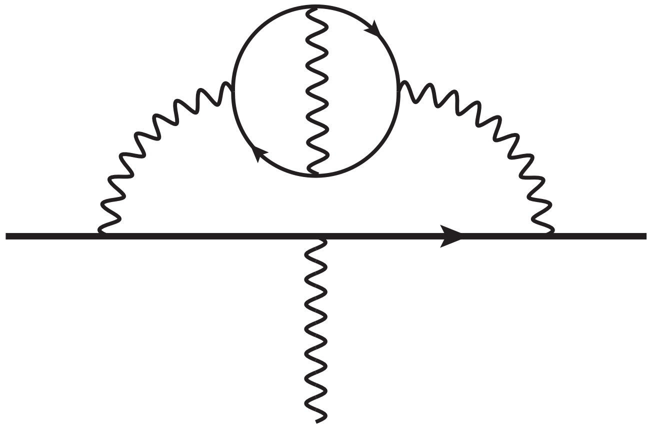
  - Because contributions are numerically small, do not need high precision to reduce their contributions to the total uncertainty to the needed sub-percent level

Isospin breaking: work in progress

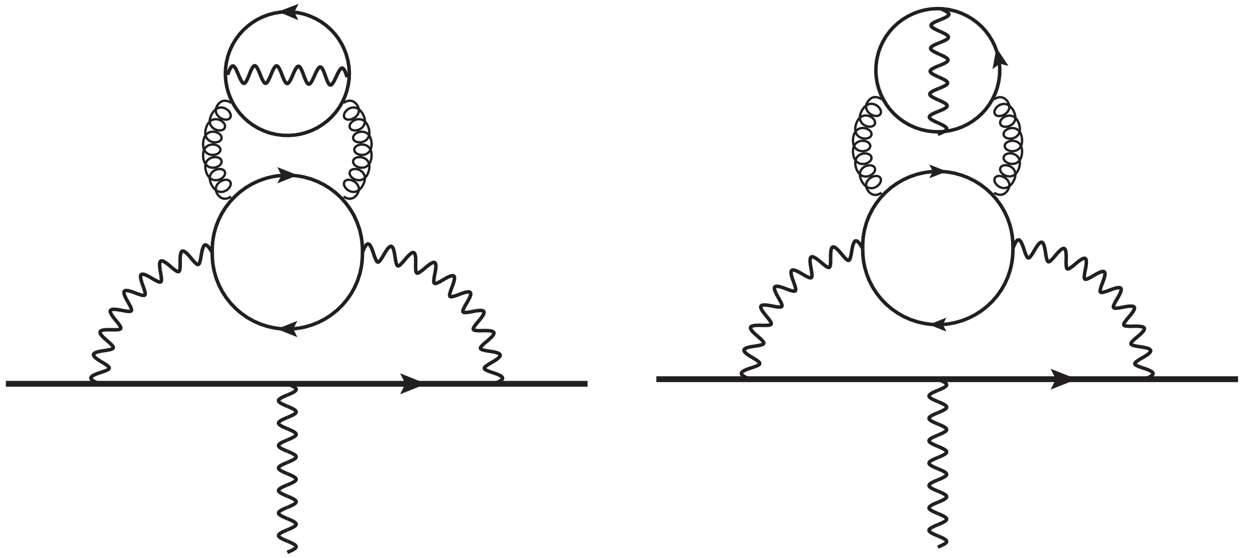
- Performing two calculations to disentangle valence and sea isospin-breaking effects
- Using UK computing resources, calculating partially-quenched vector-meson correlators on isospin-symmetric physical-mass ensemble with a~0.15 fm
  - Just completed preliminary analysis of ~1,000 configurations and three valence masses m_u, m_d, and $m_l = (m_u+m_d)=2$
  - Expect to obtain good signal for valence isospin-breaking correction to a_μ^{HVP} with sufficient precision using ~5,000 configurations.
- Generated on BNL Institutional Cluster a (1+1+1+1)-flavor ensemble with a~0.15 fm with same parameters as the isospin-symmetric ensemble above except for the ratio of the light up and down sea-quark masses fixed to their physical value $m_u/m_d=0.4582$ [PoS(LATTICE 2015)259, arXiv:1606.01228]
  - Analysis of this ensemble to determine sea isospin-breaking contribution to a_μ^{HVP} is underway

QED contributions to a_μ^{HVP}

- Leading QED corrections hadronic vacuum polarization contribution to g-2 are of $O(\alpha_{EM}^3)$



Only involves photons that couple to valence quarks ➡ **contributes in quenched QED**



Involve photons that couple to sea quarks ➡ ***can only be computed within dynamical QED***

- Although sea-photon contributions are higher-order in α_S, may not be numerically smaller than valence contributions if relevant scale at which α_S should be evaluated for this process is sufficiently small

➡ **Complete estimate of electromagnetic contributions to the muon g-2 HVP requires dynamical-QED simulations**

QED contributions: proposed plan

- MILC has submitted a proposal to USQCD to generate a dynamical QED+QCD ensemble with a~0.15 fm at the physical pion mass

PLAN A: If the SPC funds MILC’s configuration-generation proposal, we will use them immediately for our calculation of g-2

- MILC anticipates being able to generate some lattices on Indiana University's Cray Big Red, where they have been developing the dynamical QED code, so we anticipate being able to start analysis on this ensemble at the beginning of the USQCD allocation cycle in July

PLAN B: If MILC’s configuration-generation proposal is not funded, we will analyze the existing a~0.15 physical within quenched QED

- The Fermilab Muon g-2 Experiment needs results for a_μ^{HVP} by next Spring, so we should do the best job possible to estimate the EM contributions to the HVP contribution to g-2 in the next year, even if it is within the quenched approximation

Quark-disconnected contributions

- Error from quark-disconnected contributions contributes roughly half of total uncertainty on HPQCD result for $a_\mu^{HVP,LO}$
  - Estimate based on calculation using anisotropic Clover configurations with a~0.12 fm & m_π~390 MeV [HPQCD & HadSpec, PRD 93, no. 7, 074509 (2016)]
- Recently RBC/UKQCD used state-of-the-art variance-reduction techniques to perform first lattice-QCD calculation of quark-disconnected contribution $a_\mu^{HVP,LO}$ at the physical pion mass, obtaining a ~40% total uncertainty [PRL 116, no. 23, 232002 (2016)]
- **Have added necessary components for deflation to MILC code, and are currently analyzing the a~0.15 fm physical-mass ensemble with NERSC allocation that started in mid-January**

➡ **Requesting time from USQCD to analyze the a~0.12 fm physical-mass ensemble**

- Promised reduction in statistical errors will enable us to replace HPQCD's estimated bound on the size of omitted quark-disconnected contributions with a direct calculation
  - Given errors commensurate with or better than the RBC/UKQCD result, will bring error in $a_\mu^{HVP,LO}$ from quark-line disconnected u,d, & s contributions to below ~0.5%.

Resource request

(1) **zero-priority INCITE time on BG/Q Mira at Argonne to analyze ~100 configurations on the a~0.06 fm physical-mass ensemble** (additional resources could easily be used to analyze additional configurations)

(2) **GPU time on the BNL Institutional Cluster to analyze ~2,500 configurations on the new a~0.015 fm dynamical QED + QCD ensemble** (or to analyze ~5,000 configurations on existing QCD ensemble within quenched QED)

(3) **Cluster time at Fermilab to calculate quark-disconnected contributions on the a~0.12 fm physical-mass ensemble using deflation methods**

Project task	$\approx a$ (fm)	$L^3 \times T$	# configs.	time requested
Dynamical QED	0.15	$32^3 \times 48$	2,500	421k C2050 GPU-hrs
Quenched QED ("plan B")	0.15	$32^3 \times 48$	5,000	841k C2050 GPU-hrs
Disconnected contributions	0.12	$48^3 \times 64$	50	18×10^6 Jpsi core-hrs
FV + discretization error	0.06	$96^3 \times 192$	100	15% Mira zero-priority

Ongoing analysis work

Euclidean G(t) from $e^+e^- \to$ hadrons

- Spectral representation for G(t) given in terms of the spectral density **ρ(ω)**, which is related to the total $e^+e^-\to$hadrons cross section

➡ Can calculate G(t) and its moments from experimental average for R(s) [see Jegerlehner public alphaQED fortran package]

- Enables comparison of lattice-QCD results & experimental measurements at more fundamental level than a_μ

- Provides means of combining lattice-QCD and experimental information to improve Standard-Model determination of $a_\mu^{HVP,LO}$

Spectral representation of G(t) @ zero momentum

$$G(t) = \int_0^\infty d\omega\, \omega^2 \rho(\omega^2) e^{-\omega|t|}$$

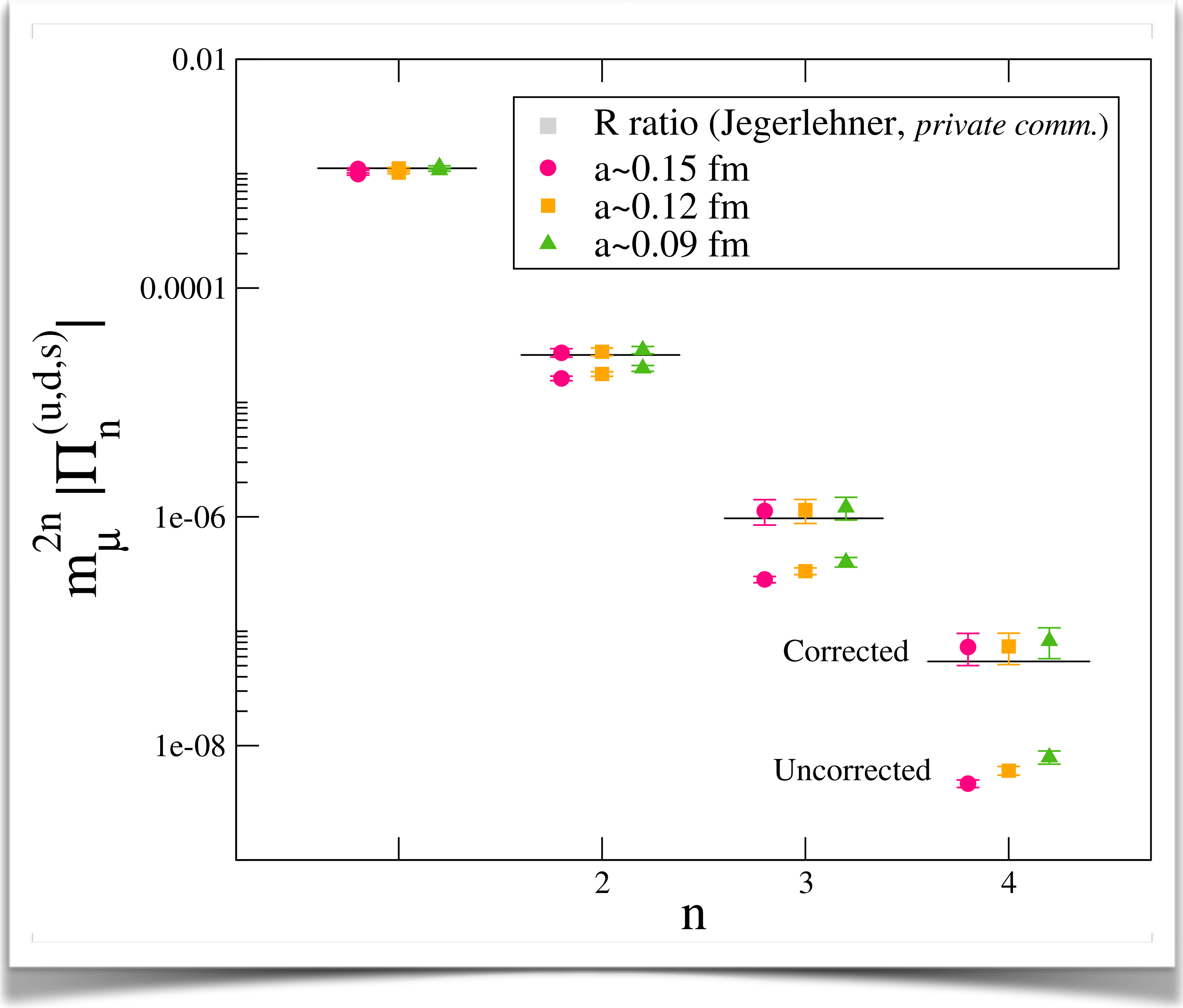
Spectral density from $e^+e^-\to$ hadrons

$$\rho(s) = \frac{R(s)}{12\pi^2}\,,$$

$$R(s) \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{4\pi\alpha(s)^2/(3s)}$$

Taylor coefficients: R-ratio vs. lattice data

- Comparison of $\Pi(q^2)$ Taylor coefficients on physical-mass ensembles with R-ratio values tests:
  - Moments + Padé approach
  - Scalar-QED calculation of finite-volume + discretization corrections
- **Corrections bring sum of u/d- and s-quark Πs into agreement with R-ratio data**



$a_\mu^{\mathrm{HVP,LO}}$ from mixed-rep. correlator

- **Can calculate a_μ directly from weighted integral Euclidean electromagnetic-current correlator**
- Provides check of and alternative to moments + Padé approach
- **Kernel $\tilde{K}(t)$ proportional to t at small t and to 1/t at large t, suppressing contributions from large times**
  - *Advantage over correlator moments, which become progressively sensitive to larger times via t^{2n} factor*

$$a_\mu^{\mathrm{HLO}} = 4\alpha^2\, m_\mu \int_0^\infty dt\, t^3\, G(t)\, \tilde{K}(t),$$

$$G(t) \equiv \int d\boldsymbol{x}\, \langle j_z^{\mathrm{em}}(t,\boldsymbol{x}) j_z^{em\,\dagger}(0)\rangle\,,$$

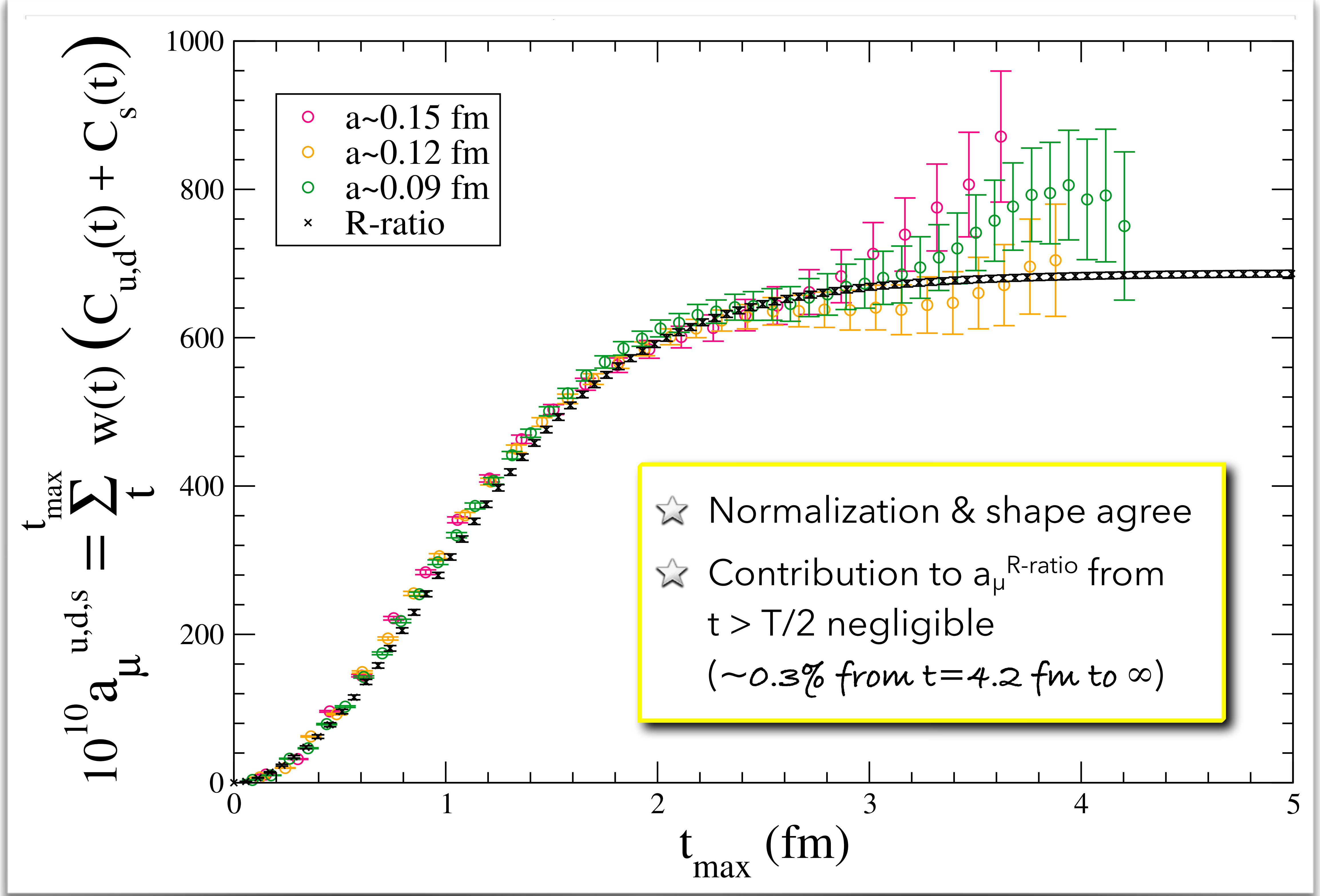
$$\tilde{K}(t) \equiv \frac{2}{m_\mu t^3} \int_0^\infty \frac{d\omega}{\omega}\, K_E(\omega^2)\left[\omega^2 t^2 - 4\sin^2\left(\frac{\omega t}{2}\right)\right]\,,$$

$$K_E(s) = \frac{1}{m_\mu^2} \cdot \hat{s} \cdot Z(\hat{s})^3 \cdot \frac{1 - \hat{s} Z(\hat{s})}{1 + \hat{s} Z(\hat{s})^2}\,,$$

$$Z(\hat{s}) = -\frac{\hat{s} - \sqrt{\hat{s}^2 + 4\hat{s}}}{2\hat{s}}, \qquad \hat{s} = \frac{s}{m_\mu^2}$$

[Bernecker & Meyer, EPJA47 (2011) 148 , arXiv:1107.4388]

$a_\mu^{(u+d+s)}$ integral: R-ratio vs. lattice data

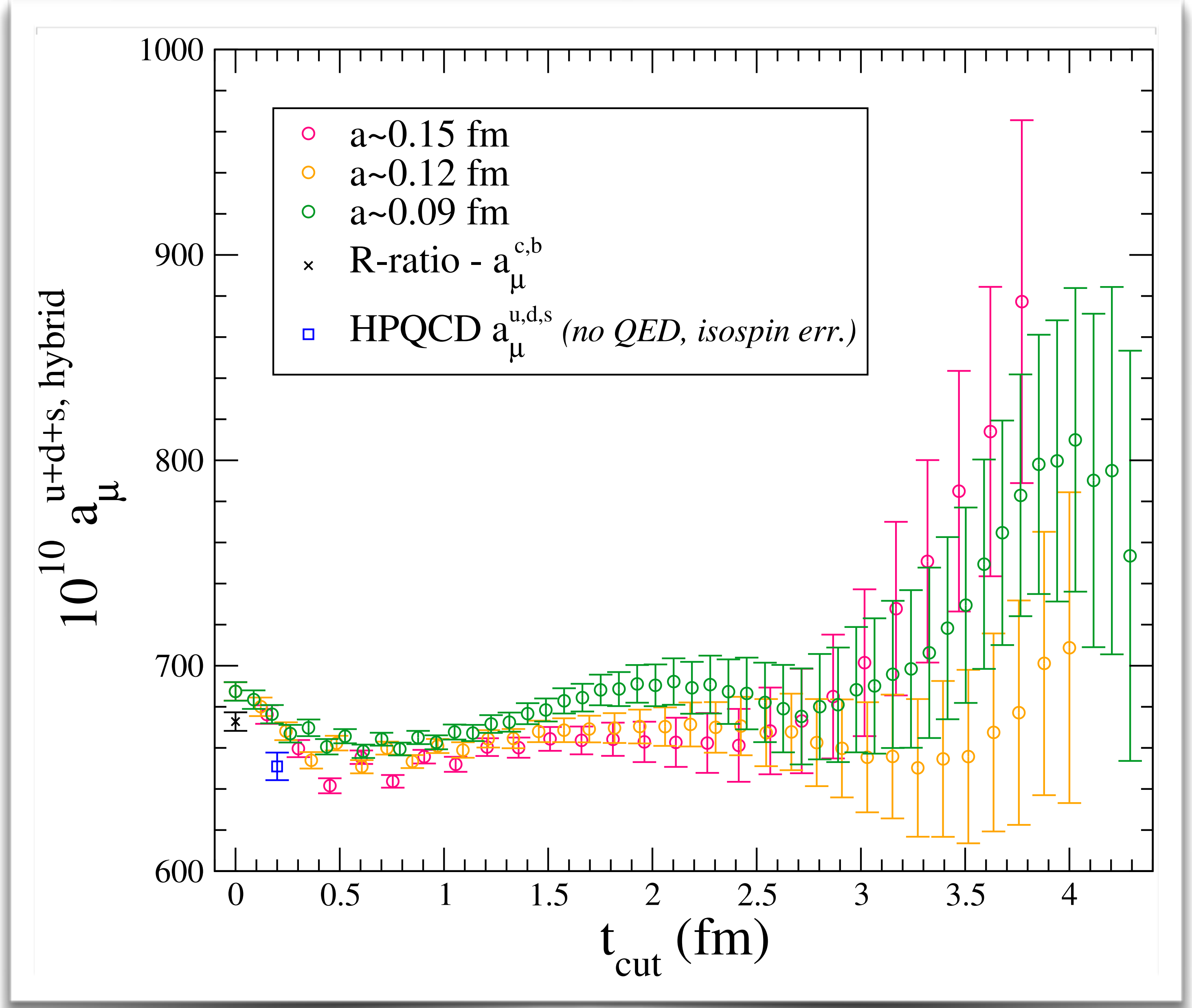


- ☆ Normalization & shape agree
- ☆ Contribution to $a_\mu^{\text{R-ratio}}$ from $t > T/2$ negligible (~0.3% from t=4.2 fm to ∞)

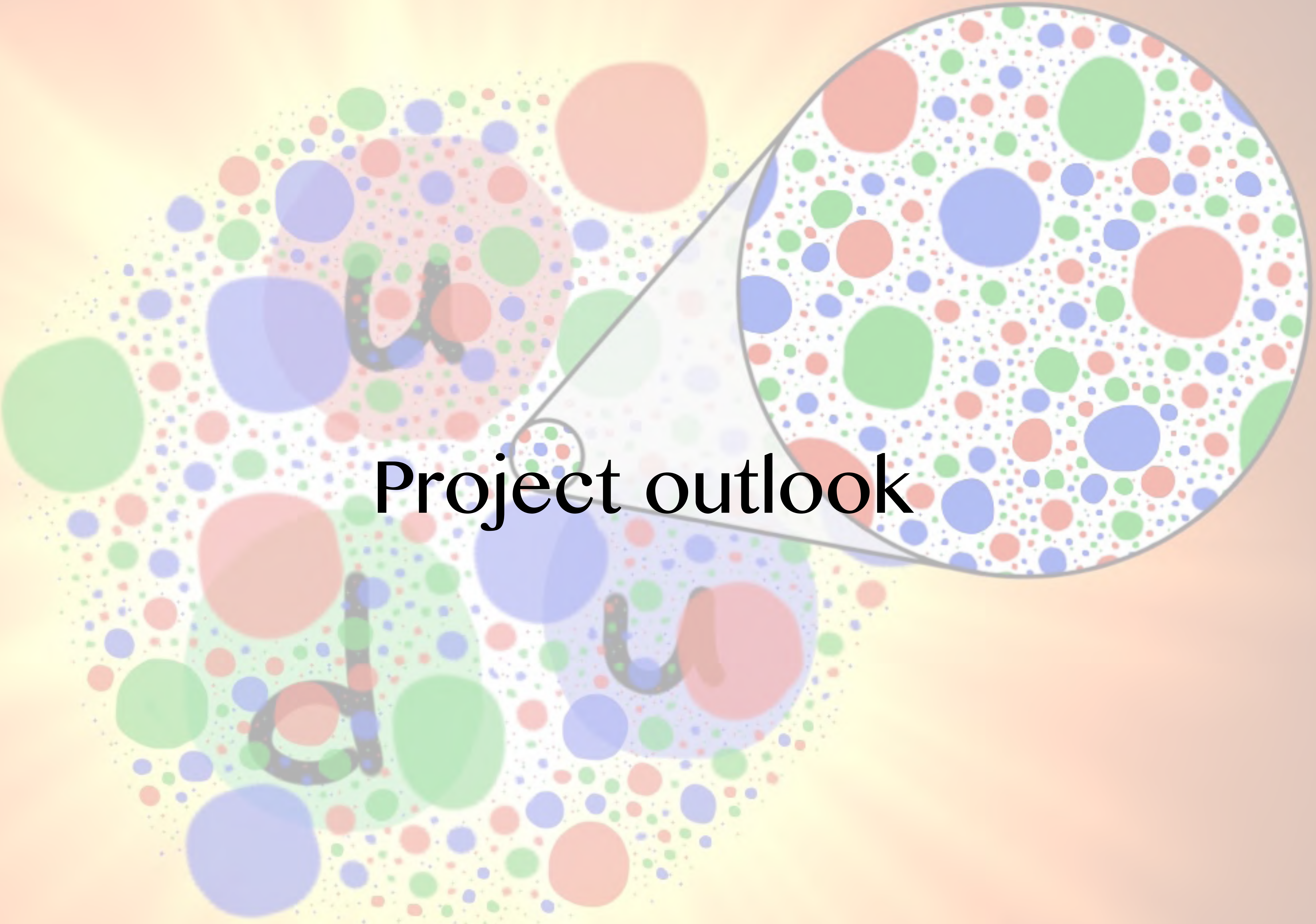
"Hybrid" lattice + experiment $a_\mu^{HVP,LO}$

$$a_\mu = \sum_{t=0}^{T} \mathrm{w}(t)\, G_{\mathrm{lat.}}(t) + \sum_{t=T+1}^{\infty} \mathrm{w}(t)\, G_{\mathrm{exp.}}(t)$$

- **Exploring hybrid approaches that combine numerical lattice-QCD data with of experimental $e^+e^- \to$ hadrons measurements** (see, e.g., Lehner 2017 APS April Meeting)
- Result would no longer be exclusively from ab-initio QCD, but may enable reaching target experimental precision sooner while we continue to reduce our lattice-calculation uncertainties



Project outlook



Outlook

- *Ab-initio* lattice-QCD calculations of the hadronic contributions are *urgently* needed to make full use upcoming Fermilab and J-PARC experiments to measure the muon g-2 to higher precision and improve the sensitivity to physics beyond the Standard Model
- Theoretical methods and algorithms are in place to obtain a percent-level calculation of the hadronic-vacuum-polarization contribution to g-2 from *ab-initio* lattice QCD
  - Using strategy outlined in this proposal, and given requested computing resources, anticipate obtaining ~1% total error on the $a_\mu^{\mathrm{HVP,LO}}$ in the next one-to-two years
  - At this point, we will begin to address the new largest sources of uncertainty, with a goal of reaching the anticipated experimental precision within five years
- **Analysis of recently generated data is in progress, and we will present first preliminary result from joint Fermilab Lattice / HPQCD / MILC effort at Lattice 2017**

▶ Thanks to USQCD for support, and stay tuned!