Precision lattice QCD calculations and predictions of fundamental physics in heavy quark systems

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SCIDAC 2006, June 2006

Particle physics: uncover the fundamental particles and interactions at the smallest distance scales

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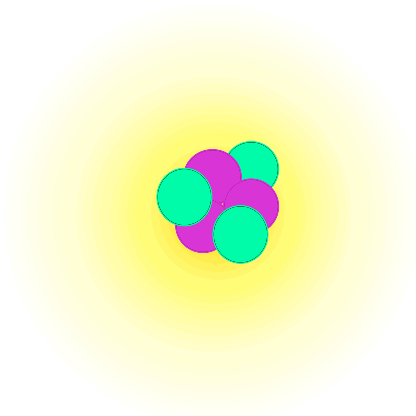
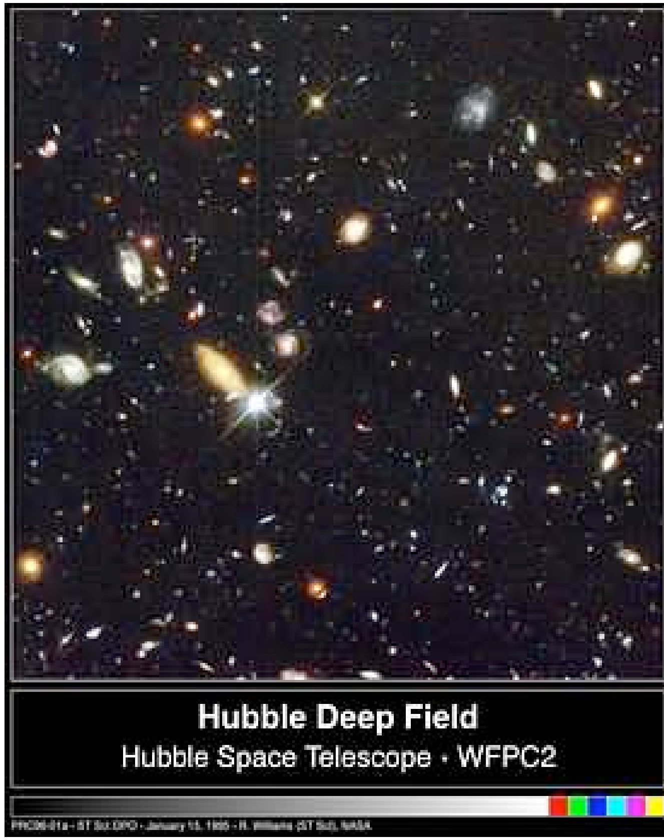
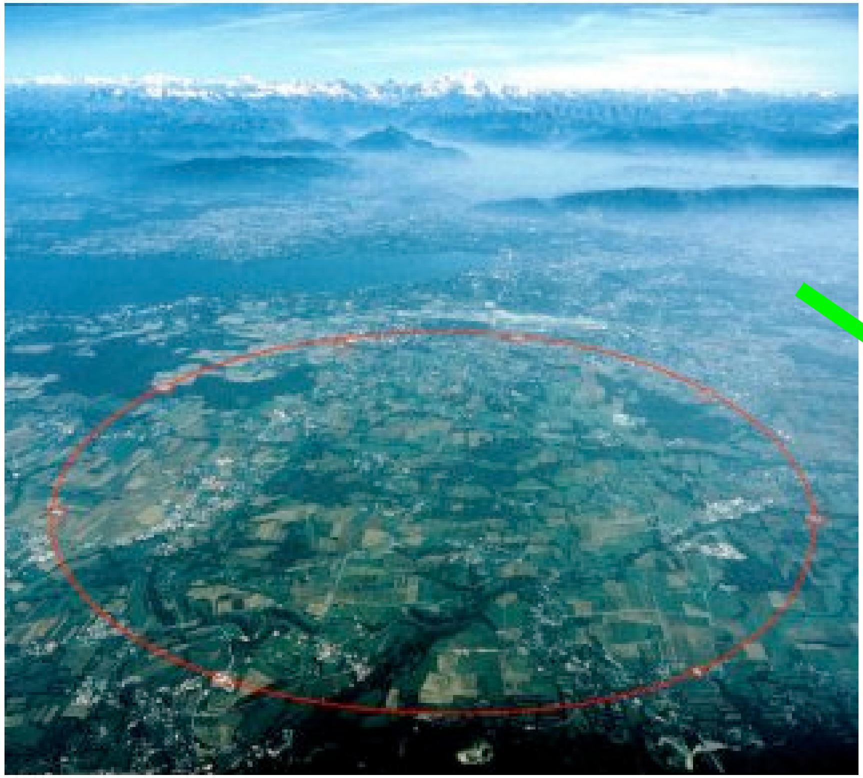


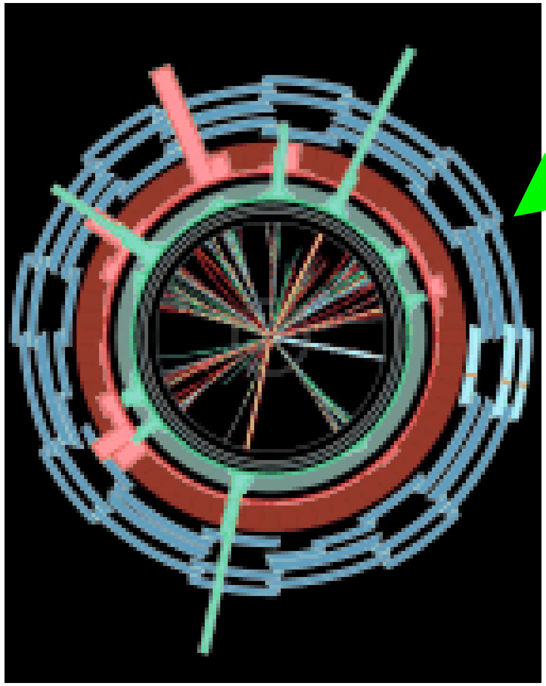


Atom - nm

Proton - fm

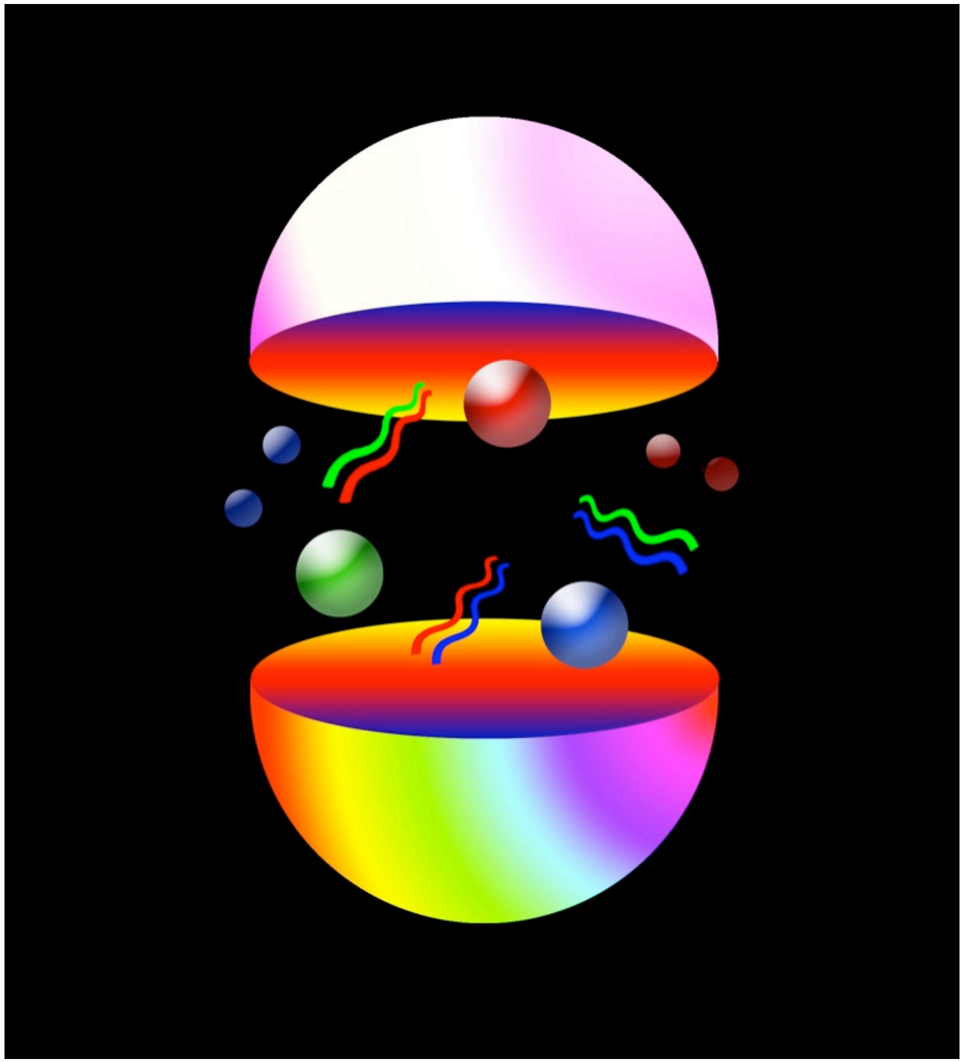


Particle physics experiment



Smashing protons together at high energy does not give subunits directly - just more particles!

Protons and other hadrons are made of quarks interacting by the strong force.



Quarks never seen as free particles - to study them need accurate expt *and* theoretical calculations.

QCD is theory of strong force - hard to calculate because strongly-coupled and nonlinear - needs numerical simulation. This is lattice QCD.

Standard Model of particle physics has:

6 quarks - $\begin{pmatrix} d \\ u \end{pmatrix} \begin{pmatrix} s \\ c \end{pmatrix} \begin{pmatrix} b \\ t \end{pmatrix}$ $\longrightarrow$ a ‘zoo’ of hadrons

6 leptons - $\begin{pmatrix} e \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$

3 forces (ignore gravity) : strong, weak, electromagnetic

Over 20 parameters, whose origin is some deeper theory with New Physics.

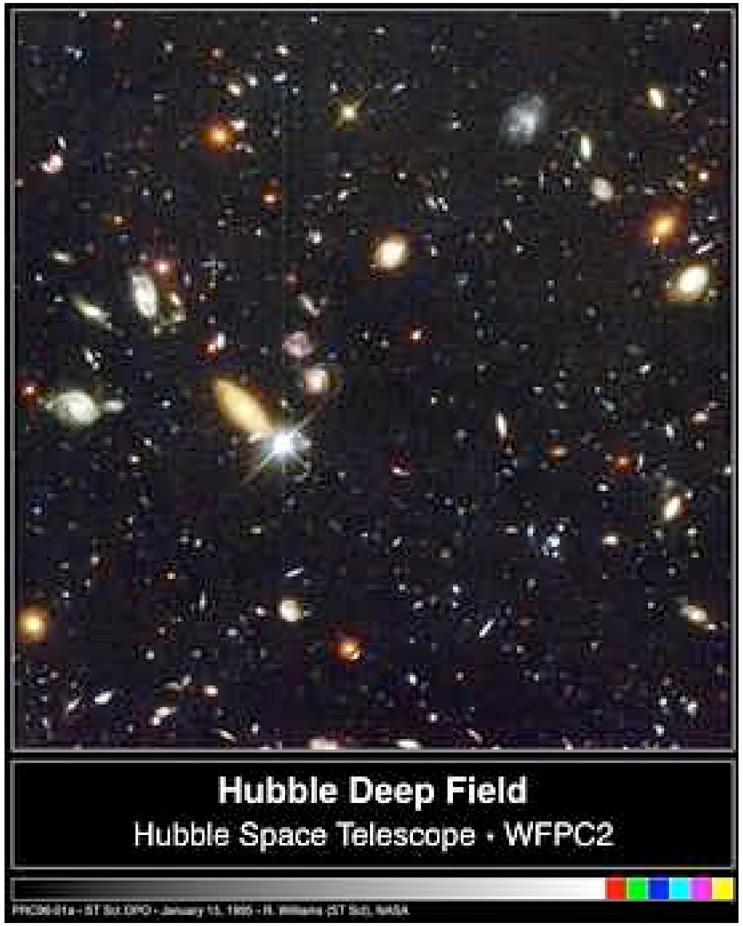
QCD is theory of strong force - mirrors QED

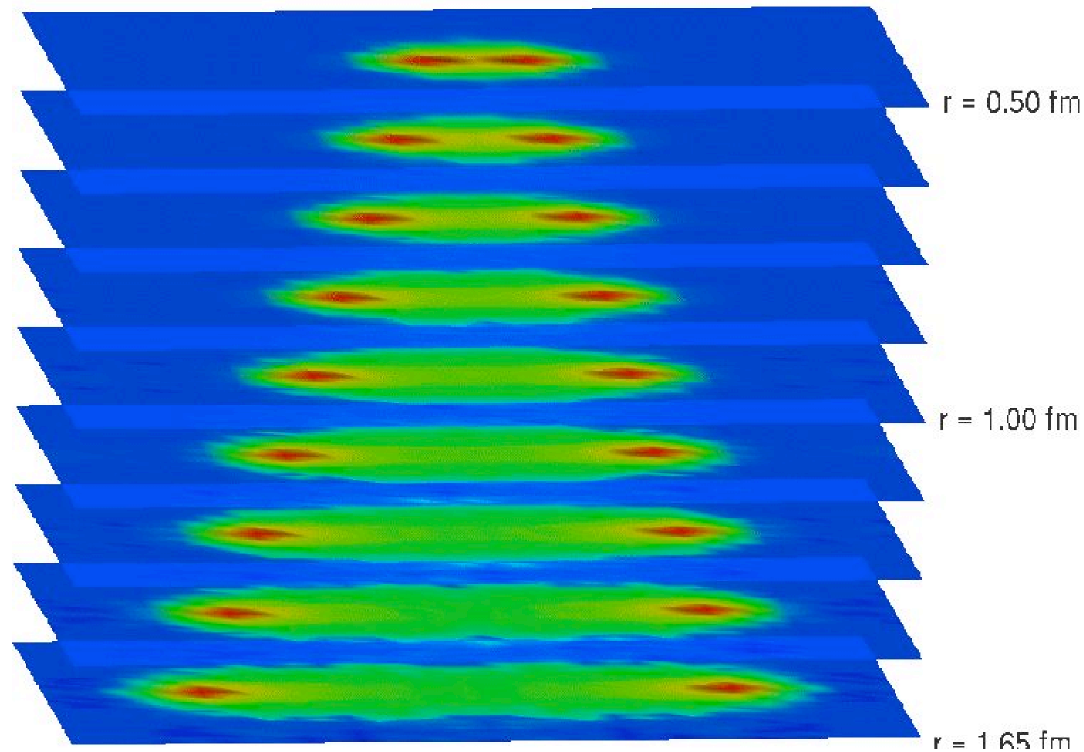
Quarks $\longleftrightarrow$ electrons

color charge RGB $\longleftrightarrow$ electric charge

gluons $\longleftrightarrow$ photons

BUT QED and QCD behaviour very different:

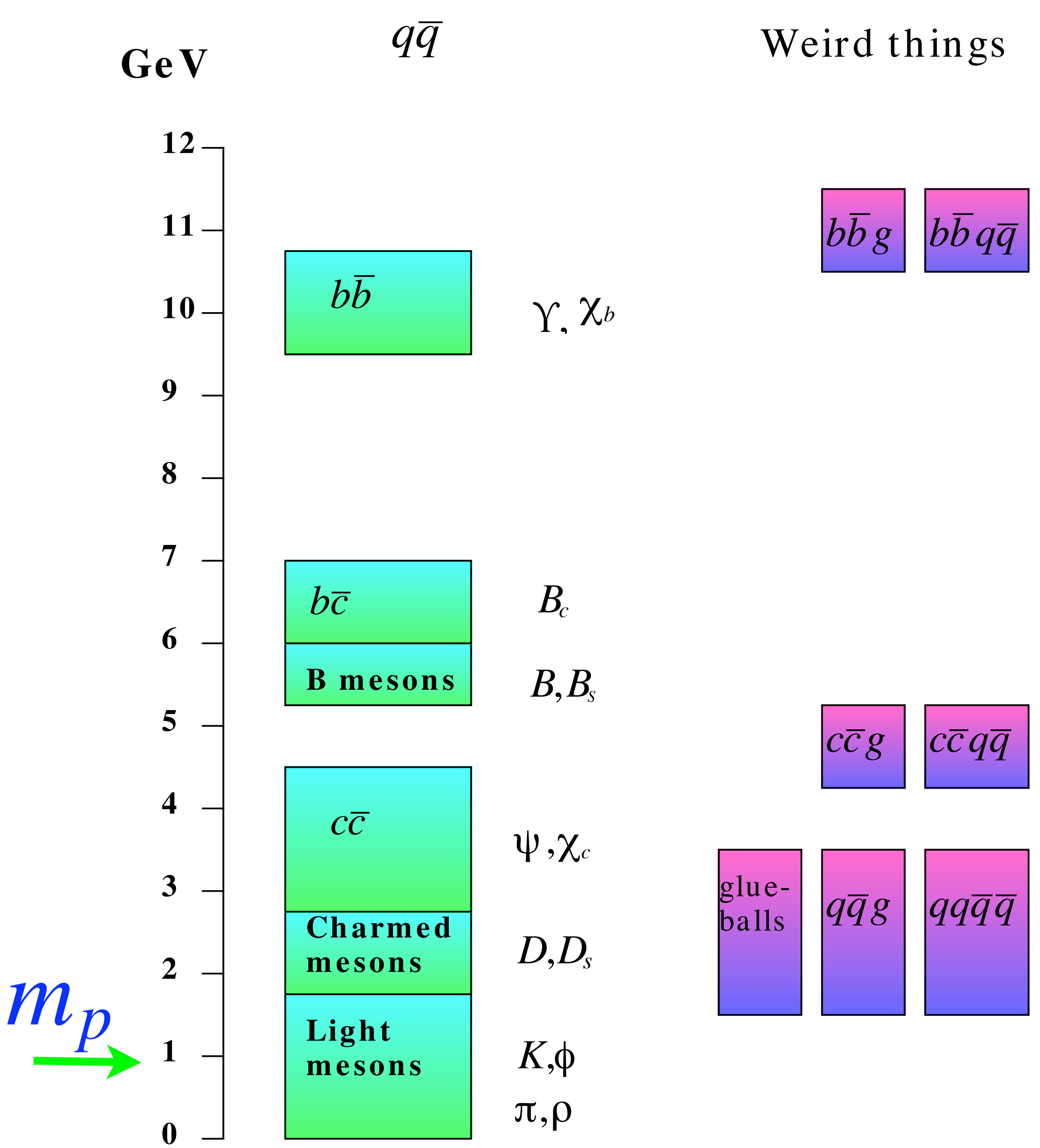




QED - uncharged photons travel freely - easy to get free electrons

QCD - gluons carry color charge - attempt to separate $q\bar{q}$, force becomes strong at large distance and quarks and gluons confined.

All info. about quarks indirect - from 'colorless' bound states, hadrons. Baryons = qqq e.g. p,n , mesons = $q\bar{q}$



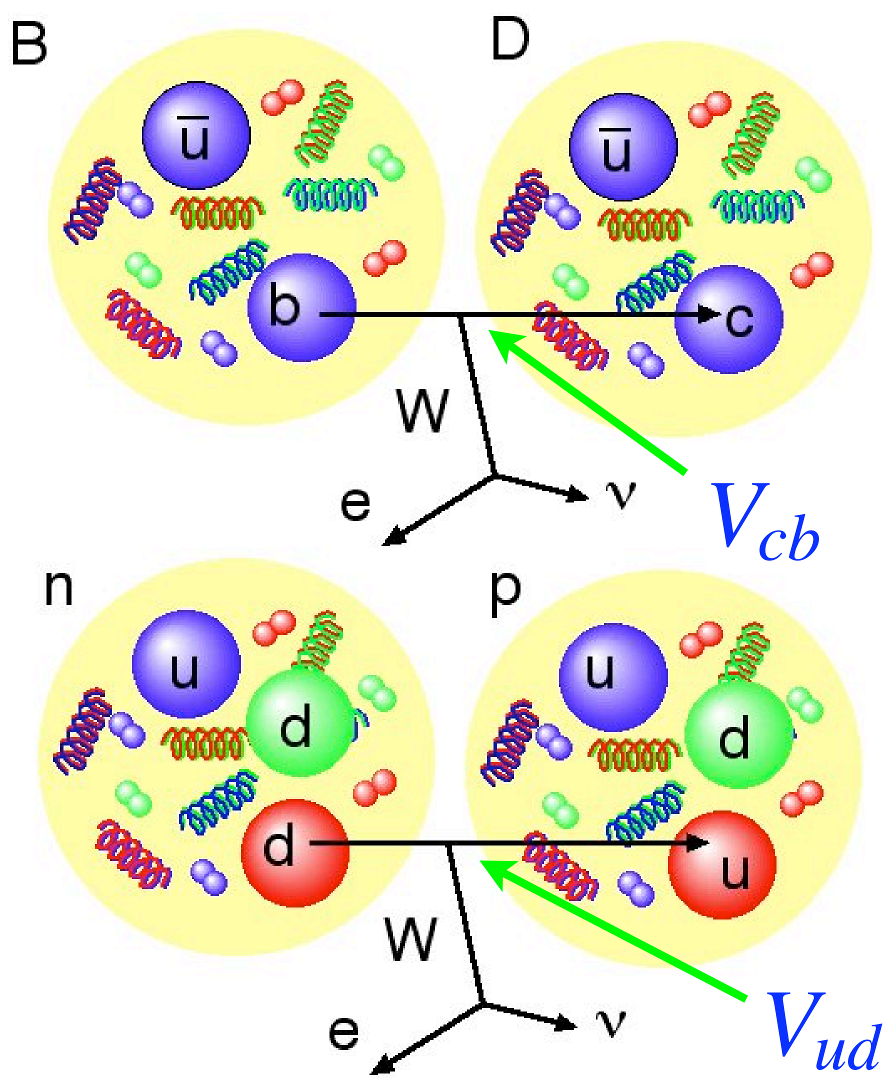
The Meson Spectrum

Rich spectrum of states - masses calculable in QCD if we can solve theory. A lot of states only exist briefly but can be seen in particle detectors and properties determined.

'Weird things' not yet unambiguously seen. Theory can help to find them.

Quarks also feel the weak force

Key to understanding CP (matter-antimatter) symmetry violation in Standard Model. (New physics?)



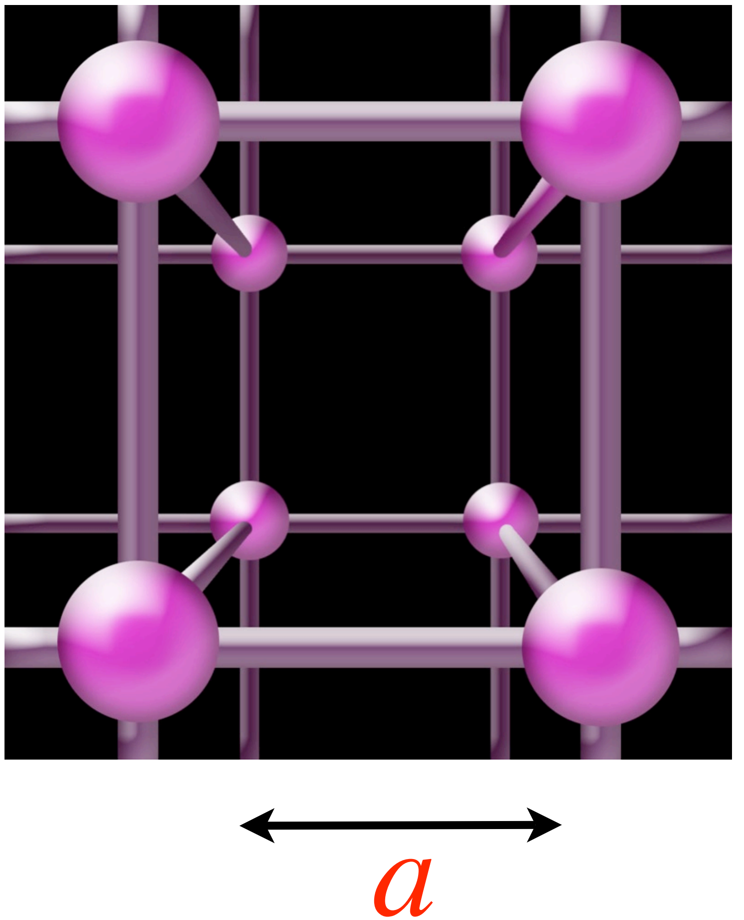
Complex couplings between quarks of different flavor and W. 3x3 CKM matrix poorly known

BUT quark weak decays occur inside hadrons. QCD effects are critical - must calculate decays of B mesons in lattice QCD

Lattice QCD

- Solve QCD by numerical evaluation of path integral:

$$\int dA_\mu d\psi d\overline{\psi} e^{-S_{QCD}}$$



- make integral finite with a space-time lattice
- Importance sampling - make gluon configs - ‘snapshots of vacuum’ and propagate quarks through them.

- ‘Measure’ e.g. hadron correlators on the gluon configs to calc. hadron masses and weak decay rates

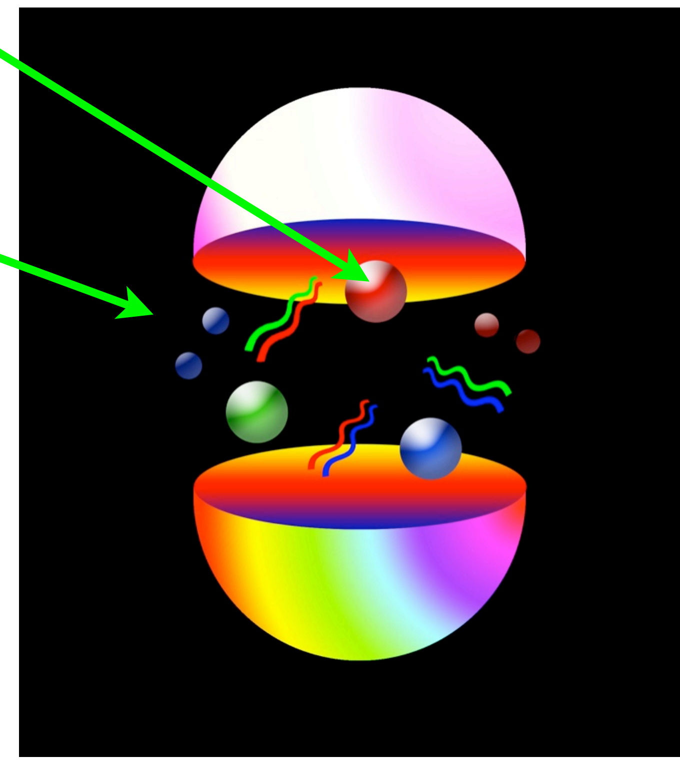
Handling light quarks is a big headache

$$L_{q,QCD} = \overline{\psi}(\gamma \cdot D + m)\psi \equiv \overline{\psi} M \psi$$

For valence quarks, need to calc. M^{-1}

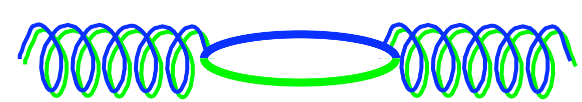
For sea quarks need to inc. $det(M)$ in making gluon configs

Very costly as $m_q \rightarrow 0$



Early calcs:

Quenched Approximation - omitted sea quarks. This is not good enough for precision required. Need to unquench with real s and light u/d.

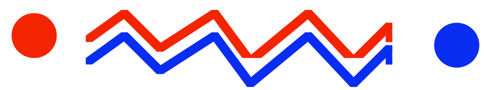


Cost of lattice QCD calc. grows as a^{-6}

must work with largest spacing possible

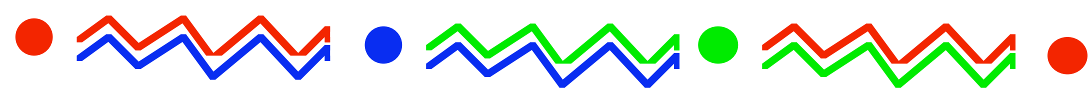
Discretisation errors are a big issue

Derivs become finite differences:



$$\frac{\partial\psi(x_j)}{\partial x} = \frac{\psi(x_j+a)-\psi(x_j-a)}{2a} + O(a^2)$$

Correct with higher order difference - but must take account of gluon radiation



'Improvement' gives much higher accuracy - results with sea quarks from 'improved staggered quark QCD action' -errors a few % at a=0.1 fm

Life as a lattice QCD theorist

Tbytes data

Large analyses - 0.5-1 Tflopyears
(Fermilab cluster)

Generate configs - 1-2 Tflopyrs
(QCDOC at Edinburgh)

Small analyses and tests - Gflopyrs
(Glasgow cluster)

People time is important too!

Take-home message

- There has been a revolution in the numerical simulation of the theory of the strong force (lattice QCD) since 2003
- Lattice QCD now delivering results : hadron masses that agree with expt; precise parameters of QCD; decay rates needed to determine the CKM matrix accurately.

Lattice QCD results

MILC collaboration gluon configurations have:

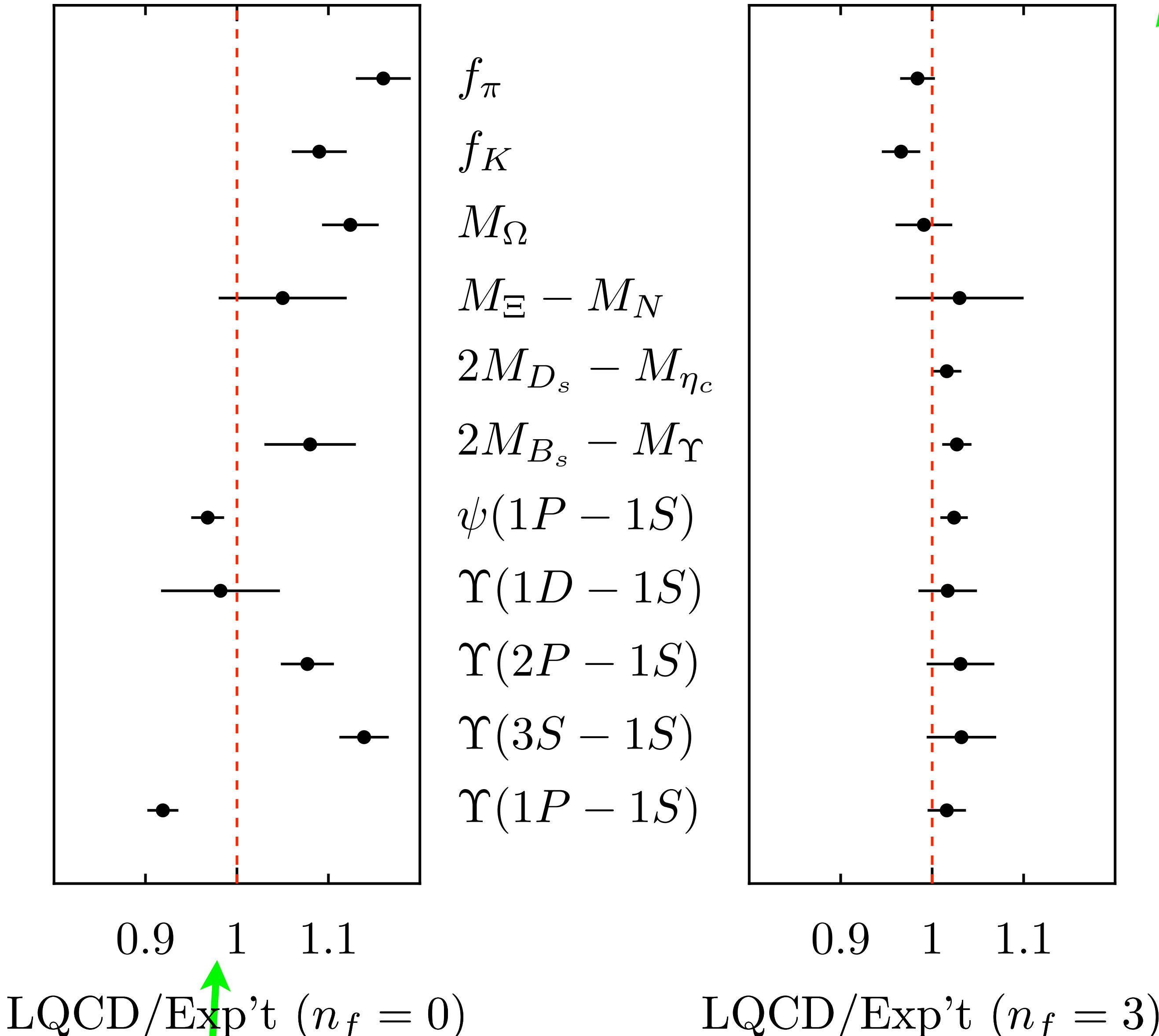
- 2+1 flavors of sea quarks, down to $m_{u/d} = m_s/10$.
- Many $m_{u/d}$ values; 2 m_s values
- 3 values of lattice spacing: 0.18fm, 0.12fm and 0.09fm
- Spatial volume exceeding $(2.5fm)^3$ so lattice size $28^3 \times 96$

QCD has 5 parameters : 4 quark masses and a bare coupling. Must fix these using 'gold-plated' (i.e. stable) hadron masses. FNAL/HPQCD/MILC/UKQCD analysis of MILC configs. Fix: $a : M_{\Upsilon'} - M_{\Upsilon}$

$m_{u/d} : M_{\pi}$ $m_s : M_K$ $m_c : M_{D_s}$ $m_b : M_{\Upsilon}$

Calculate other gold-plated hadron masses as test. Results that follow from this analysis (but not all configs).

Summary of results

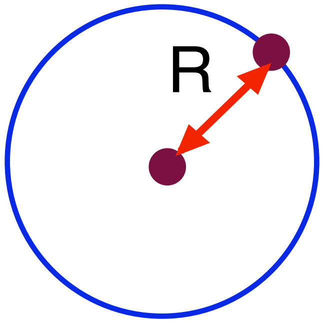


Results including u,d and s sea quarks agree with experiment *across the board* -from light to heavy hadrons. Parameters of QCD are *unambiguous*

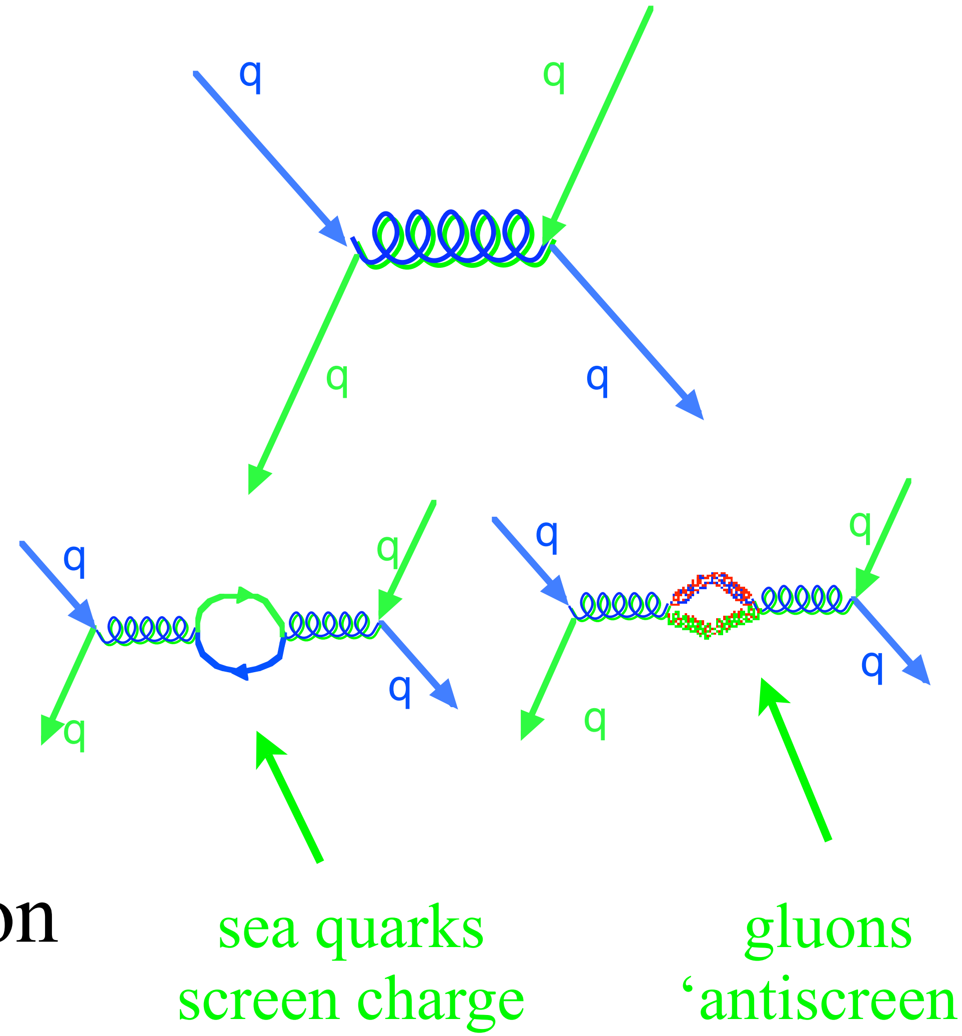
Quenched results are *wrong* and *ambiguous*

Davies et al, hep-lat/0304004,
Aubin et al, hep-lat/0407028,
Toussaint+Davies, hep-lat/0409129,
Gray et al, hep-lat/0507013,

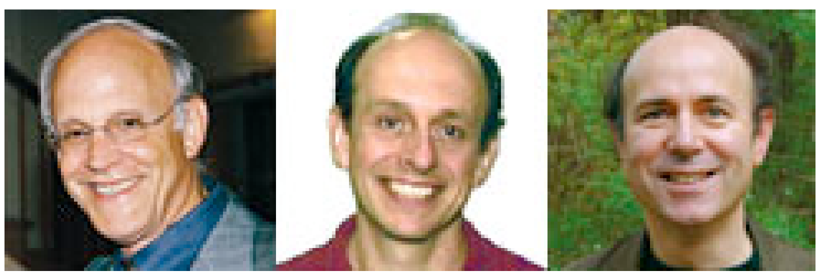
Determining Parameters of QCD : $\alpha_s = g^2/4\pi$



g = color charge. Measure with 'test charge' at distance R.



R-dependence from vacuum polarization effects



2004 Nobel Prize

In lattice QCD have g in action and 'measure' a.

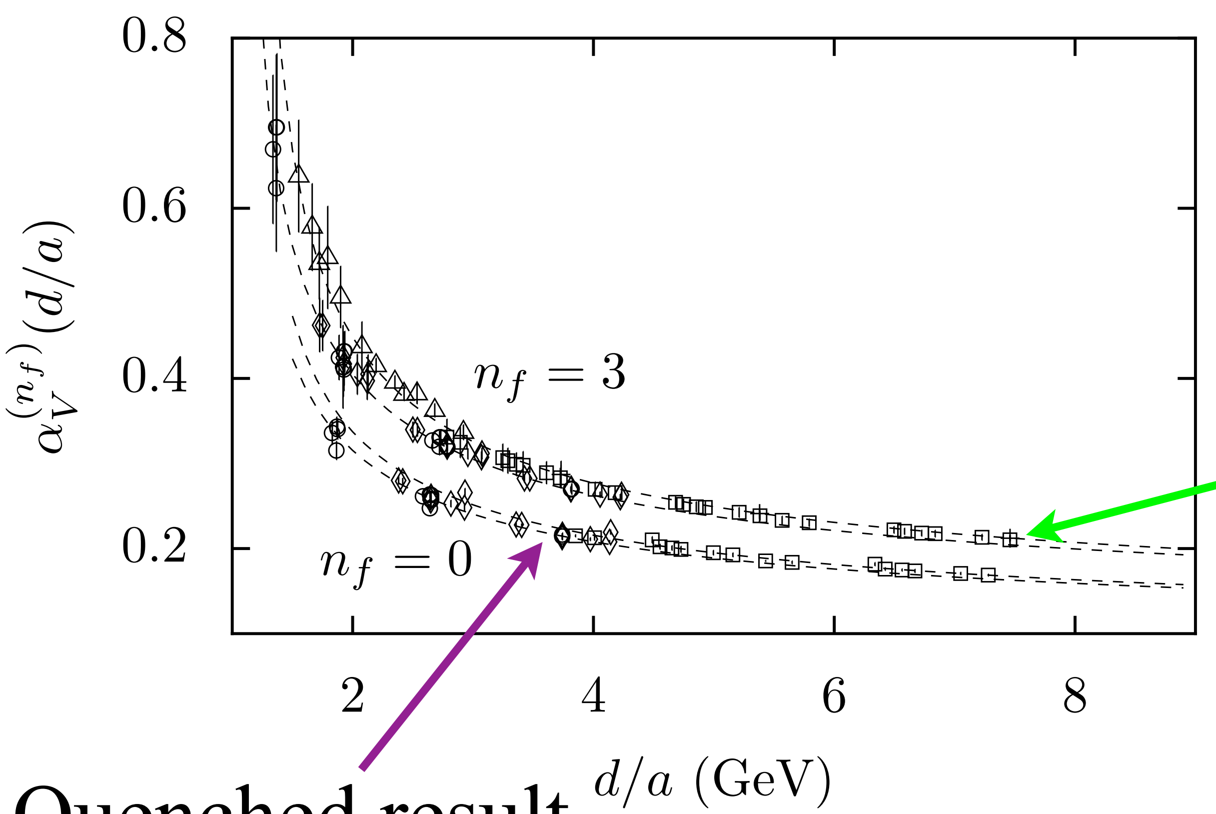
(in fact a bit harder than this to do accurately ...)

Lattice QCD result

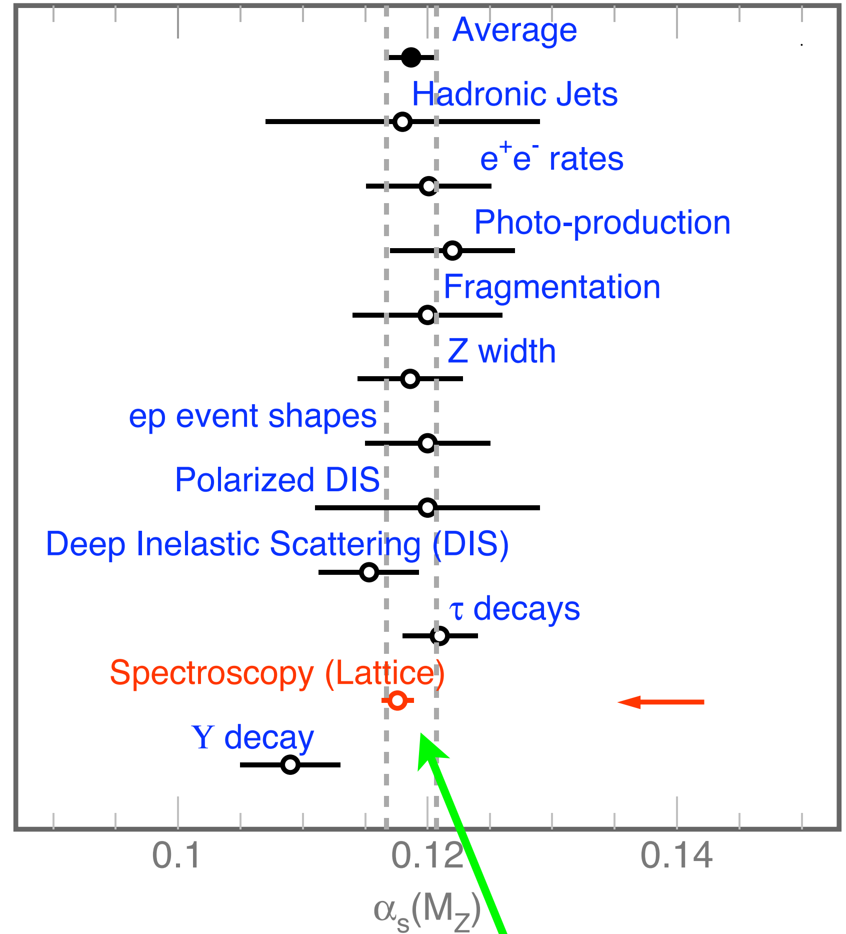
$$\alpha_{\overline{MS}}^{(5)}(M_Z) = 0.1170(12)$$

2004 PDG = 0.1187(20)

← R



Quenched result



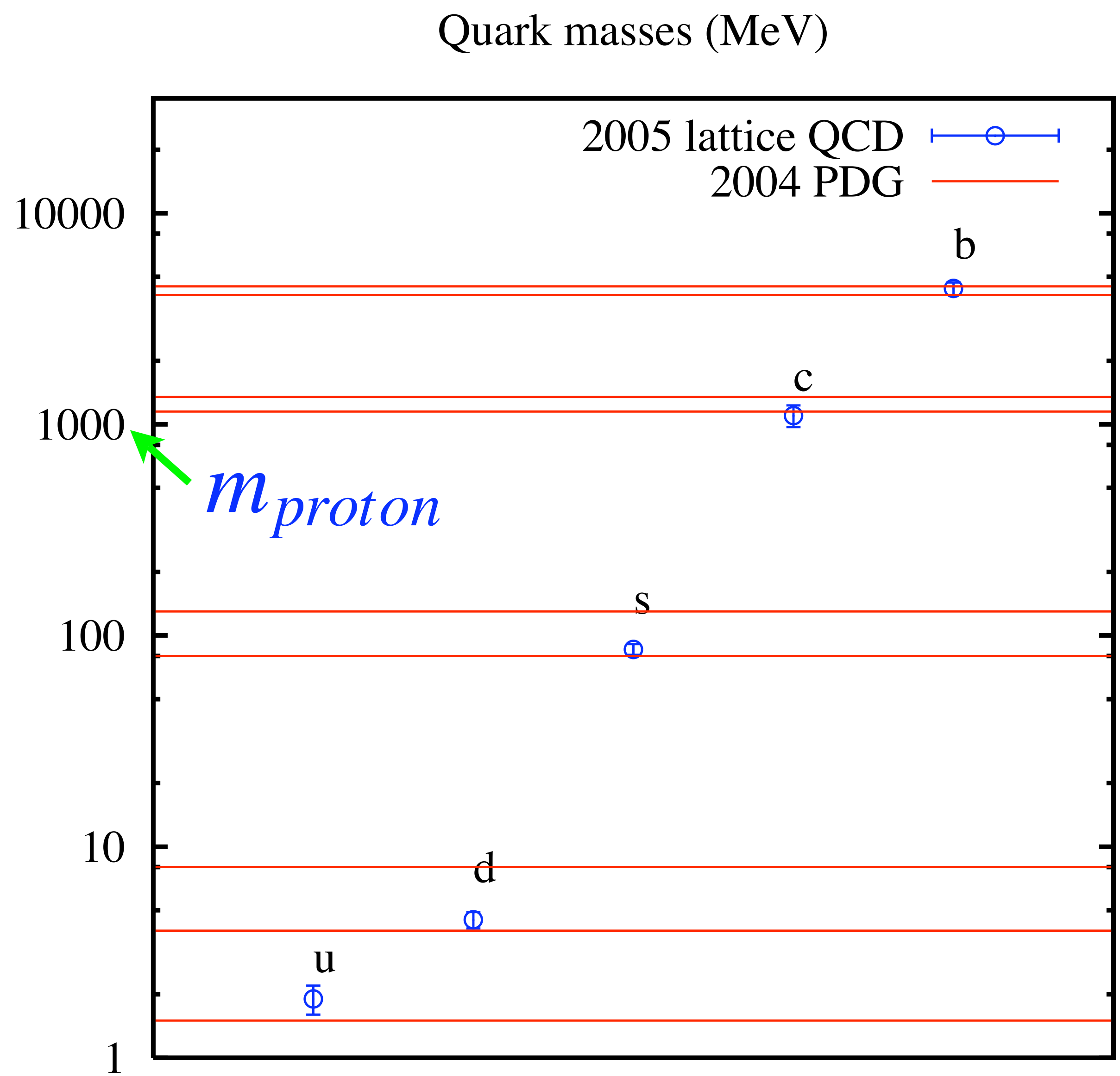
Lattice result with sea quarks

HPQCD, Mason et al, hep-lat/0503005

Determining parameters of QCD: m_q

Quarks never free so cannot measure mass directly

Masses are parameters in lattice QCD action - determine
by getting hadrons masses right.
Convert to continuum mass by (hard) analytic calcln. (gives error)



HPQCD/MILC, Aubin et al, hep-lat/0405022; HPQCD, Mason et al, hep-lat/0511160

Lattice QCD prediction! : mass of B_c meson

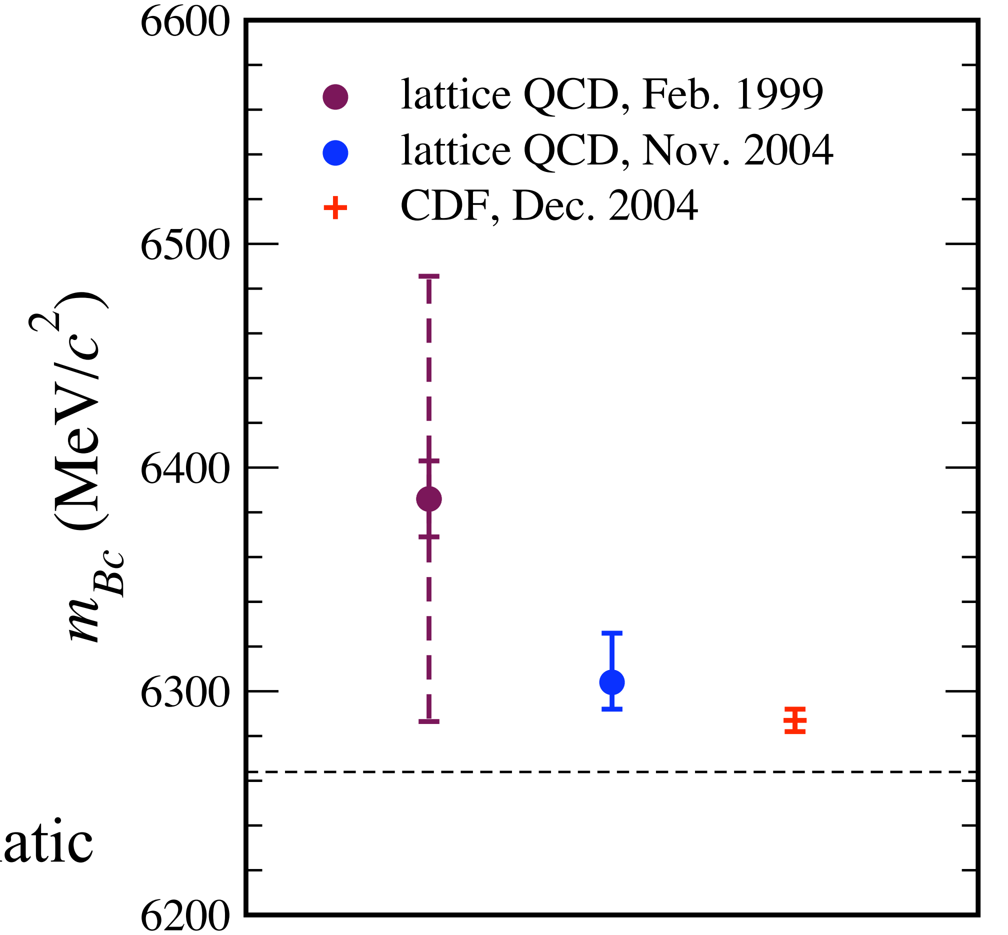
Using NRQCD formalism for *b* and FNAL formalism for *c*, calculate splitting

$$M_{B_c} - \frac{1}{2}(M_{\Upsilon} + M_{J/\psi})$$

Result:

$$M_{B_c} = 6.304(20)\mathrm{GeV}$$

Lattice systematic errors

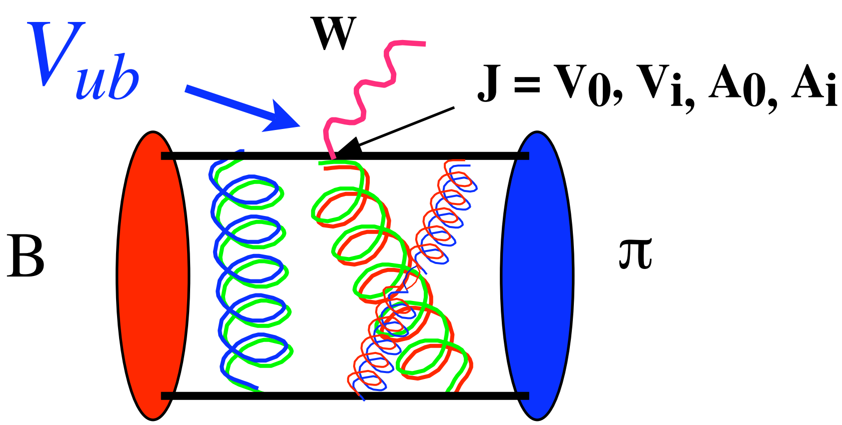


CDF (2005): 6.287(5) GeV

HPQCD/FNAL /UKQCD Allison et al, hep-lat/0411027; CDF, Acosta et al, hep-ex/0505076, Behari here.

Weak decay rates and the CKM matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to l\nu & K \to l\nu & B \to \pi l\nu \\ & K \to \pi l\nu & \\ V_{cd} & V_{cs} & V_{cb} \\ D \to l\nu & D_s \to l\nu & B \to Dl\nu \\ D \to \pi l\nu & D \to Kl\nu & \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d|\overline{B}_d\rangle & \langle B_s|\overline{B}_s\rangle & \end{pmatrix}$$



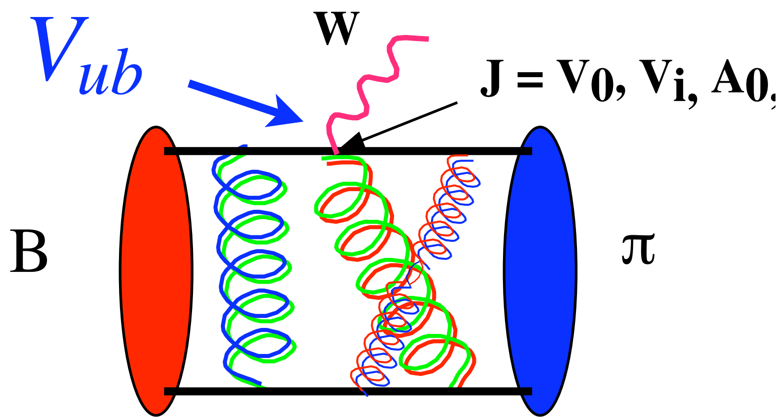
Lattice QCD calc. gives rate of basic weak decay from one *hadron to another.* CKM and lepton kinematics outside calc.

Lattice QCD can calculate decay rates for at most one ‘gold-plated’ hadron in final state.

Possible for almost every element of CKM matrix.

* Need multiple cross-checks of lattice calcs in different systems e.g. Υ, B, D, ψ etc

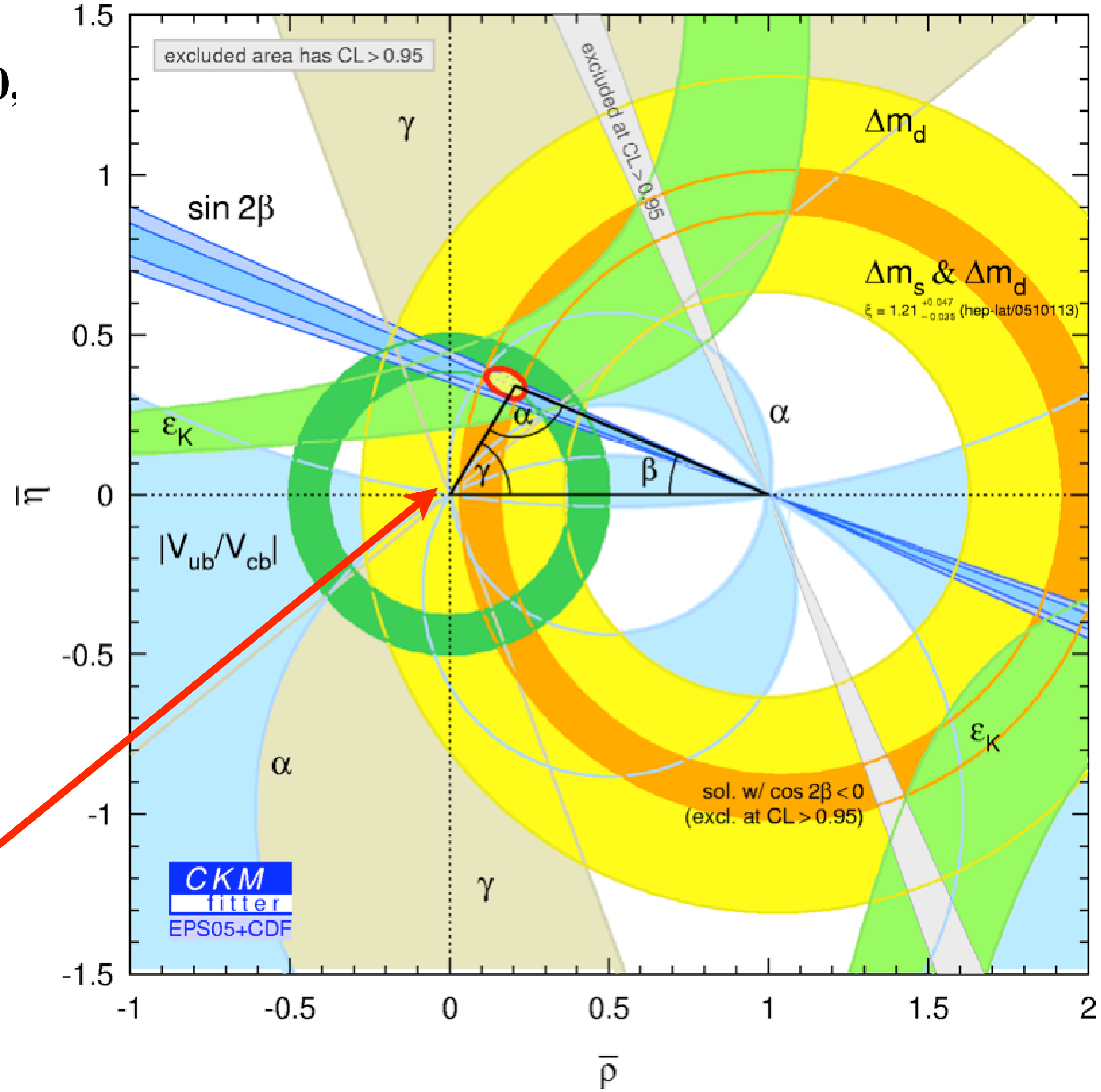
Determination of CKM matrix



weak decay of quark inside a hadron - calculate in lattice QCD

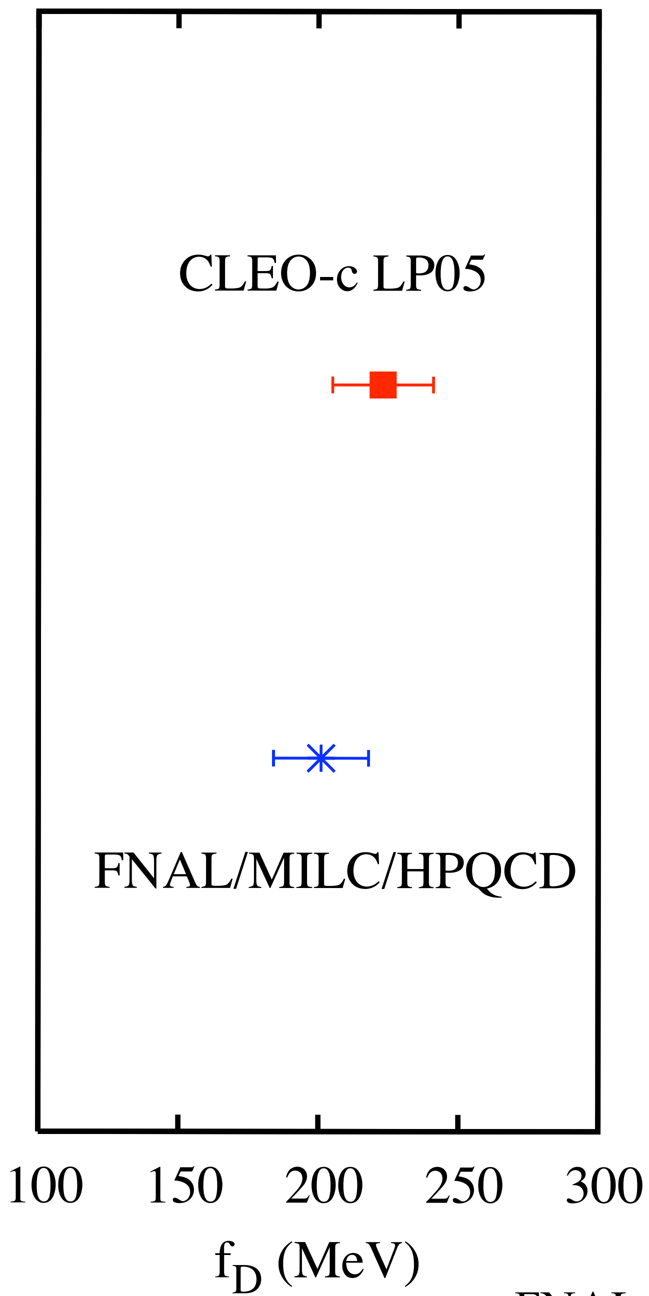
expt = CKM x lattice

unitarity triangle sides



Precision (3%) lattice QCD needed

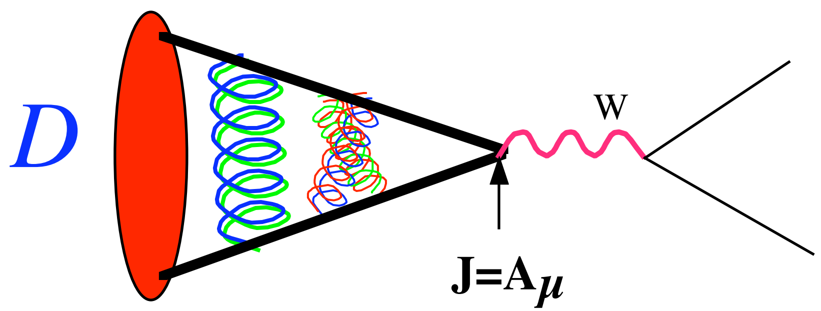
Key results from ‘B factories’ + lattice QCD



B mesons - valence quarks: b + light

Test lattice QCD by *predictions* for D (c + light).

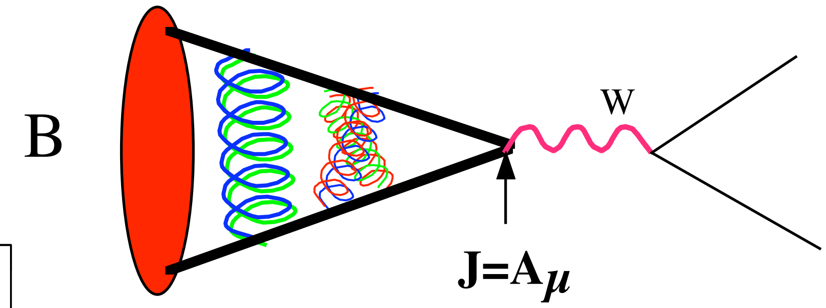
Leptonic decay rate of D from CLEO. Convert to f_D given V_{cd}

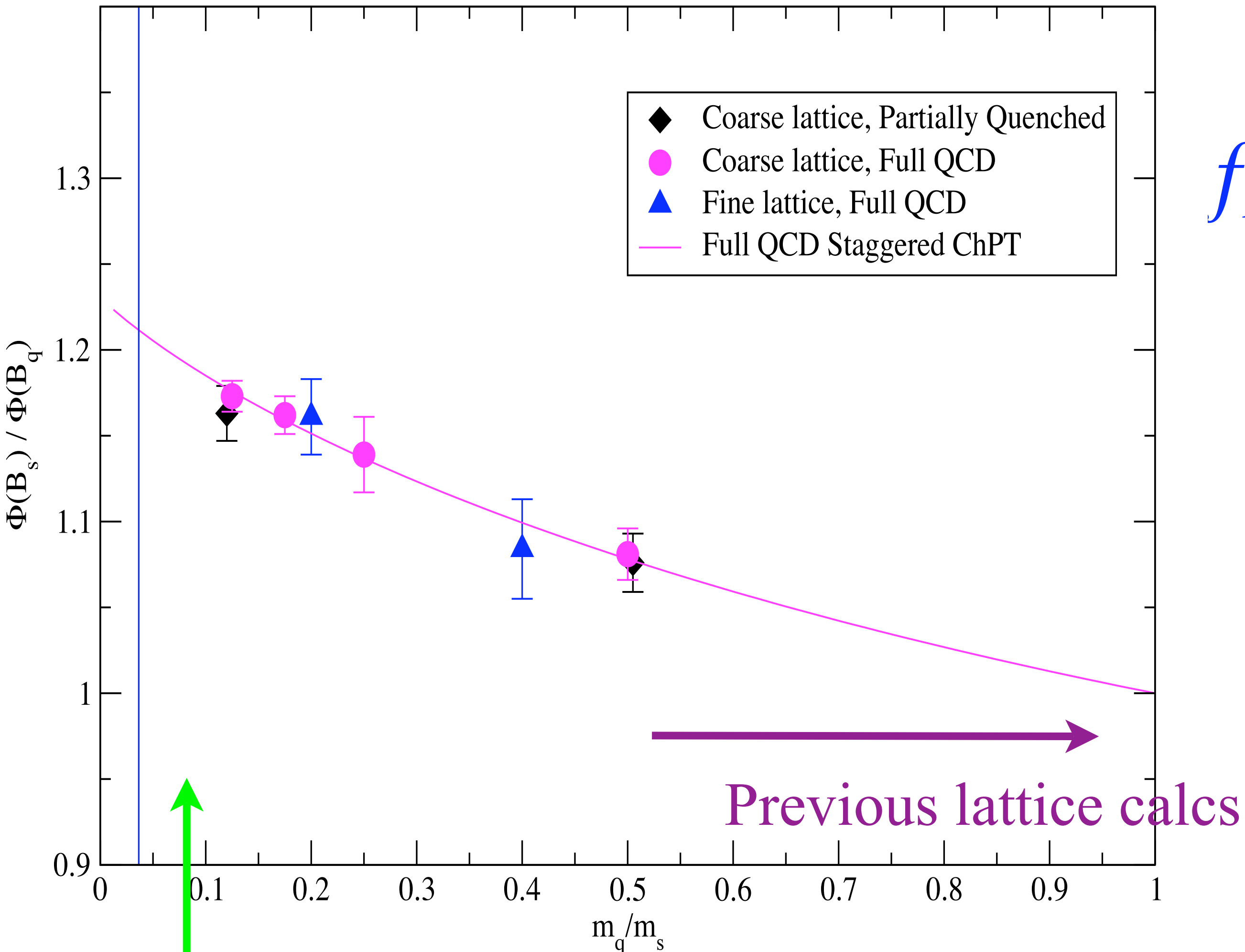


Lattice and CLEO agree but both have 8% errors - need to improve!

FNAL/MILC/HPQCD Aubin et al, hep-lat/0506030; CLEO, hep-ex/0508057

New determination of f_B, f_{B_s}





$$f_B = 216(22)\text{MeV}$$

9% error from pert matching

$$\frac{f_{B_s}}{f_B} = 1.20(3)$$

pert. error cancels. 3% from extrap. in u/d mass

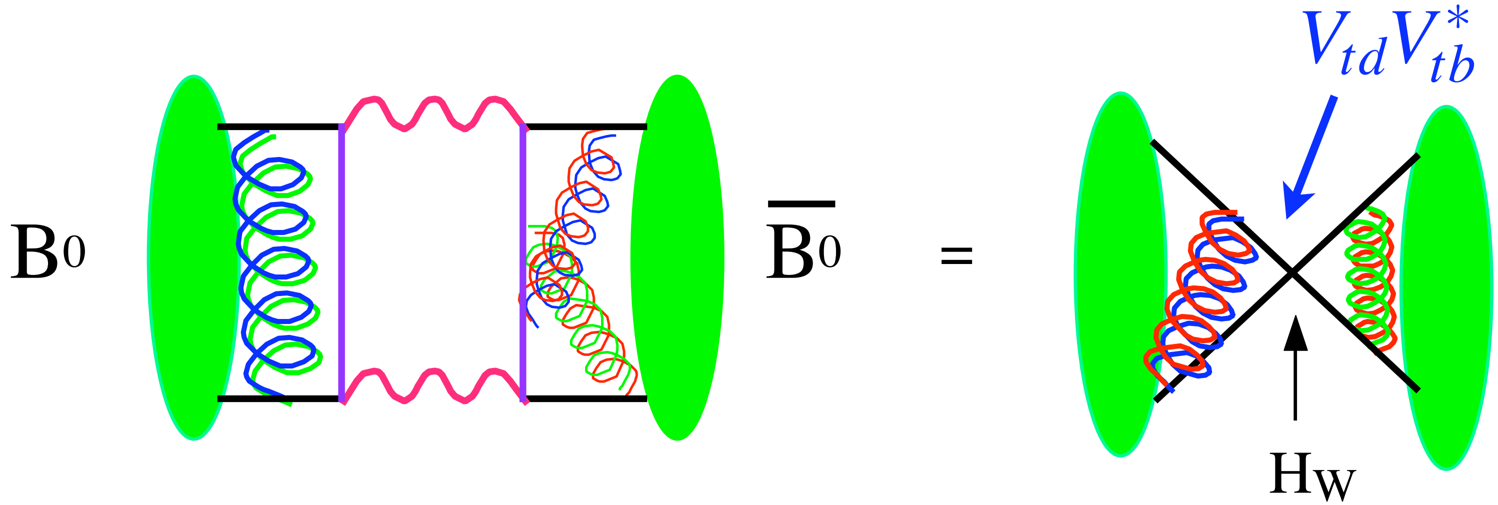
HPQCD, Gray et al, hep-lat/0507015;

log dependence on light quark mass expected

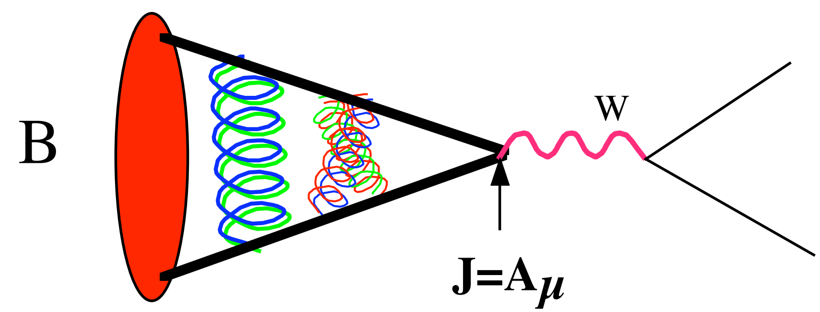
NEW Exptl result for $Br(B \to \tau\nu)$ from Belle, hep-ex/0605068

Neutral B mesons have fluctuating identity

B/B_s oscillation rate determined by box diagram. Calculate in lattice QCD as 4-q operator.



Parameterise as $f_B^2 B_B$ where f_B is decay constant.

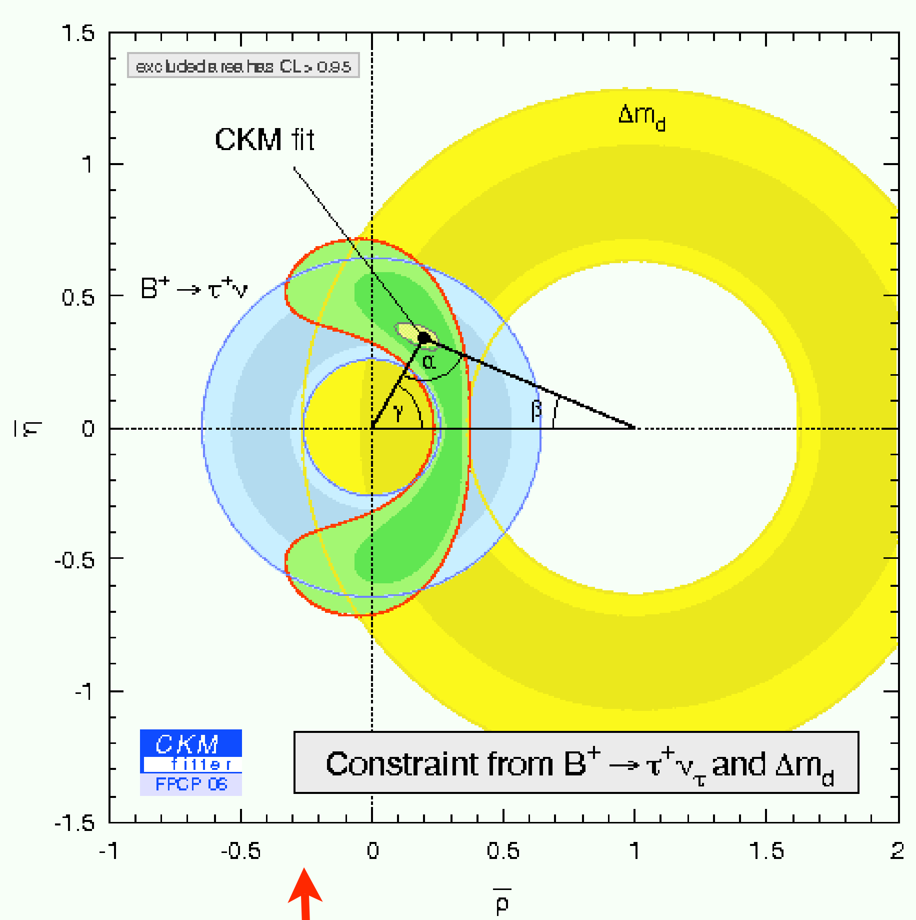


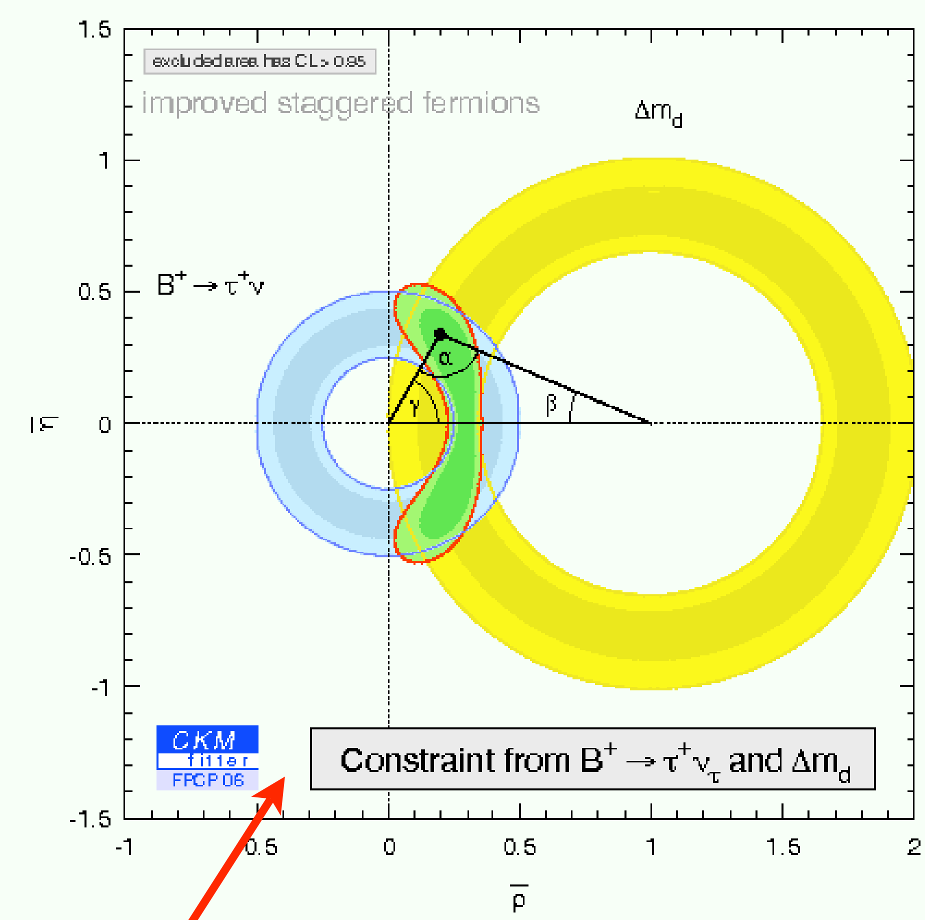
f_{B_s}/f_B is now calculated to 3% . B_{B_s}/B_B expected to be 1, but not yet clear.

NEW - measurement by CDF of B_s oscillations

Constraints on CKM unitarity triangle

Using just B leptonic decay and B/B_s oscillations





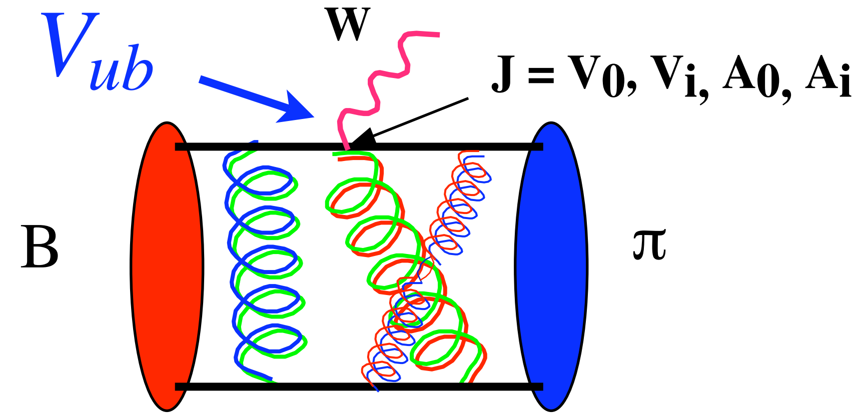
Mainly quenched +
2 flavors heavy sea clover

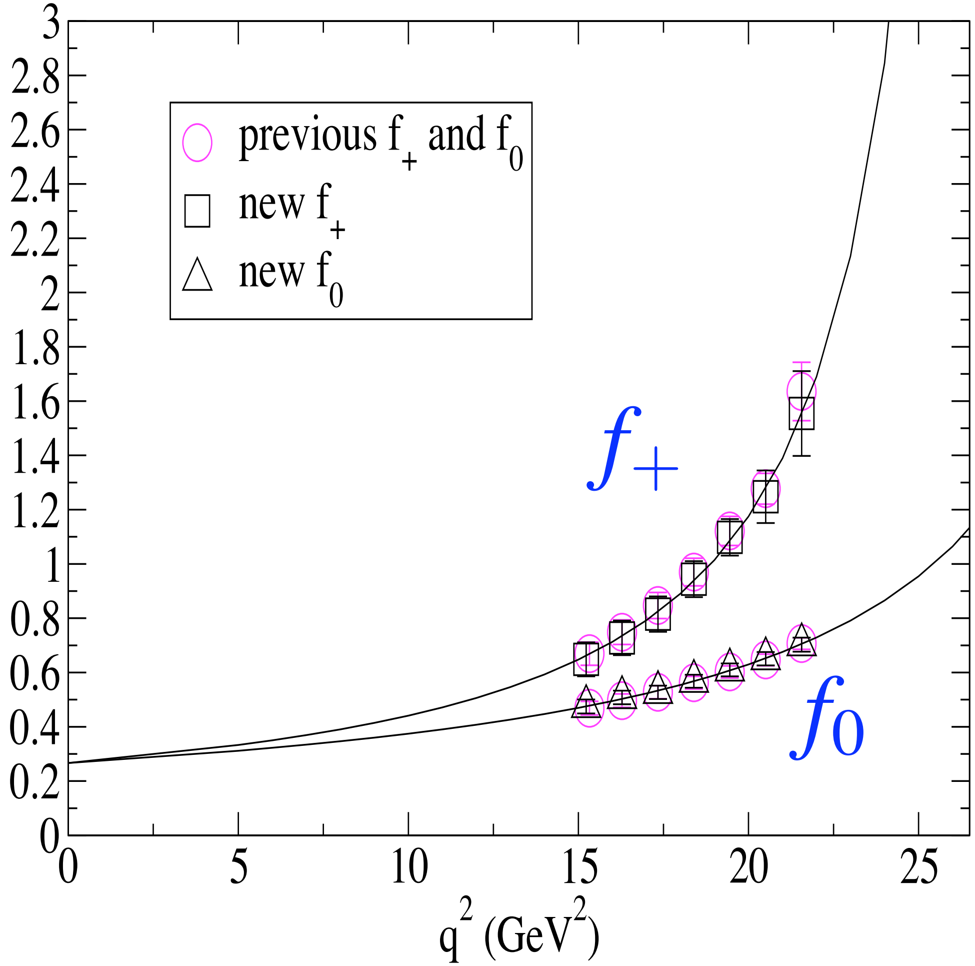
New 3-flavor imp. stagg
except for quenched B_B

CKMfitter group, FPCP 2006

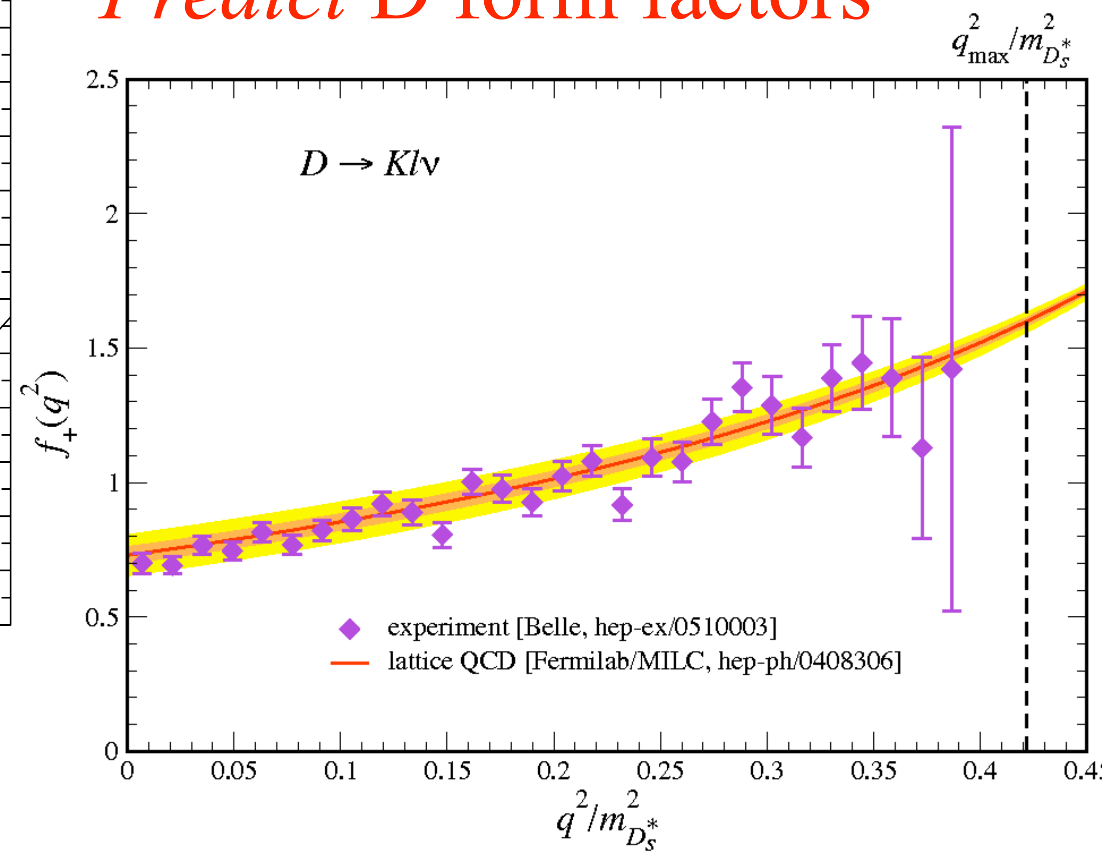
Semileptonic form factors

$B \to \pi l \nu$ and V_{ub}





Predict D form factors



HPQCD form factor gives:

$$V_{ub} = 4.22(30)(51) \times 10^{-3}$$

8% error from expt

12% error from lattice

HPQCD. hep-lat/0601021;
FNAL/MILC, hep-lat/040830;
Belle, hep-ex/0510003

Conclusions

• Accurate Lattice QCD calculations are now maturing. Tests of hadron masses and determination of parameters of QCD are at the few % level.

Future

• 10% errors on decay matrix elements for CKM. Beat down errors further and finish B oscillation calc. for impact on unitarity triangle.
• Longer term - work on harder calculations e.g. for proton structure and unstable particles.
• Calculations beginning now that use other quark formalisms that are numerically more expensive.