Quark Spin from Anomalous Ward Identify and Chiral Axial-vector Current

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On behalf of χQCD collaboration

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Definitions

Peter Hasenfratz, et. al, Nucl.Phys.B643:280-320,2002

The vector conserved current for the wilson/clover action,

$$\bar{\psi}_y \frac{1-\gamma_\mu}{2} U_\mu(y) \psi_{y+\hat{\mu}} - \bar{\psi}_{y+\hat{\mu}} U_\mu^\dagger(y) \frac{1+\gamma_\mu}{2} \psi_y$$

with $\partial_\mu^* \left[\bar{\psi}_y \frac{1-\gamma_\mu}{2} U_\mu(y) \psi_{y+\hat{\mu}} - \bar{\psi}_{y+\hat{\mu}} U_\mu^\dagger(y) \frac{1+\gamma_\mu}{2} \psi_y \right] = 0$

∂_μ^* *is the backward lattice derivative.*

That for the overlap fermion is,

$$V_\mu^a(x) = J_{R\mu}^a(x) + J_{L\mu}^a(x) = \overline{\psi} \left(P_L K_\mu(x) \hat{P}_R + P_R K_\mu(x) \hat{P}_L \right) T^a \psi$$
$$= \frac{1}{2} \overline{\psi} \left(K_\mu(x) - \gamma_5 K_\mu(x) \hat{\gamma}_5 \right) T^a \psi \,,$$

with $K_\mu(x) = -i \left. \frac{\delta D(U_\mu^{(\alpha)})}{\delta \alpha_\mu(x)} \right|_{\alpha=0}$ *and* $\hat{\gamma}_5 = \gamma_5 (1 - D) \,.$

$$U_\mu(x) \to U_\mu^{(\alpha)}(x) = e^{i\alpha_\mu(x)} U_\mu(x)$$

Peter Hasenfratz, et. al, Nucl.Phys.B643:280-320,2002

Definitions

Exact chiral axial-vector current *can be defined with* ***chiral fermions,***

$$A^a_\mu(x) = J^a_{R\mu}(x) - J^a_{L\mu}(x) = \overline{\psi}\left(P_L K_\mu(x)\hat{P}_R - P_R K_\mu(x)\hat{P}_L\right)T^a\psi$$
$$= \frac{1}{2}\overline{\psi}\left(-\gamma_5 K_\mu(x) + K_\mu(x)\hat{\gamma}_5\right)T^a\psi\,,$$

which satisfies the Anomalous Ward Identity (AWI),

$$\left\langle i\frac{\delta^a_A \mathcal{O}}{\delta\epsilon(x)}\right\rangle_{\mathrm{F}} - \left\langle \mathcal{O}\partial^*_\mu A^a_\mu(x)\right\rangle_{\mathrm{F}} + 2m\left\langle \mathcal{O}P^a(x)\right\rangle_{\mathrm{F}} - \delta^{a0}2N_f\left\langle \mathcal{O}q(x)\right\rangle_{\mathrm{F}} = 0$$

where

$$P^a(x) = \overline{\psi}\left[\frac{1}{2}E(x)\gamma_5\left(1-\frac{1}{2}D\right) + \frac{1}{4}\left(1-\frac{1}{2}D\right)\left(\hat{\gamma}_5 E(x) + E(x)\hat{\gamma}_5\right)\right]T^a\psi$$

and $$q(x) = \frac{1}{2}\mathrm{tr}(\gamma_5 D(x,x))$$

$\mathcal{O}$ *can be an arbitrary operator, likes the meson interpolation field or nucleon correlators, etc..*

Why

we need the chiral axial vector current?

On DWF 2+1 24^3x64 lattice with a=0.111(3) fm:

- Z_V from the vector charge is 1.096(6),

  Yi-Bo Yang, et. al, χQCD collaboration, Phys.Rev. D93 (2016) no.3, 034503

- Z_A from Ward identity in pion two point function is 1.105(4),

  Yi-Bo Yang, et. al, χQCD collaboration, Phys.Rev. D92 (2015) no.3, 034517

- But our improved current work shows that additional operator is needed to make the axial charge from A_i and A_4 to be the same.

  Jian Liang et. al, χQCD collaboration, arXiv: 1612.04388

$$J_\mu^A = Z_A \left(\bar{\psi}\gamma_5\gamma_\mu\hat{\psi} + f'\partial_\mu(\bar{\psi}\gamma_5\hat{\psi}) + g\bar{\psi}\gamma_5\sigma_{\mu\nu}\overleftrightarrow{D}_\nu\hat{\psi} \right)$$

No effect in the forward matrix element (the $f'\partial_\mu$ term)

The O(a) correction terms (the $f'\partial_\mu(\bar{\psi}\gamma_5\hat{\psi})$ and $g\bar{\psi}\gamma_5\sigma_{\mu\nu}\overleftrightarrow{D}_\nu\hat{\psi}$ terms)

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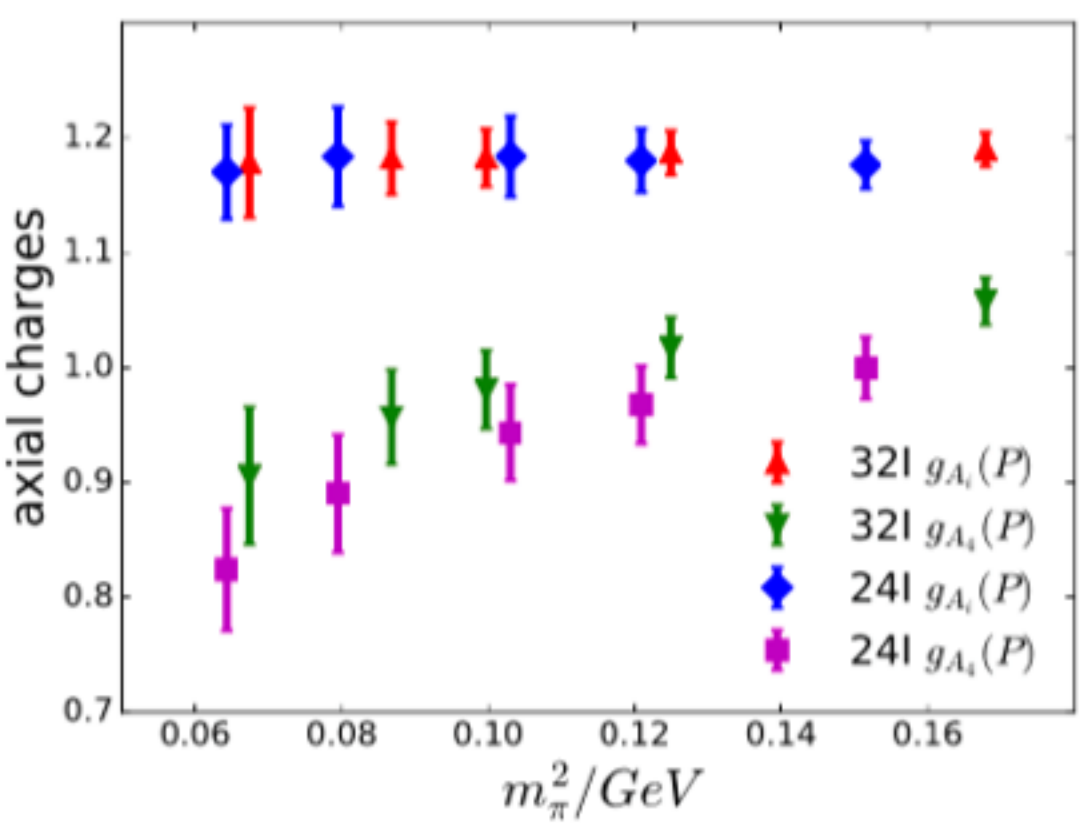

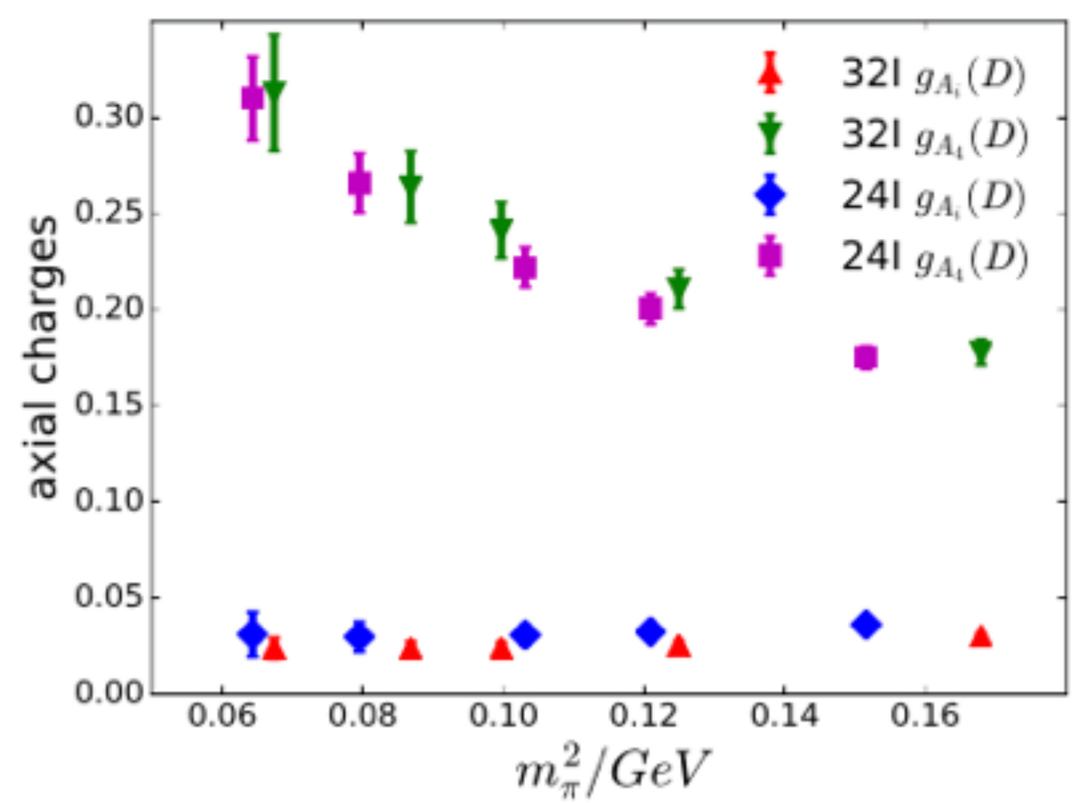

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- But our improved current work shows that additional operator is needed to make the axial charge from A_i and A_4 to be the same.

Jian Liang et. al, χQCD collaboration, arXiv: 1612.04388

  - *The improved current can enlarge the iso-vector g_A by 2%.*
  - *We will check the same quantity with* ***the chiral axial vector current***

$$A^a_\mu(x) = J^a_{R\mu}(x) - J^a_{L\mu}(x) = \overline{\psi}\left(P_L K_\mu(x)\hat{P}_R - P_R K_\mu(x)\hat{P}_L\right)T^a\psi$$
$$= \frac{1}{2}\overline{\psi}\left(-\gamma_5 K_\mu(x) + K_\mu(x)\hat{\gamma}_5\right)T^a\psi\,,$$

to confirm:

1. ***whether*** *the result is also* ***larger*** *than that with the local current.*
2. ***whether*** *the value from A_i and A_4 can be* ***the same****.*

Why

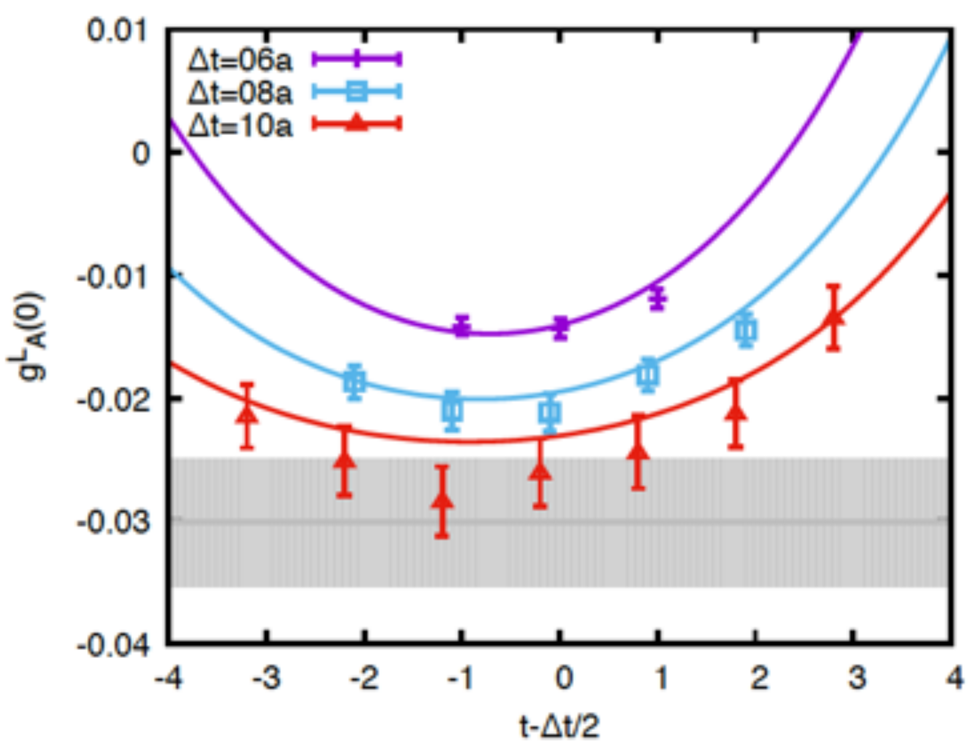
we need the chiral axial vector current?

From the Anomalous Ward Identity (AWI), $\kappa_A \partial^\mu A^0_\mu = 2\sum_{f=1}^{N_f} m_f \overline{q}_f \gamma_5 q_f + N_f 2q$

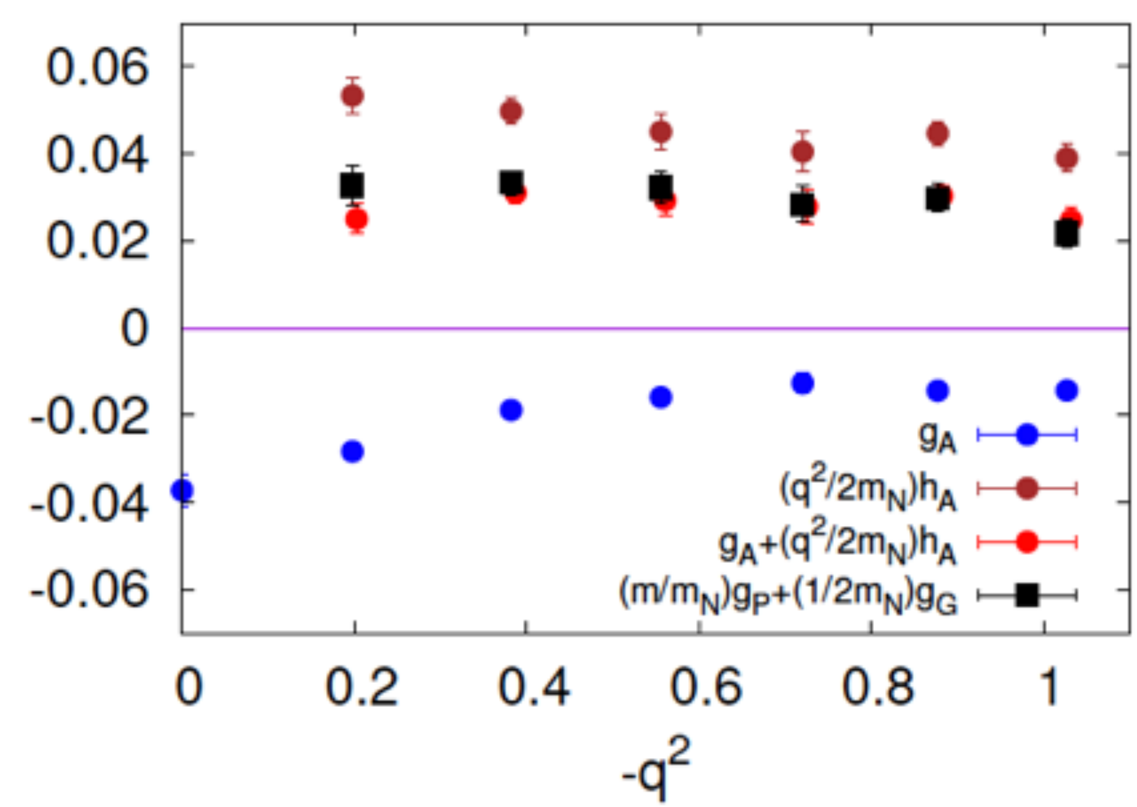
we can have the following relation,

$$\langle ps\,|\mathcal{A}_\mu|\,ps\rangle s_\mu = \lim_{\vec{q}\to 0} \frac{i|\vec{s}|}{\vec{q}\cdot\vec{s}} \langle p', s|2m_f\mathcal{P} + 2iq\,|p,s\rangle$$

$$= 2m_f\langle p,s|\int d^3x\ \vec{x}\cdot\vec{s}\ \mathcal{P}(x)|p,s\rangle + 2i\langle p,s|\int d^3x\ \vec{x}\cdot\vec{s}\ q(x)\,|p,s\rangle$$



$\kappa_A \sim 1.36$



Ming Gong, Yi-Bo Yang, et. al, χQCD collaboration, arXiv: 1511.03671

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$$\begin{aligned}\langle ps\,|\mathcal{A}_\mu|\,ps\rangle s_\mu &= \lim_{\vec{q}\to 0}\frac{i|\vec{s}|}{\vec{q}\cdot\vec{s}}\langle p', s|2m_f\mathcal{P} + 2iq\,|p,s\rangle \\ &= 2m_f\langle p,s|\int d^3x\ \vec{x}\cdot\vec{s}\,\mathcal{P}(x)|p,s\rangle + 2i\langle p,s|\int d^3x\ \vec{x}\cdot\vec{s}\ q(x)\,|p,s\rangle\end{aligned}$$

- *The improved current can enlarge the strange spin by 36%.*
- *We will check the AWI with **the chiral axial vector current***

$$\begin{aligned}A^a_\mu(x) = J^a_{R\mu}(x) - J^a_{L\mu}(x) &= \overline{\psi}\left(P_L K_\mu(x)\hat{P}_R - P_R K_\mu(x)\hat{P}_L\right)T^a\psi \\ &= \frac{1}{2}\overline{\psi}\,(-\gamma_5 K_\mu(x) + K_\mu(x)\hat{\gamma}_5)\,T^a\psi\,,\end{aligned}$$

to confirm:

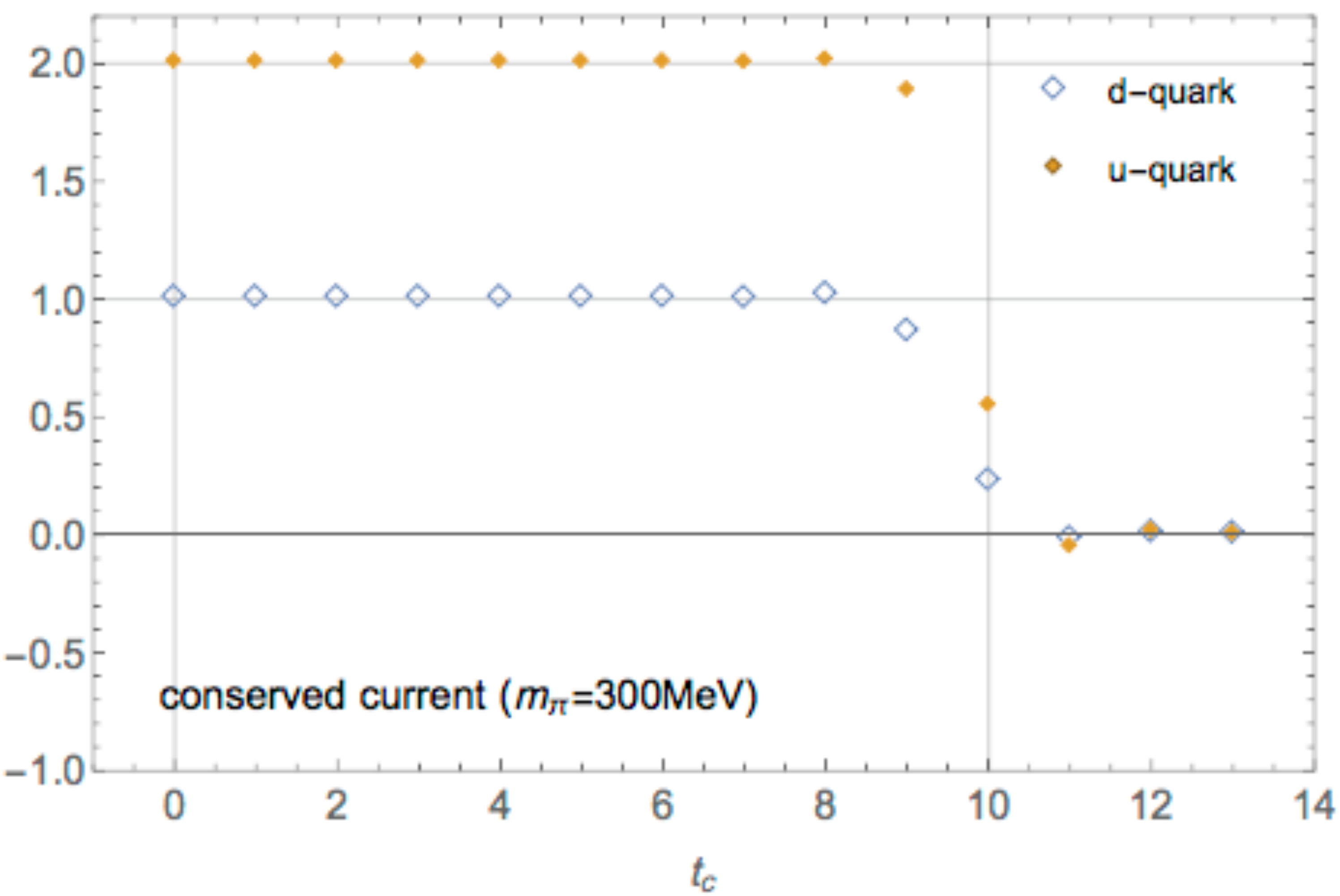
1. ***whether*** *κ_A=1 in such a case.*
2. ***whether*** *the AWI also holds perfectly in the connected insertion case.*

The test

with the conserved vector current

The vector conserved current for the overlap action,

$$V^a_\mu(x) = J^a_{R\mu}(x) + J^a_{L\mu}(x) = \overline{\psi}\left(P_L K_\mu(x)\hat{P}_R + P_R K_\mu(x)\hat{P}_L\right)T^a\psi$$
$$= \frac{1}{2}\overline{\psi}\left(K_\mu(x) - \gamma_5 K_\mu(x)\hat{\gamma}_5\right)T^a\psi\,,$$



With the conversed vector current:

- The matrix elements with V_4 are perfectly agree with the vector charges of u/d in proton,
- except the possible counteract term effect in the boundary t=10.
- We are still working on this!

Andrei Alexandru, et. al, χQCD collaboration, in preparation.

Summary

- The **Exact chiral axial-vector current** can be constructed with **the chiral fermion**.
- We need the chiral axial vector current to control the **systematic uncertainties** from kinds of the **discrepancies** we confirmed with the **local currents**.
- The **kernel** needed by the vector conserved current has been **constructed** and we are making **progress on the tests**.